

LECTURE NOTES

Power System Operation & Control

B. Tech, 6th Semester, EEE

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COURSE CONTENT

Power System Operation & Control

B. Tech, 6th Semester, EEE

➤ **Module-I**

Review of the structure of a Power System and its components. Per unit calculations. Analysis of Power Flows: Formation of Bus Admittance Matrix. Real and reactive power balance equations at a node. Load and Generator Specifications. Application of numerical methods for solution of nonlinear algebraic equations. Gauss Seidel method for the solution of the power flow equations. Newton-Raphson method for the solution of the power flow equations. Coupled and Decoupled Newton-Raphson methods for the solution of the power flow equations.

➤ **Module-II**

Economic Operation and Management of Power System. Basic Pricing Principles: Generator Cost Curves, Utility Functions, Economic Operation without Transmission losses. Economic Operation with Transmission losses. Transmission loss coefficient, Economic Dispatch, Unit Commitment. Function of Load Dispatch Centres. Demand side-management.

➤ **Module-III**

Control of Frequency and Voltage, Turbines and Speed-Governors, Frequency dependence of loads, Droop Control and Power Sharing. Automatic Generation Control. Generation and absorption of reactive power by various components of a Power System. Excitation System Control in synchronous generators. Automatic Voltage Regulators. ALFC of Single Area Systems, ALFC of Two Area Systems.

➤ **Module-IV**

Power System Stability, The Stability Problem, The Swing Equation, The Power-Angle Equation. Synchronizing Power Coefficients, Equal-Area Criterion for Stability, Multi-machine Stability Studies, Classical Representation, Step-By-Step Solution of the Swing Curve, Factors Affecting Transient Stability.

REFERENCES
Power System Operation & Control
B. Tech, 6th Semester, EEE

Books:

- [1] J. Grainger and W. D. Stevenson, “Power System Analysis”, McGraw Hill Education, 1994.
- [2] O. I. Elgerd, “Electric Energy Systems Theory”, McGraw Hill Education, 1995.
- [3] D. P. Kothari and I. J. Nagrath, “Modern Power System Analysis”, McGraw Hill Education, 4th Edition, 2011.
- [4] Power System Analysis- By Hadi Saadat, TMH, 2002 Edition, Eighth Reprint.

Digital Learning Resources:

<https://nptel.ac.in/courses/108/104/108104052/>

Per unit System:-

per unit system is defined as the ratio of actual value to the base value of the quantity.

$$p.u. = \frac{\text{Actual value of quantity}}{\text{Base value of quantity}}$$

Advantages:-

- p.u. values provides more meaning full information.
- p.u. impedance values for the apparatus falls within narrow range.
- p.u. impedance values of transformers referred to either primary or secondary side is same.
- The impedance of machines are specified by the manufacturer in terms of per unit values.
- p.u. values in terms of phase quantities or line ^{values} quantities are same.
- the computational effort is very much reduced by using of p.u. quantities. The p.u. values being in the order of unity or less can be easily handle with a digital computer.

For p.u. system four parameter are considered.

(1) voltage (V) in (kV) (2) current (I) (3) power (S) (MVA) (4) Impedance (Z)

$$S_{pu} = \frac{S}{S_B}, V_{pu} = \frac{V}{V_B}, I_{pu} = \frac{I}{I_B}, Z_{pu} = \frac{Z}{Z_B}$$

$$I_B = \frac{S_B}{\sqrt{3} V_B}, \text{ and } Z_B = \frac{V_B / \sqrt{3}}{I_B}$$

$$\Rightarrow Z_B = \frac{V_B / \sqrt{3}}{S_B / \sqrt{3} V_B} = \frac{V_B}{\sqrt{3}} \times \frac{\sqrt{3} V_B}{S_B} = \frac{V_B^2}{S_B}$$

$$Z_B = \frac{KV^2}{MVA}$$

The phase and line quantities expressed in pu are the same and the circuit laws are valid.

$$S_{pu} = V_{pu} I_{pu}^*, \quad V_{pu} = Z_{pu} I_{pu}$$

The load power at its rated voltage can also be expressed by a p.u. impedance

$$S_L(3-\phi) = 3 V_p I_p^*$$

$$= \frac{3 |V_p|^2}{Z_p^*}$$

$$Z_p = \frac{3 |V_p|^2}{S_L^*(3-\phi)}$$

$S_L(3-\phi) \rightarrow$ Complex load power.

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V_p is also represented V_{L-L} because in pu system V_p & V_L are same

$$Z_{pu} = \frac{Z_p}{Z_B} = \frac{3|V_{L-L}|^2}{S_L^*(3-\phi)} + \frac{|V_B|^2}{S_B}$$

$$= \frac{3|V_{L-L}|^2}{|V_B|^2} \times \frac{S_B}{S_L^*(3-\phi)}$$

$$Z_{pu} = 3 \frac{V_{pu}}{S_{L,pu}^*}$$

Change of Base:-

Let Z_{pu}^{old} = p.u. Impedance on the power base S_B^{old} and the voltage base V_B^{old}

$$Z_{pu}^{old} = \frac{Z_\Omega}{Z_B^{old}} = Z_\Omega \cdot \frac{S_B^{old}}{(V_B^{old})^2}$$

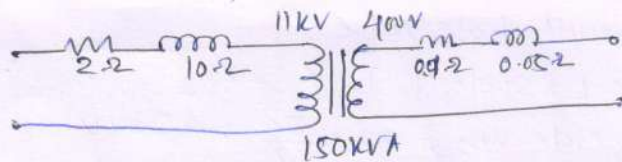
Expressing Z_Ω to a new power base and a new voltage base, results in the new p.u. Impedance.

$$Z_{pu}^{new} = \frac{Z_\Omega}{Z_B^{new}} = Z_\Omega \frac{S_B^{new}}{(V_B^{new})^2}$$

Divide $\frac{Z_{pu}^{new}}{Z_{pu}^{old}} = \frac{S_B^{new}}{S_B^{old}} \left(\frac{V_B^{old}}{V_B^{new}} \right)^2$

$$Z_{pu}^{new} = Z_{pu}^{old} \left(\frac{S_B^{new}}{S_B^{old}} \right) \left(\frac{V_B^{old}}{V_B^{new}} \right)^2$$

Q-1 Determine the pu values of the transformer



Solⁿ Total resistance referred to primary side (R_{01}) = $R_1 + R_2'$

$$R_{01} = R_1 + \frac{R_2}{K^2} = 2 + \frac{0.052}{\left(\frac{400}{11 \times 10^3}\right)^2} = 9.547 \Omega$$

$$K = \frac{V_2}{V_1} = \frac{400}{11 \times 10^3}$$

Total reactance referred to primary side (X_{01}) = $X_1 + X_2' = X_1 + \frac{X_2}{K^2} = 47.74 \Omega$

$$Z_{01} = R_{01} + jX_{01} = 9.547 + j47.74 \text{ (Actual value)}$$

$$Z_{base} = \frac{KV_b^2}{MVA} = \frac{11^2}{0.150} = 806.7 \Omega$$

$$Z_{pu} = \frac{9.547 + j47.74}{806.7} = (0.0118 + j0.0592) \text{ pu}$$

Total resistance referred to secondary side (R_{02}) = $R_2 + K^2 R_1$

$$R_{02} = 0.01 + \left(\frac{400}{11 \times 10^3}\right)^2 \times 2 = 0.012635$$

Total reactance referred to secondary side $X_{02} = X_2 + K^2 X_1$

$$X_{02} = 0.05 + \left(\frac{400}{11 \times 10^3}\right)^2 \times 10 = 0.0631769 \Omega$$

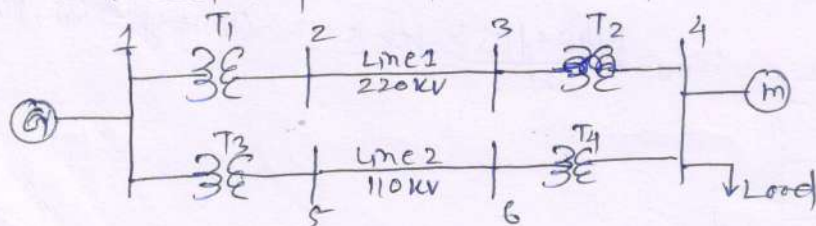
$$Z_{02} = R_{02} + jX_{02} = 0.012635 + j0.0631769$$

$$Z_{Base} = \frac{KV^2}{MVA} = \frac{N^2}{0.150} = \frac{(0.400)^2}{0.150} = 1.06667 \Omega$$

$$Z_{pu} = \frac{0.012635 + j0.0631769}{1.06667} = (0.0118 + j0.0592) \text{ pu}$$

From the above problem it is cleared that the per unit impedance on either side of the transformer is equal.

Q-2:- The one line diagram of a 3- ϕ power system is shown below. select a common base of 100 MVA and 22kV on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in p.u. The manufacturers data for each device is given as follows



G: 90 MVA 22kV $X = 18\%$

T1: 50 MVA 22/220kV $X = 10\%$

T2: 40 MVA 220/11kV $X = 6.0\%$

T3: 40 MVA 22/110kV $X = 6.4\%$

T4: 40 MVA 110/11kV $X = 8.0\%$

M: 66.6 MVA 10.45kV $X = 18.5\%$

The 3- ϕ load at bus 4 absorbs 57 MVA, 0.6 pf lagging at 10.45kV. Line 1 and Line 2 have reactances of 48.1 and 65.432 respectively.

Solⁿ Base quantity 100 MVA, 22 kV

For T₁ LV side associated with base value

The base voltage V_{B1} on the LV side of T_1 is 22 kV

The base voltage on its HV side $V_{B2} = 22 \times \frac{220}{22} = 220 \text{ kV}$

For T₂ $V_{B3} = 220 \text{ kV}$ on HV of T_2

$V_{B4} = 220 \times \frac{11}{220} = 11 \text{ kV}$ LV of T_2

Bus 4 is also having base of $V_{B4} = 11 \text{ kV}$

For T₃ $V_{B5} = 22 \times \frac{110}{22} = 110 \text{ V}$

For T₄ $V_{B6} = 110 \text{ V}$

Reactance calculation

$$Z_{pu}^{new} = Z_{pu}^{old} \frac{S_B^{new}}{S_B^{old}} \times \left(\frac{V_B^{old}}{V_B^{new}} \right)^2$$

For generator G: $X_{pu}^{new} = 0.18 \times \frac{100}{90} \times = 0.2 \text{ pu}$

for T/Fs T_1 : $X_{T1,pu} = 0.1 \times \frac{100}{50} \times = 0.2 \text{ pu}$

T_2 : $X_{T2,pu} = 0.06 \times \frac{100}{40} = 0.15 \text{ pu}$

T_3 : $X_{T3,pu} = 0.064 \times \frac{100}{40} = 0.16 \text{ pu}$

T_4 : $X_{T4,pu} = 0.08 \times \frac{100}{40} = 0.2 \text{ pu}$

For motor: $X_{m,pu}^{new} = 0.185 \times \frac{100}{66.5} \times \left(\frac{10.45}{11} \right)^2 = 0.25 \text{ pu}$

For line-1: Actual $x = 48.4 \Omega$ $X_{L1,pu} = \frac{48.4}{484} = 0.1 \text{ pu}$
 $X_{Base} = \frac{kV^2}{MVA} = \frac{220^2}{100} = 484 \Omega$

For line-2: Actual $x = 65.43 \Omega$ $X_{L2,pu} = \frac{65.43}{121} = 0.54 \text{ pu}$
 $X_{Base} = \frac{kV^2}{MVA} = \frac{110^2}{100} = 121$

The load apparent power at 0.6 p.f. lagging is

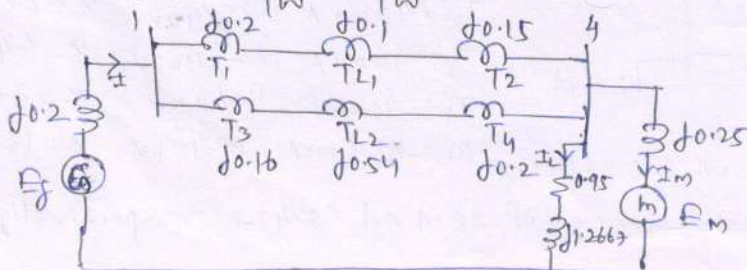
$$S_{L(3-\phi)} = 5 \angle \cos^{-1} 0.6 = 5 \angle 53.13^\circ \text{ MVA}$$

Actual Load impedance

$$Z_L = \frac{(V_{LL})^2}{S_{L(3-\phi)}} = \frac{10.54^2}{5 \angle -53.13^\circ} = 1.914 \angle 53.13^\circ = 1.148 + j1.53 \Omega$$

$$Z_{Base} = \frac{11^2}{1W} = \frac{121}{1W} = 1.21 \Omega$$

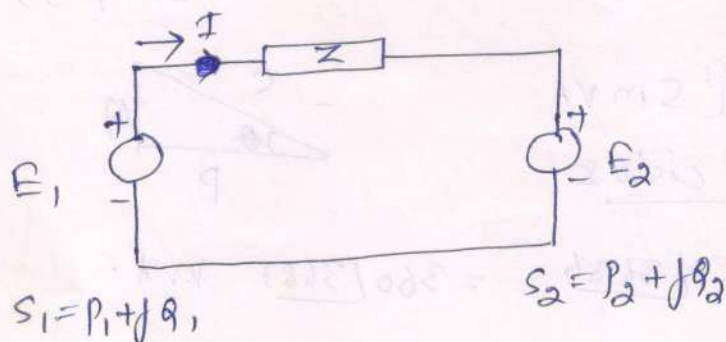
$$Z_{pu} = \frac{1.148 + j1.53}{1.21} = 0.95 + j1.266 \text{ pu}$$



Q Two ideal voltage sources designed as m/c 1 and 2 are connected as shown in figure. If $E_1 = 100 \angle 0^\circ \text{ V}$, $E_2 = 100 \angle 30^\circ \text{ V}$ and $Z = 0 + j5 \Omega$. Determine (a) whether each machine is generating or consuming real power and the amount.

(b) whether each m/c is receiving or supplying reactive power and the amount.

(c) the p and q absorbed by the impedance.



$$\therefore I = \frac{E_1 - E_2}{Z} = \frac{100 + j0 - 86.6 - j50}{j5} = \frac{13.4 - j50}{j5} = -10 - j2.68 = 10.35 \angle 108^\circ \text{ A}$$

$$S_1 = E_1 (-I)^* = P_1 + jQ_1$$

$$= 100 (10 + j2.68)^* = 1000 - j268 \text{ VA}$$

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$$S_2 = E_2 I^* = P_2 + jQ_2$$

$$= (86.6 + j50) (-10 - j2.68) = -1000 - j268 \text{ VA}$$

The reactive power absorbed in series impedance is

$$I^2 X = 10.35^2 \times 5 = 536 \text{ var}$$

Here

m/c 1 may be gen.

P_1 is +ve
& Q_1 is -ve

So m/c - 1 consumes energy at the rate of 1000 W
and supplies reactive power of 268 var

Hence it is a motor.

Here m/c 2 may be expected to be a motor

P_2 is -ve
 Q_2 is -ve

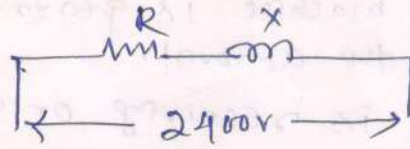
So m/c - 2 generates energy @ 1000 W
and supplies reactive power 268 var

Hence it is a generator

(c) Supplied reactive power $268 + 268 = 536 \text{ var}$ is required by the inductive reactance of S_2 . Since $R=0$ no power is consumed by Z and all the watts generated by m/c 2 are transferred to m/c 1.

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Q1 An inductive load consisting of R and X in series feeding from a 2400 V rms supply absorbs 288 kW at lagging p.f of 0.8 . Determine R and X .



$$P = 288\text{ kW} \quad 0.8 (\text{lag})$$

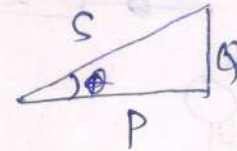
$$P = VI \cos \phi$$

$$Q = VI \sin \phi$$

$$P = S \cos \phi \quad \therefore S \text{ in VA}$$

$$S = \frac{P}{\cos \phi} \quad \angle \cos 0.8$$

$$= \frac{288}{0.8} \angle 36.86 = 360 \angle 36.87 \text{ kVA}$$



$$Z = R + jX \quad Z = \frac{V}{I}$$

$$S = VI^*$$

$$I^* = \frac{S}{V} = \frac{360 \angle 36.87}{2400 \angle 0} = 0.15 \angle -36.87 \text{ kA}$$

$$I = 0.15 \times 10^3 \angle -36.87 \text{ A}$$

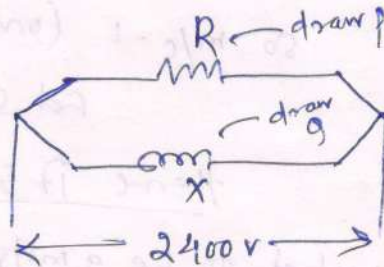
$$Z = \frac{V}{I} = \frac{2400 \angle 0}{150 \angle -36.87} = 16 \angle 36.87 = 12.79 + j9.6$$

$$R = 12.79 \Omega \quad X = 9.6 \Omega$$

Q2:- An inductive load consisting of R and X in parallel feeding from a 2400 V rms supply absorbs 288 kW at a lagging p.f of 0.8 . Determine R and X .

$$S = \frac{V^2}{Z}$$

$$P = \frac{V^2}{R} \quad Q = \frac{V^2}{X}$$



$$S = P + jQ$$

$288\text{ kW}, 0.8 \text{ pf (lag)}$

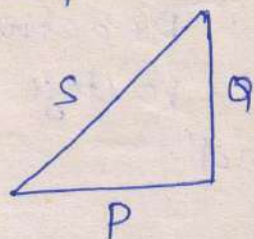
$$S = \frac{P}{\cos \phi} \angle \cos 0.8 = 360 \angle 36.87 \text{ kVA} = 287.9 + j216$$

$$P = 288\text{ kW}$$

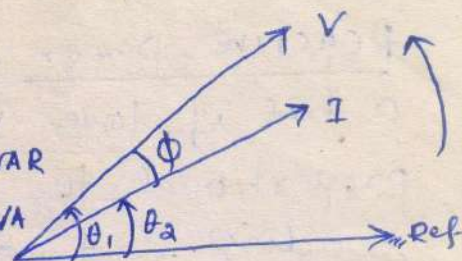
$$S = P + jQ \quad Q = 216 \text{ kVAR}$$

$$R = \frac{V^2}{P} = \frac{2400^2}{288 \times 10^3} = 20 \Omega \quad X = \frac{V^2}{Q} = \frac{2400^2}{216 \times 10^3} = 26.67 \Omega$$

Complex power:-



$P \rightarrow$ Real power, kW
 $Q \rightarrow$ Reactive power, kVAR
 $S \rightarrow$ Apparent power, kVA



Let us consider lagging power factor. In the vector diagram, the voltage leads the Ref. by θ_1 .
 Vector I leads the same reference by θ_2 .

Complex power is defined as the product of voltage and conjugate of current or it is the product of conjugate voltage and current.

$$(i) \quad \dot{S} = \dot{V} \dot{I}^*$$

$$\dot{V} = |V| \angle \theta_1$$

$$\dot{I} = |I| \angle \theta_2$$

$$\dot{I}^* = |I| \angle -\theta_2$$

$$\dot{S} = \dot{V} \cdot \dot{I}^*$$

$$= |V| \angle \theta_1 \cdot |I| \angle -\theta_2$$

$$= |V| |I| \angle \theta_1 - \theta_2$$

$$= |V| |I| \angle \phi$$

$$S = VI \cos \phi + jVI \sin \phi$$

$$S = P + jQ$$

$$\boxed{VI^* = P + jQ}$$

$$(ii) \quad \dot{S} = \dot{V}^* \dot{I}$$

$$\dot{V} = |V| \angle \theta_1$$

$$\dot{I} = |I| \angle \theta_2$$

$$\dot{V}^* = |V| \angle -\theta_1$$

$$\dot{S} = \dot{V}^* \dot{I} = |V| \angle -\theta_1 \times |I| \angle \theta_2$$

$$= |V| |I| \angle \theta_2 - \theta_1$$

$$= |V| |I| \angle -\phi$$

$$= V(\cos \phi - jV \sin \phi)$$

$$\dot{S} = P - jQ$$

$$\boxed{V^* I = P - jQ}$$

* In a power system real power flows from a bus with higher angle to a bus with a lower angle. Real power flow does not depend on the difference of the magnitude of the sending end and receiving end voltage.

$$P_R \approx \frac{|V_S| |V_R|}{|X|} \sin \delta$$

$$P = \frac{VE}{X} \sin(\delta - \theta)$$

Economic operation of power system:-

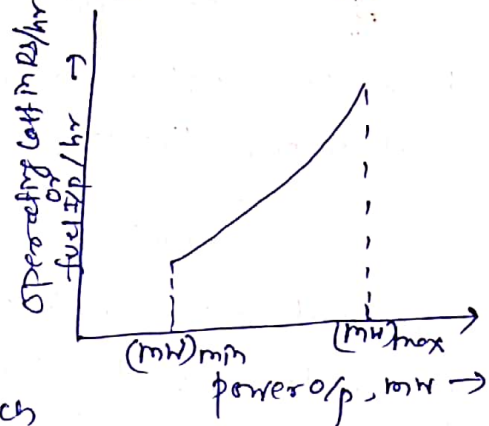
- Economy operation is in determining allocation of generation to each station for various system load levels.
- Economy operation is very important for a power system to return a profit on the capital invested.

Generator operating cost:-

The major component of generator operating cost is the fuel input/hr

The input-output curve of a unit can be expressed in terms of rupees/hr versus output in megawatts. The cost curve can be determined experimentally.

A typical curve is shown in figure



where

$(MW)_{min}$ → minimum power output below which it is uneconomical to run the unit.

$(MW)_{max}$ → max^m power output
max^m capacity of a unit.

The analytical expression of operating cost

$$C_i(P_{Gi}) = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + d_i \quad \text{Rs/hr}$$

Suffix i → unit no.

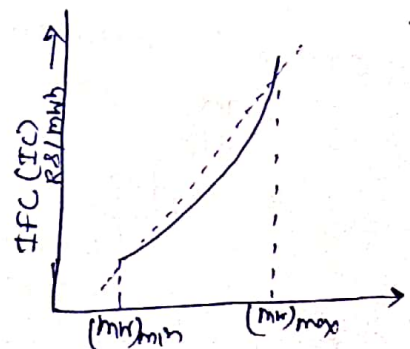
C_i → operating cost.

P_{Gi} → Active power generation of a unit.

a_i, b_i, c_i → constant.

The slope of the cost curve, $\frac{dC_i}{dP_{Gi}}$ is called incremental fuel cost (ICF).

$$\text{I.C. or I.F.C.} \quad \boxed{\frac{dC_i}{dP_{Gi}} = a_i P_{Gi} + b_i} \quad \text{Rs/MWh}$$



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Optimal Generator allocation with line losses neglected:-

Optimal operation:-

Let us assume that it is known a priori which generators are to run to meet a particular load demand on the station

$$\sum_{i=1}^K P_{Gi \max} \geq P_D$$

where $P_{Gi \max} \rightarrow$ Rated real power capacity of the i^{th} generator.

$P_D \rightarrow$ Total power demand on the station.

The load on each generator is to be constrained within lower and upper limits.

$$P_{Gi \min} \leq P_{Gi} \leq P_{Gi \max}, \quad i = 1, 2, \dots, K.$$

The optimal manner in which the load demand P_D must be shared by the generators on the bus is by minimising the operating cost.

$$C = \sum_{i=1}^K C_i(P_{Gi})$$

Under the equality constraint of meeting the load demand, i.e.

$$\sum_{i=1}^K P_{Gi} - P_D = 0 \quad \text{where } K \rightarrow \text{no. of generator on the bus.}$$

Lagrange multipliers method:-

The loading of each generator is constrained by the inequality constraints

Since $C_i(P_{Gi})$ is non linear

and C_i is independent of P_{Gj} ($j \neq i$), this is a separable non-linear programming problem.

$$L = \sum_{i=1}^K C_i P_{Gi} - \lambda \left[\sum_{i=1}^K P_{Gi} - P_D \right]$$

where λ is the Lagrange multiplier.

In order to minimise the operating cost,

$$\frac{dL}{dP_{Gi}} = 0$$

$$\Rightarrow \frac{dC_i}{dP_{Gi}} - \lambda = 0$$

$$\boxed{\frac{dC_i}{dP_{Gi}} = \lambda}$$

$$\boxed{\frac{dC_1}{dP_{G1}} = \frac{dC_2}{dP_{G2}} = \frac{dC_3}{dP_{G3}} = \dots = \lambda} \rightarrow \text{Condition for minimise operating cost.}$$

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Optimum Generation Scheduling:-

Economic load dispatch including transmission losses:-

The load shared among various plants including line losses, the objective is to minimize the overall cost of generation

$$C = \sum_{i=1}^K C_i(P_{Gi})$$

Taking into account the equality constraint of load demand and transmission losses

$$\text{ie } \sum_{i=1}^K P_{Gi} - P_D - P_L = 0$$

where K = total no of generating plants.

P_{Gi} = Active generation of i^{th} plant.

P_D = sum of load demand at all buses (system load demand)

P_L = Total system transmission losses.

The Lagrangian as

$$L = \sum_{i=1}^K C_i(P_{Gi}) - \lambda \left[\sum_{i=1}^K P_{Gi} - P_D - P_L \right]$$

For optimum real power dispatch

$$\frac{\partial L}{\partial P_{Gi}} = 0 ; i = 1, 2, \dots, K$$

$$\Rightarrow \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda + \lambda \frac{\partial P_L}{\partial P_{Gi}} = 0$$

$$\Rightarrow \frac{dC_i(P_{Gi})}{dP_{Gi}} = \lambda \left(1 - \frac{\partial P_L}{\partial P_{Gi}} \right)$$

$$\frac{\frac{dC_i(P_{Gi})}{dP_{Gi}}}{\left(1 - \frac{\partial P_L}{\partial P_{Gi}} \right)} = \lambda \Rightarrow$$

$$\boxed{\frac{dC_i(P_{Gi})}{dP_{Gi}} L_i = \lambda}$$

(min^m fuel cost is obtained when the I.F.C. of each plant multiplied by its L_i is the same for all the plants)

where $\frac{1}{\left(1 - \frac{\partial P_L}{\partial P_{Gi}} \right)} = L_i$ = penalty factor of the i^{th} plant

$$\boxed{(I.C.)_i \times L_i = \lambda}$$

where λ is in RS/mwh,
and fuel cost in RS/hr.

$$\boxed{(I.C.)_i = \lambda [1 - (TL)_i]} \rightarrow \text{Exact Co-ordination eqn.}$$

where $\frac{\partial P_L}{\partial P_{Gi}}$ = incremental transmission loss; associated with i^{th} generating plant.
(I.T.L)

Thus it is clear that to solve the optimum load scheduling problem, it is necessary to compute ITL for each plant, and therefore we must determine the functional dependence of transmission loss on real powers of generating plants. One of the methods of expressing transmission loss as a function of generator power is through B-coefficients. This method is reasonably adequate for treatment of loss co-ordination in economic scheduling of load between plants.

The general form of the loss formula using B-coefficient is

$$P_L = \sum_{m=1}^K \sum_{n=1}^K P_{Gm} B_{mn} P_{Gn}$$

$\downarrow$ Transmission loss $\downarrow$ Loss coeff. $\downarrow$ Real power generation of n^{th} unit.

Real power generation of m^{th} unit

$$\frac{\partial P_L}{\partial P_{Gi}} = \frac{\partial}{\partial P_{Gi}} \left[P_{Gi} B_{ii} P_{Gi} + \sum_{\substack{m=1 \\ m \neq i}}^K P_{Gm} B_{mi} P_{Gi} + \sum_{\substack{n=1 \\ n \neq i}}^K P_{Gi} B_{in} P_{Gn} \right]$$

$$= 2 P_{Gi} B_{ii} + \sum_{\substack{m=1 \\ m \neq i}}^K P_{Gm} B_{mi} + \sum_{\substack{n=1 \\ n \neq i}}^K P_{Gn} B_{in} \quad (\because B_{ij} = B_{ji})$$

$$\boxed{\frac{\partial P_L}{\partial P_{Gi}} = \sum_{j=1}^K 2 P_{Gj} B_{ij}}$$

$$\frac{dC_i}{dP_{Gi}} = \lambda \left(1 - \frac{\partial P_L}{\partial P_{Gi}} \right) \Rightarrow \frac{dC_i}{dP_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \lambda$$

The quadratic plant cost curve is $C_i(P_{Gi}) = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + d_i$

The incremental cost is $\frac{dC_i}{dP_{Gi}} = a_i P_{Gi} + b_i$

$$\Rightarrow a_i P_{Gi} + b_i + \lambda \sum_{j=1}^K 2 P_{Gj} B_{ij} = \lambda$$

$$a_i P_{Gi} + 2\lambda \sum_{\substack{j=1 \\ j \neq i}}^K B_{ij} P_{Gj} = \lambda - b_i$$

$$P_{Gi} (a_i + 2\lambda \sum_{\substack{j=1 \\ j \neq i}}^K B_{ij}) = \lambda - b_i$$

$$P_{Gi} = \frac{\lambda - b_i - \lambda \sum_{\substack{j=1 \\ j \neq i}}^K 2 B_{ij} P_{Gj}}{a_i + 2\lambda \sum_{j=1}^K B_{ij}}$$

$$\boxed{P_{Gi} = \frac{1 - \frac{b_i}{\lambda} - \sum_{\substack{j=1 \\ j \neq i}}^K 2 B_{ij} P_{Gj}}{\frac{a_i}{\lambda} + 2 \sum_{j=1}^K B_{ij}}}$$

$i = 1, 2, \dots, K$

A constant load of 300 MW is supplied by two 200 MW generator
 no. 1 & 2 for which the respective incremental fuel cost are

$$\frac{dF_1}{dP_1} = 0.1P_1 + 20 \text{ Rs/MWh}, \quad \frac{dF_2}{dP_2} = 0.12P_2 + 15 \text{ Rs/MWh}$$

where P in MW and Cost F in Rs/hr

Determine

- (i) The most economical division of load between the generators.
- (ii) The saving in Rs/day there by obtain compare to equal load sharing between generators.

Solⁿ (i) $P_1 + P_2 = 300$ — (i)

$$0.1P_1 + 20 = 0.12P_2 + 15 \text{ — (ii)}$$

$$P_1 = 140.9 \text{ MW}$$

$$P_2 = 151.9 \text{ MW}$$

(ii) Integrally $F_1 = \frac{0.1P_1^2}{2} + 20P_1$
 $F_2 = \frac{0.12P_2^2}{2} + 15P_2$

Saving due to 1st unit,

$$\Delta F_1 = \frac{0.1}{2} [150^2 - 140.9^2] + 20 [150 - 140.9] = 314.36 \text{ Rs/hr}$$

Saving due to 2nd unit,

$$\Delta F_2 = \frac{0.12}{2} [150^2 - 151.9^2] + 15 [150 - 151.9] = -305.27 \text{ Rs/hr}$$

$$\text{Total saving, } \Delta F_1 + \Delta F_2 = 314.36 - 305.27 = 9.09 \text{ Rs/hr}$$

$$\text{Saving / day} = 9.09 \times 24 = 218.16 \text{ Rs/day}$$

Q-2:- The IFC in Rs/MWh for a plant consisting of 2 units are given by

$$\frac{dF_1}{dP_1} = 0.0080P_1 + 8.0, \quad \frac{dF_2}{dP_2} = 0.0096P_2 + 6.4$$

The total load varies between 250 MW to 1250 MW and that max^m & min^m loads for both the units are 625 MW and 100 MW respectively.

- (i) Calculate the allocation of load betwⁿ the units for obtaining min^m cost during various total loads.
- (ii) Determine the saving in fuel cost in Rs/hr for economic distribution of a total load of 900 MW between the 2 units of the plants while compared with equal distribution of same total load.

$$\text{Sol}^n \quad \frac{df_1}{df_1} = 0.008p_1 + 8.0 \quad \text{--- (1)}$$

$$\frac{df_2}{df_2} = 0.0096p_2 + 6.4 \quad \text{--- (2)}$$

$$\left. \frac{df_1}{df_1} \right|_{p_1=100} = 0.008 \times 100 + 8 = 8.8 \text{ RS/mwh}$$

$$\left. \frac{df_2}{df_2} \right|_{p_2=100} = 0.0096 \times 100 + 6.4 = 7.36 \text{ RS/mwh}$$

$$I_{C2} < I_{C1}$$

Load transfer from 1 to 2
for I.C to be same i.e. 8.8 RS/mwh

$$0.0096 \times p_2 + 6.4 = 8.8$$

$$p_2 = \frac{8.8 - 6.4}{0.0096} = 250$$

$$\text{Fr } p_1 = 200 \quad 0.008 \times 200 + 8.0 = 9.6$$

but $I_{C1} = I_{C2}$ for balance

$$\text{So } I_{C2} = 9.6, \quad 0.0096 \times p_2 + 6.4 = 9.6$$

$$p_2 = \frac{9.6 - 6.4}{0.0096} = 333.33$$

$$\text{Fr } p_1 = 300 \quad 0.008 \times 300 + 8.0 = 10.4$$

$$0.0096 \times p_2 + 6.4 = 10.4$$

$$p_2 = \frac{10.4 - 6.4}{0.0096} = 416.67$$

$$\text{For } p_1 = 400$$

$$0.008 \times 400 + 8 = 11.2$$

$$0.0096 p_2 + 6.4 = 11.2$$

$$p_2 = \frac{11.2 - 6.4}{0.0096} = 500$$

$$\text{Fr } p_1 = 500$$

$$0.008 \times 500 + 8 = 12$$

$$0.0096 p_2 + 6.4 = 12$$

$$p_2 = \frac{12 - 6.4}{0.0096} = 583.33$$

$$\text{For } p_1 = 550$$

$$0.008 \times 550 + 8 = 12.4$$

$$0.0096 \times p_2 + 6.4 = 12.4$$

$$p_2 = \frac{12.4 - 6.4}{0.0096} = 625$$

$$\text{Fr } p_1 = 625$$

$$0.008 \times 625 + 8 = 13$$

$$0.0096 \times p_2 + 6.4 = 13$$

$$p_2 = \frac{13 - 6.4}{0.0096} = 687.5 \leq 825$$

$$p_1 + p_2 = 625 + 687.5 = 1312.5 \leq 1250$$

Limit violation is observed for p_2 & $p_1 + p_2$.

1) From 900 MW for economic operation of the system the load sharing betwⁿ unit 1 and unit 2 should be 400 MW & 500 MW respectively.

The increase in cost for unit 1 = 400 to 450

$$\int_{400}^{450} (0.008 P_1 + 8) dP_1 = \left[0.008 \frac{P_1^2}{2} + 8 P_1 \right]_{400}^{450}$$

$$= \frac{0.008}{2} (450^2 - 400^2) + 8(450 - 400) = 750 \text{ Rs/hr}$$

Similarly for unit 2

$$\int_{500}^{450} (0.0096 P_2 + 6.4) dP_2 = \left[0.0096 \frac{P_2^2}{2} + 6.4 P_2 \right]_{500}^{450}$$

$$= \frac{0.0096}{2} (450^2 - 500^2) + 6.4(450 - 500) = -548 \text{ Rs/hr}$$

$$\text{Net increase in cost} = 570 - 548 = 22 \text{ Rs/hr}$$

$$\text{Savng/hr} = 22 \text{ Rs/hr}$$

Q-3:- The incremental cost characteristics of a two plant system are

$$I.C_1 = 1.0 P_1 + 85 \text{ Rs/mwh}$$

$$I.C_2 = 1.2 P_2 + 72 \text{ Rs/mwh}$$

where P_1 and P_2 are in MW

The loss coefficient matrix in mwh^{-1} is $\begin{bmatrix} 0.015 & -0.001 \\ -0.001 & 0.02 \end{bmatrix}$

Compute the optimal scheduling with $\lambda = 15 \text{ Rs/mwh}$.

$$\text{Sol}^n \quad \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} 0.015 & -0.001 \\ -0.001 & 0.02 \end{bmatrix}$$

$$P_L = \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$= \begin{bmatrix} (B_{11} P_1 + B_{21} P_2) & (B_{12} P_1 + B_{22} P_2) \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$= B_{11} P_1^2 + B_{21} P_1 P_2 + B_{12} P_1 P_2 + B_{22} P_2^2$$

$$P_L = B_{11} P_1^2 + 2 B_{12} P_1 P_2 + B_{22} P_2^2$$

$$\frac{\partial P_L}{\partial P_1} = 2 B_{11} P_1 + 2 B_{12} P_2$$

$$= (2 \times 0.015) P_1 + (2 \times -0.001) P_2$$

$$\frac{\partial P_L}{\partial P_1} = 0.03 P_1 - 0.002 P_2$$

$$\frac{\partial P_L}{\partial P_2} = 2B_{12}P_1 + 2B_{22}P_2$$

$$= (2 \times -0.001)P_1 + (2 \times 0.002)P_2$$

$$\frac{\partial P_L}{\partial P_2} = -0.002P_1 + 0.004P_2$$

$$\frac{dC_1}{dP_1} \times \frac{1}{(1 - \frac{\partial P_L}{\partial P_1})} = \lambda \Rightarrow \frac{1P_1 + 85}{1 - (0.003P_1 - 0.002P_2)} = 15 \quad \text{--- (1)}$$

$$\frac{dC_2}{dP_2} \times \frac{1}{(1 - \frac{\partial P_L}{\partial P_2})} = \lambda \Rightarrow \frac{1.2P_2 + 72}{1 - (-0.002P_1 + 0.004P_2)} = 15 \quad \text{--- (2)}$$

Q-4 For a 2 plant system IFC are given by

$$\frac{dF_1}{dP_1} = 0.01P_1 + 2$$

$$\frac{dF_2}{dP_2} = 0.01P_2 + 1.5$$

take loss coefficient as $B_{11} = 0.0015$, $B_{12} = -0.0005$, $B_{22} = 0.0025$.
Determine the operating schedule corresponding to $\lambda = 2.6$ solve using iterative method.

$$P_{gi} = \frac{1 - \frac{b_i}{\lambda} - \sum_{j=1}^K 2B_{ij}P_{gj}}{\frac{a_i}{\lambda} + 2B_{ii}}$$

$$P_1 = \frac{1 - \frac{b_1}{\lambda} - 2B_{12}P_2}{\frac{a_1}{\lambda} + 2B_{11}} = \frac{1 - \frac{2}{2.6} - 2(-0.0005)P_2}{\frac{0.01}{2.6} + 2(0.0015)}$$

$$P_1 = \frac{0.230769 + 0.001P_2}{0.00684615} \quad \text{--- (I)}$$

$$P_2 = \frac{1 - \frac{b_2}{\lambda} - 2B_{21}P_1}{\frac{a_2}{\lambda} + 2B_{22}} = \frac{1 - \frac{1.5}{2.6} - 2(0.0005)P_1}{\frac{0.01}{2.6} + 2(0.0025)}$$

$$P_2 = \frac{0.423077 + 0.001P_1}{0.00884615} \quad \text{--- (II)}$$

Generation corresponding to equal incremental cost of production is calculated as

$$0.01P_1 + 2 = 2.6 \Rightarrow P_1 = 60 \text{ MW}$$

$$0.01P_2 + 1.5 = 2.6 \Rightarrow P_2 = 110 \text{ MW}$$

put P_2 eqn (I) $\Rightarrow P_1 = 49.77 \text{ MW}$

put P_1 eqn (II) $\Rightarrow P_2 = 54.61 \text{ MW}$

and iterating $P_1 = 41.8 \text{ MW}$

$P_2 = 52.53 \text{ MW}$

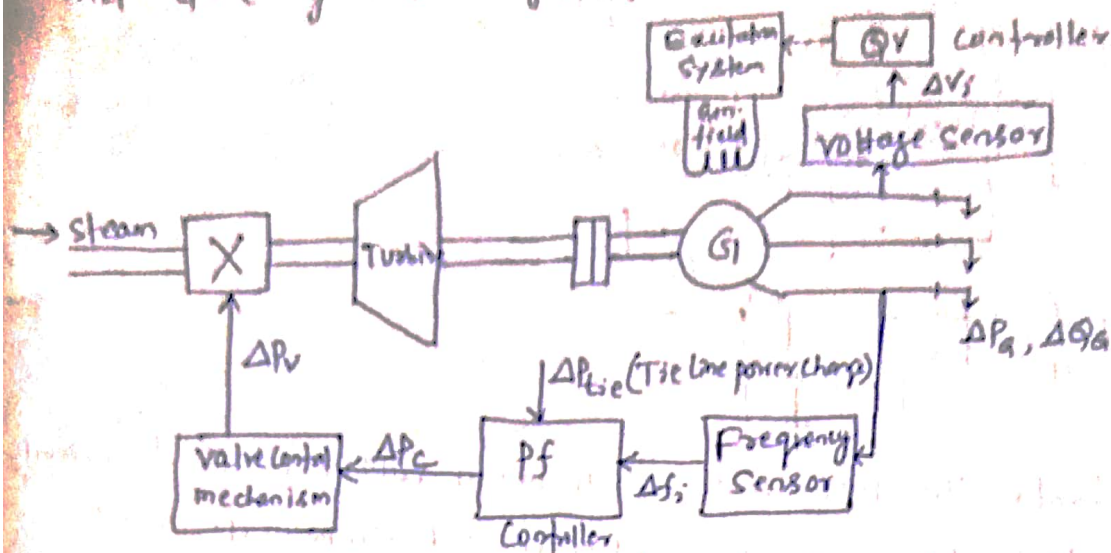
3rd iteration $P_1 = 41.38 \text{ MW}$

$P_2 = 52.50 \text{ MW}$

4th iteration $P_1 = 41.37 \text{ MW}$

$P_2 = 52.5 \text{ MW}$

Explain with neat diagram the dynamic interaction between pf (negative feedback) and QV (mega var voltage) loops.



MW frequency or pf loop or pf control:-

The control of frequency simultaneously control the real power exchange via outgoing lines. The frequency error Δf_i , change in tie line power ΔP_{tie} are sensed which give information about the incidental change in rotor angle $\Delta \delta_i$. This sensor signal are amplified, mixed, transformed and fed as ΔP_c (real power command signal) to the valve control mechanism. The valve control mechanism provide signal proportionately how much steam is to be fed to the turbine. As a result a change in real power generation ΔP_g is obtain.

MVAR voltage or QV control:-

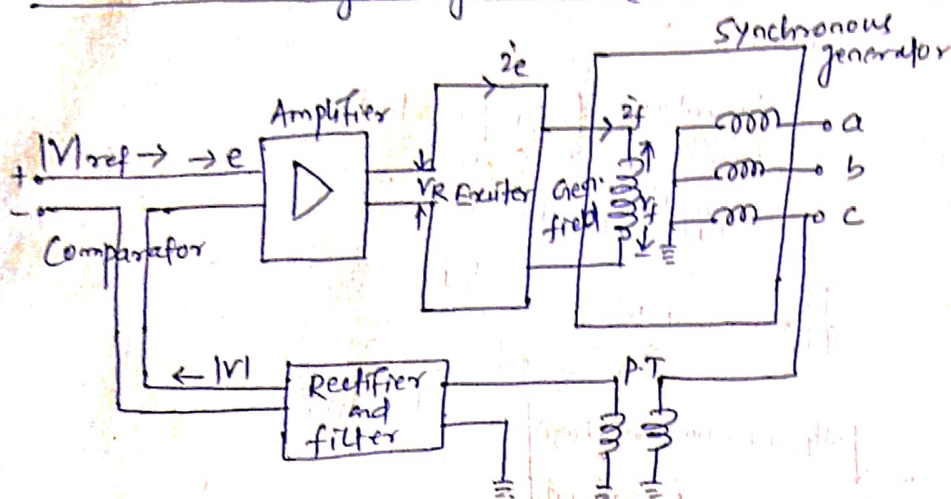
Here the voltage error ΔV_i is sensed and transformed in to reactive power command signal which is fed to the excitation system to change the field current in order to achieve desired generator emf.

The excitation system time constant is much smaller than the prime mover time constant and its transient delay much faster and doesnot affect the pf dynamics. Thus the cross-coupling between pf loop and QV loop is negligible.

Explain the term Coherancy.

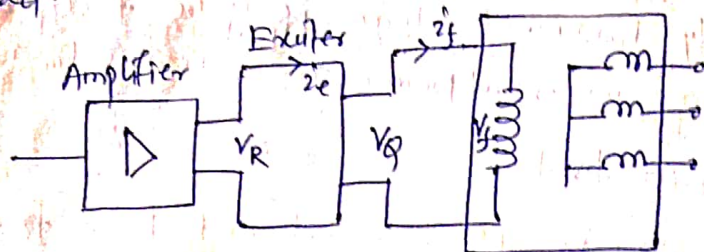
If in an N bus system two or more buses are catagorized by equal Δf_i (frequency change), then the swing in unison or coherantly.

Automatic voltage Regulator (AVR loop) :-



AVR loop controls the magnitude of terminal voltage V . The terminal voltage is continuously sensed, rectified and smoothed. This DC signal being proportional to $|V|$, is compared with a dc reference voltage $|V|_{ref}$. The resulting error voltage after amplification and signal shaping, serves as the input to the exciter which finally delivers the voltage V_f to the generator field winding.

The basic role of AVR is to provide constancy of the generator terminal voltage during normal small and slow changes in load.



Amplifier modeling :-

If the terminal voltage $|V|$ decreases, error voltage 'e' increases

$$\Delta |V|_{ref} - \Delta |V| = \Delta e \quad \text{--- (1)}$$

$$\Delta |V|_{ref}(s) - \Delta |V|(s) = \Delta e(s)$$

And $\Delta V_R = K_A \Delta e$

$$\Delta V_R(s) = K_A \Delta e(s)$$

$$\boxed{\frac{\Delta V_R(s)}{\Delta e(s)} = K_A = G(A)} \Rightarrow \text{Amplifier transfer function}$$

Instantaneous operation

$$\frac{\Delta V_R(s)}{\Delta e(s)} = G(A) = \frac{K_A}{1 + sT_A}$$

$T_A \rightarrow$ Amplifier time constant.

Exciter modeling:-

of R_e and L_e represent resistance and inductance of the exciter

$$\Delta V_R = R_e \Delta i_e + L_e \frac{d\Delta i_e}{dt}$$

measured across the mainfield the exciter produces K_1 armature volts per ampere of field current i_e

$$\Delta V_f = K_1 \Delta i_e$$

$$\Delta V_f(s) = K_1 \Delta i_e(s)$$

$$\Delta V_R(s) = R_e \Delta i_e(s) + s L_e \Delta i_e(s)$$

$$\Delta V_R(s) = R_e \cdot \frac{\Delta V_f(s)}{K_1} + s L_e \frac{\Delta V_f(s)}{K_1}$$

$$= \Delta V_f(s) \left[\frac{R_e}{K_1} + \frac{s L_e}{K_1} \right]$$

$$\Delta V_R(s) = \Delta V_f(s) \left[\frac{R_e + s L_e}{K_1} \right]$$

$$\frac{\Delta V_f(s)}{\Delta V_R(s)} = \frac{K_1}{R_e + s L_e} = \frac{K_1}{R_e \left(1 + \frac{s L_e}{R_e} \right)}$$

$$\Rightarrow \frac{\Delta V_f(s)}{\Delta V_R(s)} = \frac{K_1 / R_e}{1 + s T_e}$$

where $K_e = \frac{K_1}{R_e}$

$$\Rightarrow \boxed{G_e = \frac{\Delta V_f(s)}{\Delta V_R(s)} = \frac{K_e}{1 + s T_e}}$$

and $T_e = \frac{L_e}{R_e}$

Generator modeling:-

of R_f and L_f represents resistance and inductance

$$\Delta V_f = R_f \Delta i_f + L_f \frac{d(\Delta i_f)}{dt} \quad \text{--- (1)}$$

$$e = -\frac{d\phi_f}{dt} \quad \begin{cases} \phi_f = \phi_m \sin \omega t \\ \frac{d\phi_f}{dt} = \omega \phi_m \cos \omega t \end{cases}$$

$$e = \omega \phi_m \sin \left(\omega t - \frac{\pi}{2} \right)$$

Rms value of $|e| = \frac{\omega \phi_m}{\sqrt{2}} = \frac{\omega L_f i_f}{\sqrt{2}}$

$$\begin{cases} L \frac{di}{dt} = \frac{d\phi}{dt} \\ Li = \phi \end{cases}$$

$$\Rightarrow \Delta E = \frac{\omega L_f \Delta i_f}{\sqrt{2}}$$

$$\Delta i_f = \frac{\sqrt{2}}{\omega L_f} \Delta E$$

sub in eqⁿ (1), $\Delta V_f = R_f \Delta i_f + L_f \frac{d(\Delta i_f)}{dt} = R_f \frac{\sqrt{2}}{\omega L_f} \Delta E + L_f \frac{\sqrt{2}}{\omega L_f} \frac{d}{dt} \Delta E$

$$\Delta V_f = \frac{\sqrt{2}}{\omega L_f} \left[R_f \Delta E + L_f \frac{d}{dt} \Delta E \right]$$

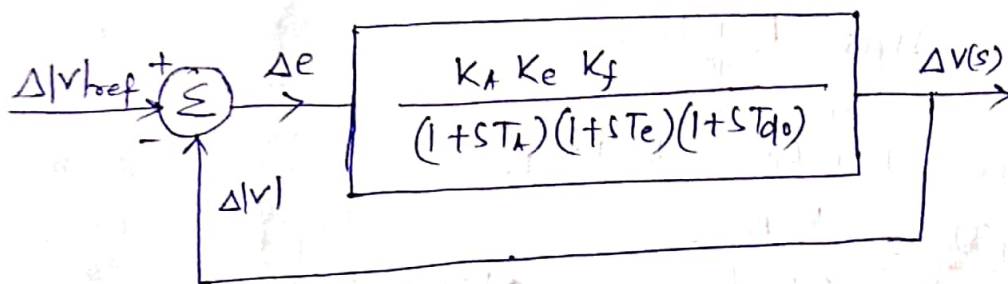
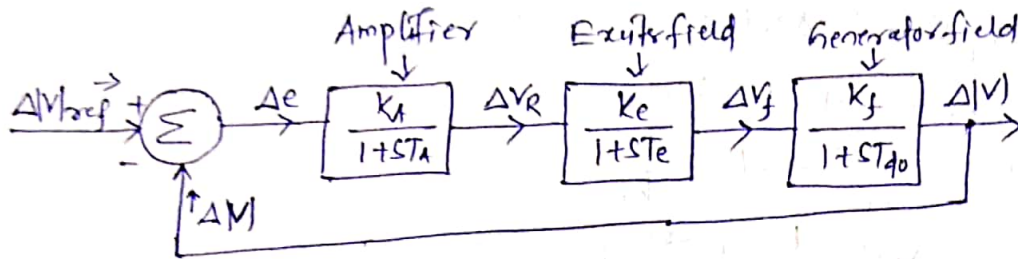
$$\Delta V_f(s) = \frac{\sqrt{2}}{\omega L_f} \left[R_f \Delta E(s) + s L_f \Delta E(s) \right]$$

$$\Delta V_f(s) = \frac{V_2}{\omega L_f} \Delta E(s) [R_f + sL_f]$$

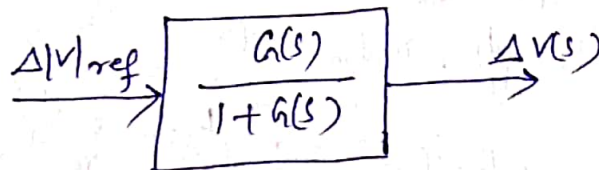
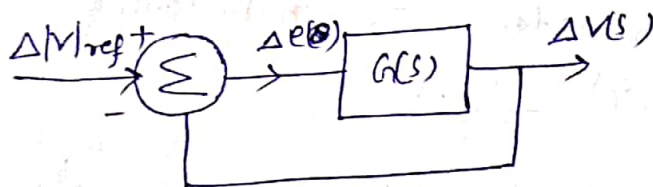
$$\frac{\Delta E(s)}{\Delta V_f(s)} = \frac{\omega L_f}{\sqrt{2} (R_f + sL_f)} = \frac{\omega L_f}{\sqrt{2} R_f (1 + \frac{sL_f}{R_f})}$$

Transfer function of Generator

$$\frac{\Delta E(s)}{\Delta V_f(s)} = \frac{K_f}{1 + sT_{d0}} \quad K_f = \frac{\omega L_f}{\sqrt{2} R_f}, \quad T_{d0} = \frac{L_f}{R_f}$$



The open loop T.F, $G(s) = \frac{K_A K_e K_f}{(1 + sT_A)(1 + sT_e)(1 + sT_{d0})}$



$$[\Delta V_{ref}(s) - \Delta V(s)] G(s) = \Delta V(s)$$

$$\Rightarrow \Delta V_{ref}(s) \cdot G(s) - \Delta V(s) \cdot G(s) = \Delta V(s)$$

$$\Delta V(s) [1 + G(s)] = \Delta V_{ref}(s) \cdot G(s)$$

$$\frac{\Delta V(s)}{\Delta V_{ref}(s)} = \frac{G(s)}{1 + G(s)}$$

Static performance of AVR loop:-

(5)

The AVR loop must

1. Regulate the terminal voltage $|V|$ to within required static accuracy limit
2. Have sufficient speed of response.
3. be stable.

For a constant (subscript 0) reference input the constant error Δe_0 must be less than some specified percentage P of the reference.

$$\Delta V|_{\text{ref},0} - \Delta V|_0 < \frac{P}{100} \Delta V|_{\text{ref},0}$$

For a constant input the transfer function is obtained by setting $s=0$

$$\Delta e_0 = \Delta V|_{\text{ref},0} - \frac{G(0)}{1+G(0)} \Delta V|_{\text{ref},0} < \frac{P}{100} \Delta V|_{\text{ref},0}$$

$$\Rightarrow \frac{(1+K) \Delta V|_{\text{ref},0} - K \Delta V|_{\text{ref},0}}{1+K} < \frac{P}{100} \Delta V|_{\text{ref},0}$$

$$\Rightarrow \frac{\Delta V|_{\text{ref},0}}{1+K} < \frac{P}{100} \Delta V|_{\text{ref},0}$$

$$K+1 > \frac{100}{P}$$

$$\boxed{K > \frac{100}{P} - 1}$$

(Static error decreases with increased loop gain)

If we specify static error at less than 1% the open loop gain K must exceed 99.

Dynamic Response of AVR loop:-

$$\frac{\Delta V(s)}{\Delta V|_{\text{ref}}(s)} = \frac{G(s)}{1+G(s)}$$

$$\Rightarrow \Delta V(s) = \frac{G(s)}{1+G(s)} \Delta V|_{\text{ref}}(s)$$

$$\Delta V(t) = \mathcal{L}^{-1} \left[\frac{G(s)}{1+G(s)} \cdot \Delta V|_{\text{ref}}(s) \right]$$

Mathematically the response depends upon the eigen values or closed loop poles which are obtained from characteristics eqn, $1+G(s)=0$. Since $G(s)$ is 3rd order, we obtained 3 poles, s_1, s_2 & s_3 . If these are distinct & real, the transient response are of the form, $A_1 e^{s_1 t}, A_2 e^{s_2 t}, A_3 e^{s_3 t}$, for the

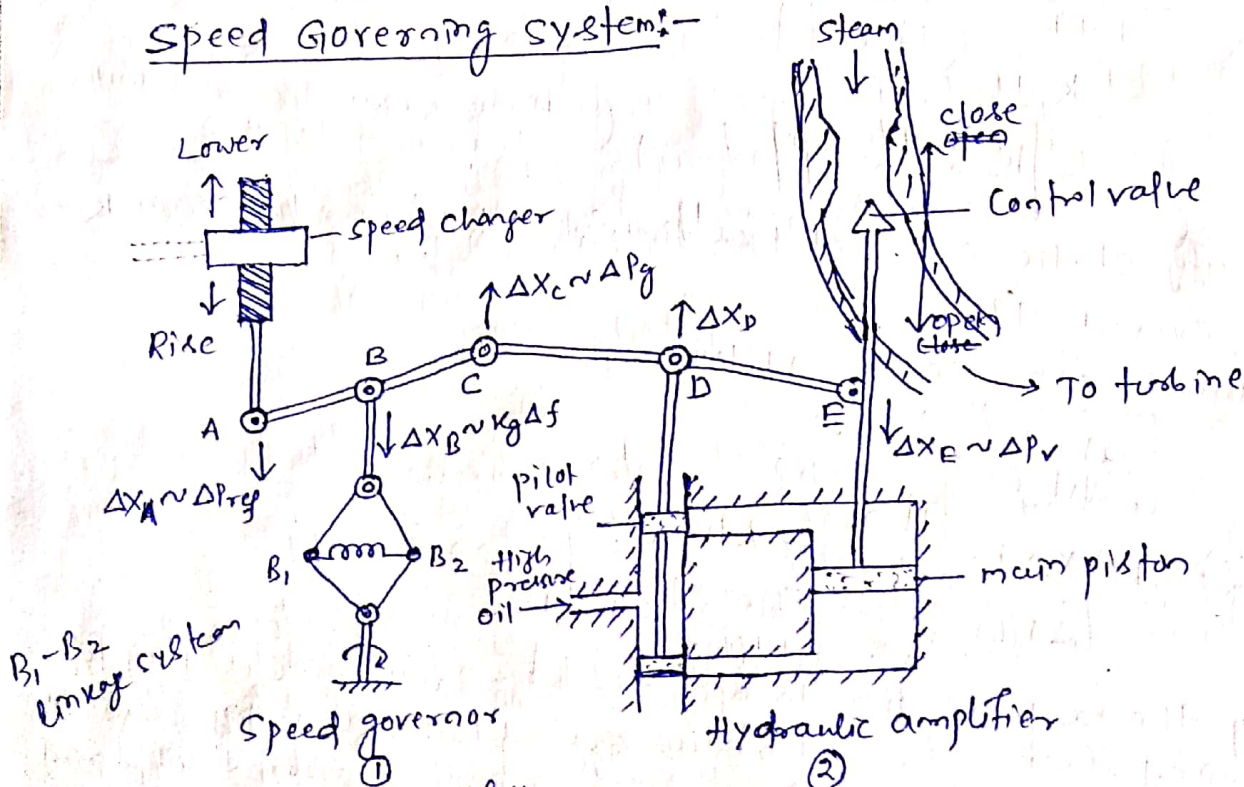
AVR loop to be stable the transient component must vanish with time. Thus we must require 3 closed loop poles to lie on left half hand side of the s -plane.

Automatic Load frequency Control (ALFC)

⑥

- The basic role of ALFC is to maintain desired megawatt output of a generator unit and assist in controlling the frequency of the larger interconnection.
- The ALFC also helps to keep the net interchange of power between pool members at predetermined values.
- Chief objective of ALFC is to control the real power (P_g) output of synchronous generator and simultaneously control the frequency of the interconnected network.
- The ALFC loop will maintain control only during normal (small and slow) changes in load and frequency.
- It is unable to provide adequate control during emergency situations when large megawatt imbalances occur.

Speed Governing system:-



(Simplified functional diagram of the Primary ALFC loop)

- The governor of Command ΔP_g is measured by the position change ΔX_c . The governor has two inputs
1. Changes ΔP_{ref} in the reference power setting.
 2. Changes Δf in the speed of frequency of the generator as measured by ΔX_b .

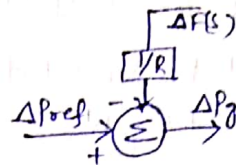
An increase in ΔP_g results from an increase in ΔP_{ref} and a decrease in Δf . Thus we can write for small increments

$$\Delta P_g = \Delta P_{ref} - \frac{1}{R} \Delta f$$

Taking Laplace transform,

$$\Delta P_g(s) = \Delta P_{ref}(s) - \frac{1}{R} \Delta F(s)$$

where R is in Hz/mW and is referred as Regulation or droop.



Hydraulic valve Actuator:-

The input position Δx_D of the valve actuator increases as a result of an increased command ΔP_g but decreases due to increased valve output ΔP_v . Equal increases in both ΔP_g and ΔP_v should result in $\Delta x_D = 0$, thus we can write

$$\Delta x_D = \Delta P_g - \Delta P_v \quad \text{mW}$$

$$\Delta x_D(s) = \Delta P_g(s) - \Delta P_v(s) \quad \text{--- (1)}$$

$$\Delta P_v = K_H \int \Delta x_D dt \quad K_H \text{ (positive constant) or hydraulic amplifier constant depends upon orifice and cylinder geometry and fluid pressure.}$$

$P_g \rightarrow$ Governor o/p, $P_v \rightarrow$ Control valve o/p

$$\Delta P_v(s) = \frac{K_H \Delta x_D(s)}{s}$$

For small changes Δx_D , the oil flow in to the hydraulic motor is proportional to position Δx_D of the pilot valve.

Sub in eqn (1), $\frac{s \Delta P_v(s)}{K_H} = \Delta P_g(s) - \Delta P_v(s)$

$$\Delta P_v(s) \left(1 + \frac{s}{K_H} \right) = \Delta P_g(s)$$

T.F of Hydraulic valve actuator $H(s)$

$$\frac{\Delta P_v(s)}{\Delta P_g(s)} = \frac{1}{\left(1 + \frac{s}{K_H} \right)} = \frac{1}{1 + sT_H}$$

$$\Delta P_g(s) \rightarrow \left[\frac{K_H(s)}{1 + sT_H} \right] \rightarrow \Delta P_v(s)$$

$$T_H = \frac{1}{K_H} = 0.1 \text{ sec}$$

hydraulic time constant

Turbine model

This model represents changes in power o/p of Steam turbine to changes in its steam valve opening (Δx_E) where ($\Delta x_E \propto \Delta P_v$)

Consider here a non-reheat turbine with a single gain factor K_T and time constant T_T .

one of the mode of representation of turbine is

T.F of Turbine $G_T(s) = \frac{\Delta P_T(s)}{\Delta P_v(s)} = \frac{K_T}{1 + sT_T}$

where T_T lies in the range of 0.2 to 2 sec
For steady state analysis $K_T = 1$

$$\Delta P_v(s) \rightarrow \left[\frac{K_T}{1 + sT_T} \right] \rightarrow \Delta P_T(s)$$

Generator load model :-

(8)

$$\Delta P_G - \Delta P_D \Rightarrow$$

① To increase K.E of rotor of the generator.

② To increase load consumption.

① To increase K.E of rotor of the gen:

Let initially K.E, W_0 (Before increasing generation)

After increasing K.E, W

$$W_0 \propto f_0^2, W \propto (f_0 + \Delta f)^2$$

$$\frac{W}{W_0} = \left(\frac{f_0 + \Delta f}{f_0} \right)^2$$

$$W = W_0 \left(\frac{f_0 + \Delta f}{f_0} \right)^2 = W_0 \left(1 + \frac{\Delta f}{f_0} \right)^2 = W_0 \left(1 + \frac{2\Delta f}{f_0} \right)$$

The rate of increase of K.E $\boxed{\frac{dW}{dt} = \frac{2W_0}{f_0} \frac{d(\Delta f)}{dt}}$

② To increase load consumption:

By an increase load consumption, all typical load experiences an increase $D = \frac{\partial P_D}{\partial f}$ MW/Hz with speed or frequency Δf

As frequency changes, motor load changes being sensitive to speed, The rate of change of load w.r.t frequency i.e.

$\frac{\partial P_D}{\partial f}$ can be regarded as nearly constant for small change in frequency Δf and can be expressed as

$$\frac{\partial P_D}{\partial f} \cdot \Delta f = D \Delta f \text{ (Increase load consumption)}$$

$$\Delta P_G - \Delta P_D = \frac{2W_0}{f_0} \frac{d(\Delta f)}{dt} + D \Delta f$$

if H is the inertia constant of the generator in MW-sec/MVA and P is rating in MW

$$\text{Then } W_0 = HP$$

$$\Delta P_G - \Delta P_D = \frac{2HP}{f_0} \frac{d(\Delta f)}{dt} + D \Delta f$$

$$\Rightarrow \Delta P_G(\text{pu}) - \Delta P_D(\text{pu}) = \frac{2H}{f_0} \frac{d(\Delta f)}{dt} + D \Delta f$$

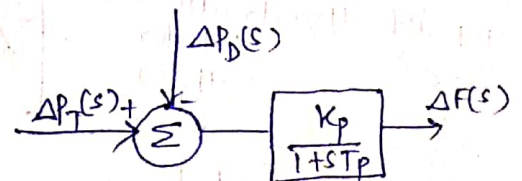
$$\Rightarrow \Delta P_G(s) - \Delta P_D(s) = \frac{2H}{f_0} s \Delta F(s) + D \Delta F(s) = \Delta F(s) \left[\frac{2Hs}{f_0} + D \right]$$

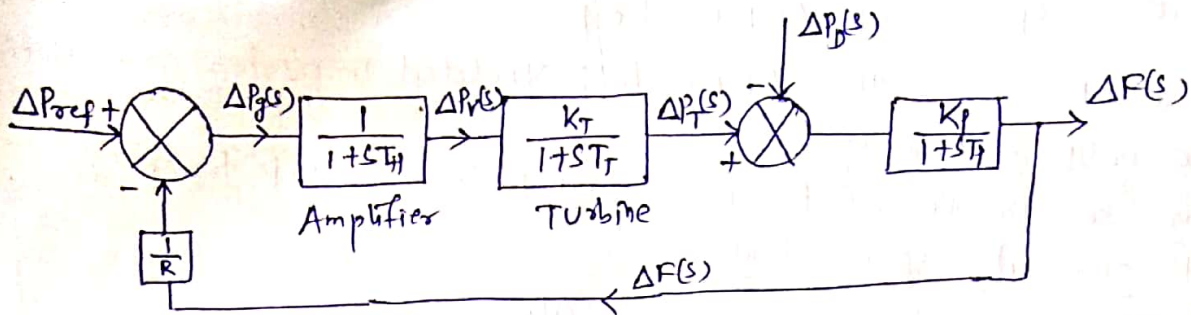
$$\frac{\Delta F(s)}{\Delta P_G(s) - \Delta P_D(s)} = \frac{1}{D + \frac{2Hs}{f_0}} = \frac{1}{D \left(1 + \frac{2Hs}{Df_0} \right)}$$

$$\boxed{\frac{\Delta F(s)}{\Delta P_G(s) - \Delta P_D(s)} = \frac{K_p}{1 + sT_p}}$$

where $K_p = \frac{1}{D} \rightarrow$ power system gain Hz/pu MW

$T_p = \frac{2H}{f_0 D} \rightarrow$ power system time const.





(Complete block diagram representing load frequency control)

Steady state Analysis of ALFC loop :- (Static Response of primary ALFC loop)

There are two important incremental inputs to the load frequency control system.

i.e. $\Delta P_{ref} \rightarrow$ The change in speed changer setting

and $\Delta P_D \rightarrow$ The change in load demand.

Let us consider a constant speed changer i.e. $\Delta P_{ref} = 0$ and

Load demands changes

i.e. Step changing load, $\Delta P_D(s) = \frac{\Delta P_D}{s}$

This is known as free governor operation

$$\left[\Delta P_{ref}(s) - \frac{1}{R} \Delta F(s) \right] \frac{K_T}{(1+sT_H)(1+sT_T)} - \Delta P_D(s) \left] \frac{K_p}{1+sT_p} = \Delta F(s)$$

$\Delta P_{ref} = 0$

$$\Rightarrow \left[-\frac{1}{R} \Delta F(s) \frac{K_T}{(1+sT_H)(1+sT_T)} - \frac{\Delta P_D}{s} \right] \frac{K_p}{1+sT_p} = \Delta F(s)$$

$$\Rightarrow \Delta F(s) \left[1 + \frac{K_T K_p}{R (1+sT_H)(1+sT_T)(1+sT_p)} \right] = -\frac{\Delta P_D}{s} \times \frac{K_p}{1+sT_p}$$

$$\Rightarrow \Delta F(s) = \frac{-\frac{\Delta P_D}{s} \times \frac{K_p}{1+sT_p}}{1 + \frac{(K_T K_p)/R}{(1+sT_H)(1+sT_T)(1+sT_p)}} = \frac{-K_p \times \frac{\Delta P_D}{s}}{(1+sT_p) + \frac{(K_T K_p)/R}{(1+sT_H)(1+sT_T)}}$$

$$\Delta F(s) = \frac{-K_p \frac{\Delta P_D}{s}}{(1+sT_p) + \frac{K_p/R}{(1+sT_H)(1+sT_T)}}$$

$K_T = 1$ (Turbine gain or constant)

$$\Delta f(\theta) \Big|_{\text{steady state error}} = \lim_{s \rightarrow 0} s \Delta F(s) = \lim_{s \rightarrow 0} \frac{s \times -K_p \times \frac{\Delta P_D}{s}}{(1+sT_p) + \frac{K_p/R}{(1+sT_H)(1+sT_T)}}$$

$$= \frac{-K_p \Delta P_D}{1 + \frac{K_p/R}{1 + \frac{1}{D}}} = \frac{-\Delta P_D}{\frac{1}{K_p} + \frac{1}{R}} = \frac{-\Delta P_D}{D + \frac{1}{R}} = -\frac{\Delta P_D}{\beta}$$

$$\boxed{\Delta f(\theta) = -\frac{\Delta P_D}{\beta}}$$

$D = \frac{1}{K_p} = D_{droop}$, pu mW/Hz or mW/Hz

where $\beta = (D + \frac{1}{R})$ pu mW/Hz, Area frequency response characteristics (AFRC)

Dynamic Response of ALFC Loop:-

(10)

The static response of the ALFC loop yielded important information about frequency accuracy.

The dynamic response of the loop will inform about tracking ability and stability of the loop.

$$\Delta F(s) = \frac{-K_p}{(1+sT_p) + \frac{(K_p K_T)/R}{(1+sT_H)(1+sT_T)}} \times \frac{\Delta P_D}{s}$$

Turbine and Governor response as instantaneous, $T_H = T_T = 0$, $K_T = 1$

$$\text{Now } \Delta F(s) = \frac{-K_p \frac{\Delta P_D}{s}}{(1+sT_p) + \frac{K_p}{R}}$$

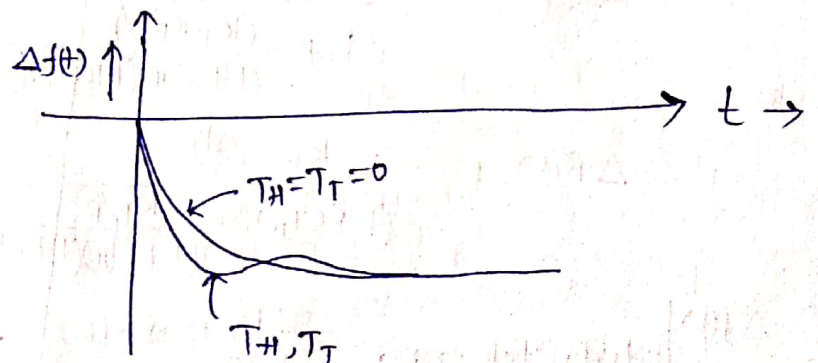
$$= \frac{-R K_p \Delta P_D}{s [R + s R T_p + K_p]}$$

$$= \frac{-R K_p \Delta P_D}{R T_p \times s \left[s + \frac{R + K_p}{R T_p} \right]}$$

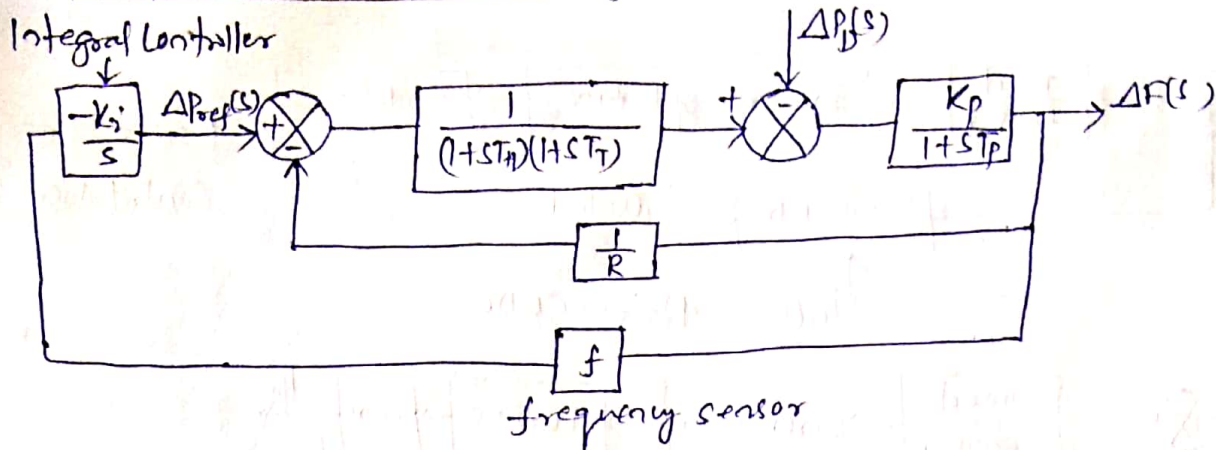
$$= \frac{-R K_p \Delta P_D \times R T_p}{R T_p (R + K_p)} \left[\frac{1}{s} - \frac{1}{s + \frac{R + K_p}{R T_p}} \right]$$

$$\Delta F(s) = \frac{-R K_p \Delta P_D}{R + K_p} \left[\frac{1}{s} - \frac{1}{s + \frac{R + K_p}{R T_p}} \right]$$

$$\Delta f(t) = - \frac{R K_p \Delta P_D}{R + K_p} \left(1 - e^{-\frac{R + K_p}{R T_p} t} \right)$$



Proportional plus Integral Controller:-



$$\Delta F(s) = \frac{-K_p}{(1+ST_p) + \left(\frac{1}{R} + \frac{K_i}{s}\right)K_p} \cdot \frac{\Delta P_d}{s}$$

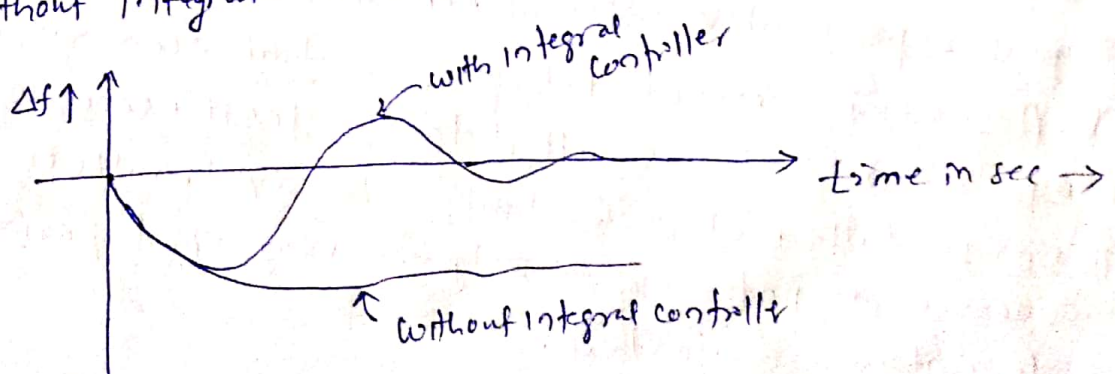
$$= \frac{-K_p}{(1+ST_p) + \frac{(RK_i + s)K_p}{RS(1+ST_H)(1+ST_T)}} \cdot \frac{\Delta P_d}{s}$$

$$= \frac{-RS(1+ST_H)(1+ST_T)K_p}{SR(1+ST_p) + (RK_i + s)K_p} \times \frac{\Delta P_d}{s}$$

$$|\Delta F|_{\text{steady state}} = \lim_{s \rightarrow 0} s \Delta F(s) = 0$$

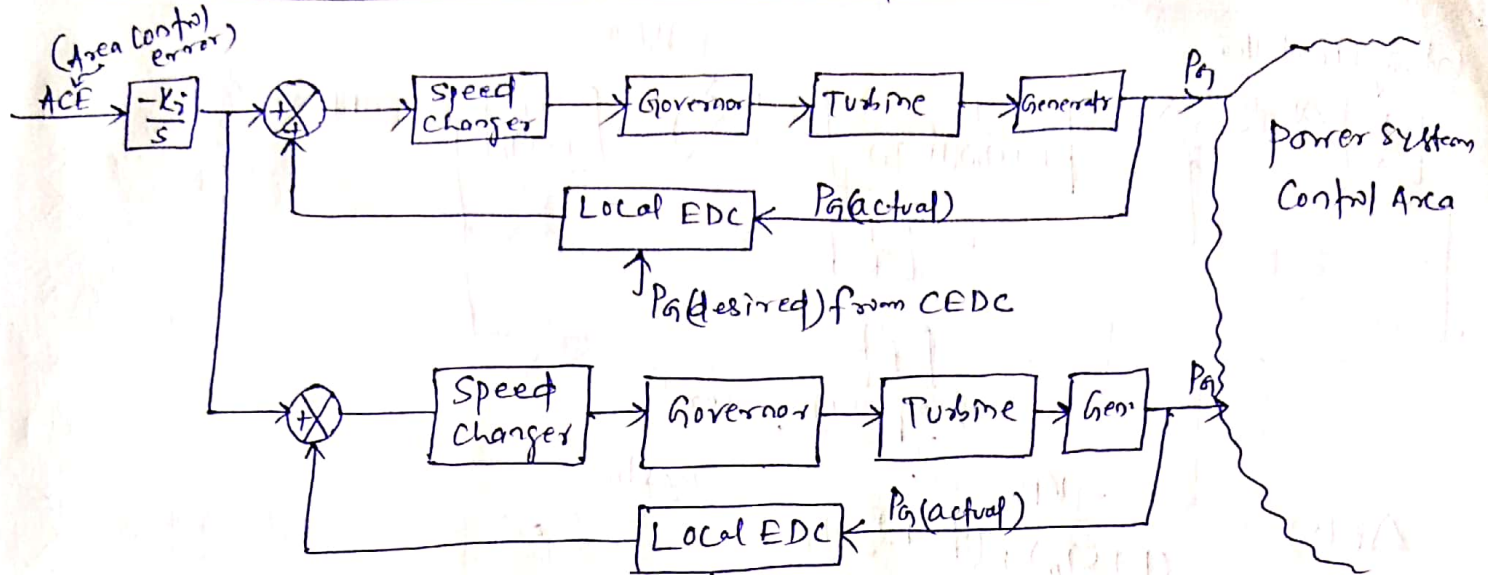
So it is seen that the steady state change in frequency has been reduced to zero by the addition of Integral Controller.

The dynamics of proportional plus integral controller can be studied numerically only. The system being of 4th order, the order of system has increased by one with addition of integral loop. The dynamics responses of proportional plus integral controller with $K_i = 0.9$ for a $\Delta P_d = 0.01$ pu (step load disturbances) obtained through digital computer are plotted in figure. For comparison the dynamic response without integral control is also plotted in same figure.



Load frequency control & Economic dispatch control:-

(12)



EDC $\rightarrow$ Economic dispatch controller.

CEDC $\rightarrow$ Central Economic dispatch computer.

Load frequency control with integral controller achieves zero steady state frequency error and a fast dynamic response, but it exercises no control over the relative loadings of various generating stations (i.e. economic dispatch of the control area).

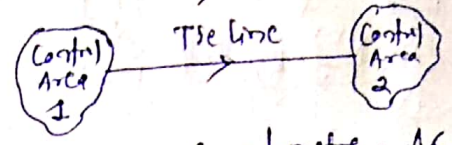
A satisfactory solution is achieved by using independent controls for load frequency and economic dispatch. While the load frequency controller is a fast acting control (a few seconds) and regulates the system around an operating point; the economic dispatch controller is a slow acting control, which adjusts the speed-changer setting every minute (or half a minute) in accordance with a command signal generated by the central economic dispatch computer (CEDC). The signal to change the speed-changer setting is constructed in accordance with economic dispatch error, $[P_g(\text{desired}) - P_g(\text{actual})]$, suitable modified by the signal representing integral ACE at that instant of time. The signal $P_g(\text{desired})$ is computed by the central economic dispatch computer (CEDC) and is transmitted to the local economic dispatch controller (EDC) installed at each station. The system thus operates with economic dispatch error only for very short periods of time before it is readjusted.

Load frequency control of two area system :-

(13)

An extended power system can be divided into a no. of load-frequency control area interconnected by means of tie lines.

The control objective now is to regulate the frequency of each area and to simultaneously regulate the tie line power as per ^{integral} inter-area power contracts. As in the case of frequency, proportional plus integral controller will be installed so as to give zero steady state error in tie line power flow as compared to the contracted power.



It is conveniently assumed that each control area can be represented by an equivalent turbine, generator and governor system.

Modeling the Tie-line:-

In normal operation the power on the tie-line

$$P_{12} = \frac{|V_1||V_2|}{X} \sin(\delta_1 - \delta_2)$$

where δ_1 & δ_2 are the angles of end voltages V_1 & V_2 respectively.

For small deviations in the angle δ_1 and δ_2 the tie line power changes with

$$\Delta P_{12} = \frac{|V_1||V_2|}{X} \cos(\delta_1 - \delta_2) (\Delta\delta_1 - \Delta\delta_2) \text{ MW}$$

$$\Delta P_{12} = T (\Delta\delta_1 - \Delta\delta_2)$$

where $T = \frac{|V_1||V_2|}{X} \cos(\delta_1 - \delta_2) \rightarrow$ Electrical stiffness of the line or synchronizing coefficient of the line.

We know that, $\omega = 2\pi f$

$$\frac{d\delta}{dt} = 2\pi f \quad \text{where } \delta \rightarrow \text{load angle}$$

$$f = \frac{1}{2\pi} \frac{d\delta}{dt}$$

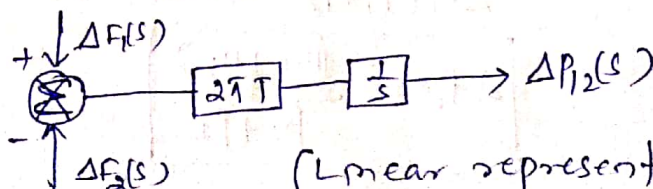
$$\Delta f = \frac{1}{2\pi} \frac{d(\Delta\delta)}{dt}$$

$$\Delta\delta = 2\pi \int_0^t \Delta f dt$$

$$\Delta P_{12} = T \times 2\pi \left[\int_0^t \Delta f_1(t) dt - \int_0^t \Delta f_2(t) dt \right]$$

$$\Delta P_{12}(s) = 2\pi T \left[\frac{\Delta F_1(s)}{s} - \frac{\Delta F_2(s)}{s} \right]$$

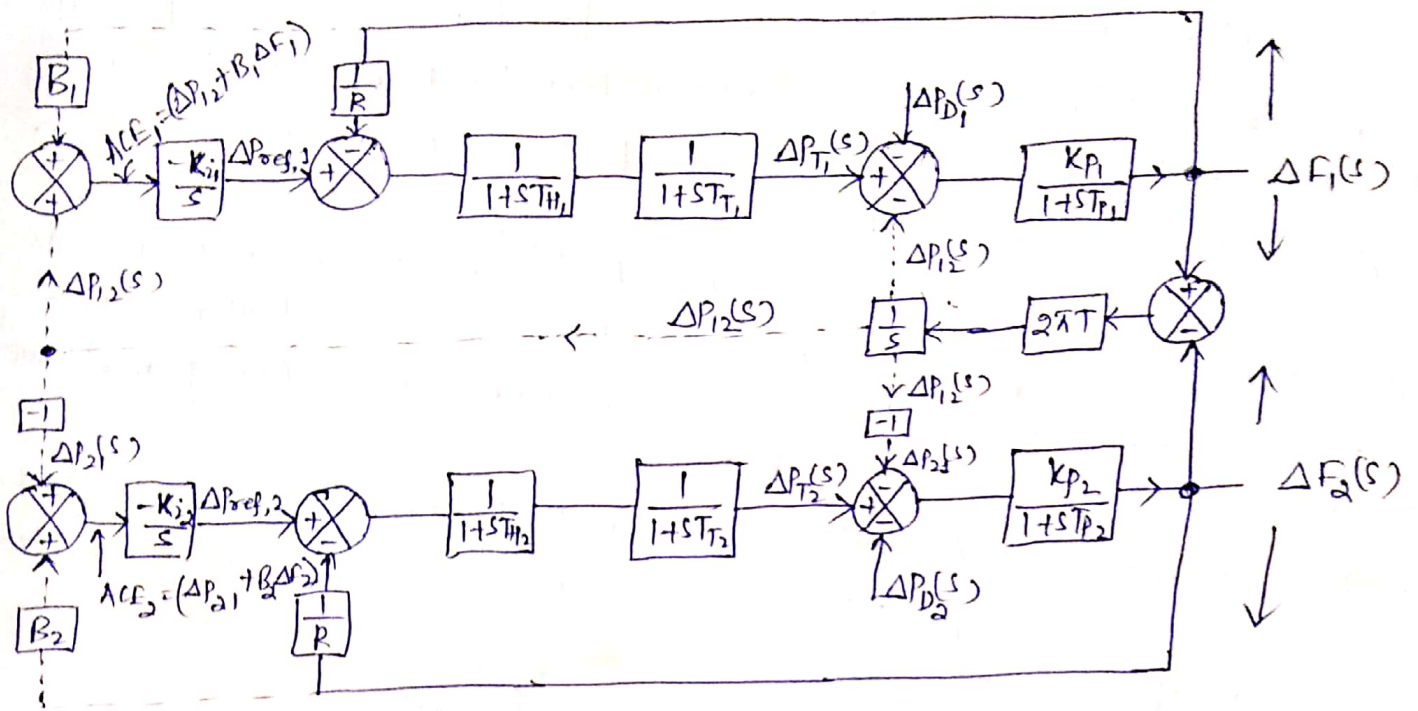
$$\Delta P_{12}(s) = \frac{2\pi T}{s} [\Delta F_1(s) - \Delta F_2(s)]$$



(Linear representation of tie-line)

Linear model of Two-area system:-

(14)



Static Response of Two-Area system:-

We shall investigate the static response of the two area system with fixed speed changer setting or position

$$\Delta P_{ref,1} = \Delta P_{ref,2} = 0$$

We assume that the loads in each area are suddenly increased by the constant incremental steps.

$$\Delta P_{D,1} = m_1 \text{ and } \Delta P_{D,2} = m_2$$

Assuming amplifier & turbine behaviour instantaneous (i.e. T_H & $T_T = 0$)

$$\Delta P_{T1,0} = -\frac{1}{R} \Delta f_0$$

Area 1 speed changer setting is zero

$$\Delta P_{T2,0} = -\frac{1}{R} \Delta f_0$$

$$\Delta P_{T1,0} - m_1 - \Delta P_{12} = D_1 \Delta f_0 \Rightarrow \Delta f_0 (D_1 + \frac{1}{R_1}) = -m_1 - \Delta P_{12}$$

$$\Delta P_{T2,0} - m_2 + \Delta P_{12} = D_2 \Delta f_0 \Rightarrow \Delta f_0 (D_2 + \frac{1}{R_2}) = -m_2 + \Delta P_{12}$$

$$\Delta f_0 \beta_1 = -m_1 - \Delta P_{12}$$

$$\Delta f_0 \beta_2 = -m_2 + \Delta P_{12}$$

$$\Delta f_0 (\beta_1 + \beta_2) = -(m_1 + m_2)$$

$$\Delta f_0 = \frac{-(m_1 + m_2)}{\beta_1 + \beta_2}$$

$$-\Delta P_{12} = (D_1 + \frac{1}{R_1}) \Delta f_0 + m_1$$

$$= \beta_1 \Delta f_0 + m_1 = \beta_1 \left[\frac{-(m_1 + m_2)}{\beta_1 + \beta_2} \right] + m_1$$

$$-\Delta P_{12} = \frac{-\beta_1 m_1 - \beta_1 m_2 + \beta_2 m_1 + \beta_2 m_1}{\beta_1 + \beta_2}$$

$$\Delta P_{12} = \frac{\beta_1 m_2 - \beta_2 m_1}{\beta_1 + \beta_2} \text{ pu MW}$$

Assuming identical Area parameter

$$D_1 = D_2 = D, \beta_1 = \beta_2 = \beta, R_1 = R_2 = R$$

$$\Delta f_0 = \frac{-(m_1 + m_2)}{2\beta} \text{ Hz}$$

$$\Delta P_{12} = \frac{\beta(m_2 - m_1)}{2\beta} = \frac{m_2 - m_1}{2} \text{ pu MW}$$

* If a step load change occur only in area 2

$$\Delta f_0 = \frac{-m_2}{2\beta} (\because m_1 = 0)$$

$$\Delta P_{12} = \frac{m_2}{2}$$

These last two eqn tell about the advantages of pool operation.

i) 50% of the added load in area 2 will be supplied by area 1 via the tie line.

ii) The frequency drop will be only half that which would be experienced in the areas were operating alone.

Dynamic response of two area system:-

Assuming two area to be identical

$$K_A = K_T = 1$$

$$D_1 = D_2 = 0 \text{ (Load not to vary with frequency)}$$

$$\frac{K_{P1}}{1 + sT_{P1}} = \frac{\frac{1}{D_1}}{1 + s \frac{2H}{fD_1}} = \frac{\frac{1}{D_1}}{\frac{fD_1 + s2H}{fD_1}} = \frac{f}{fD_1 + s2H} = \frac{f}{s2H} (\because D_1 = 0)$$

$$\text{Similarly, } \frac{K_{P2}}{1 + sT_{P2}} = \frac{f}{s2H} (H_1 = H_2 = H)$$

From the block diagram,

(16)

$$\left[-\frac{1}{R} \Delta F_1(s) - \Delta P_{D1}(s) - \Delta P_{12}(s) \right] \frac{f}{s2H} = \Delta F_1(s) \quad \text{--- (1)}$$

$$\left[-\frac{1}{R} \Delta F_2(s) - \Delta P_{D2}(s) + \Delta P_{12}(s) \right] \frac{f}{s2H} = \Delta F_2(s) \quad \text{--- (2)}$$

subtracting eqⁿ (2) from eqⁿ (1)

$$\frac{1}{R} [\Delta F_2(s) - \Delta F_1(s)] \frac{f}{s2H} + [\Delta P_{D2}(s) - \Delta P_{D1}(s)] \frac{f}{s2H} - 2\Delta P_{12}(s) \times \frac{f}{s2H} = \Delta F_1(s) - \Delta F_2(s)$$

$$\Rightarrow [\Delta P_{D2}(s) - \Delta P_{D1}(s)] \frac{f}{s2H} = \frac{\Delta P_{12}(s) \times s}{2\pi T} + \frac{\Delta P_{12}(s) \times f}{sH} + \frac{\Delta P_{12}(s) \times s}{2\pi T} \times \frac{f}{s2H} \times \frac{1}{R}$$

$$\Rightarrow [\Delta P_{D2}(s) - \Delta P_{D1}(s)] \frac{f}{s2H} = \Delta P_{12}(s) \frac{s}{2\pi T} + \Delta P_{12}(s) \frac{f}{sH} + \Delta P_{12}(s) \frac{f}{4\pi T R H}$$

$$= \Delta P_{12}(s) \left[\frac{s}{2\pi T} + \frac{f}{sH} + \frac{f}{4\pi T R H} \right]$$

$$= \Delta P_{12}(s) \left[\frac{2RHs^2 + 4\pi TRf + sf}{4\pi s T R H} \right]$$

$$= \frac{\Delta P_{12}(s)}{4\pi s T R H} \times 2RH \left[s^2 + \frac{f}{2RH} s + \frac{4\pi TRf}{2RH} \right]$$

$$= \frac{\Delta P_{12}(s)}{2\pi s T} \left[s^2 + \frac{f}{2RH} s + \frac{2\pi T f}{H} \right]$$

$$\Rightarrow \Delta P_{12}(s) = 2\pi s T \times \frac{f}{s2H} \frac{\Delta P_{D2}(s) - \Delta P_{D1}(s)}{s^2 + \left(\frac{f}{2RH}\right)s + \left(\frac{2\pi T f}{H}\right)}$$

$$\Delta P_{12}(s) = \frac{\pi T f}{H} \frac{\Delta P_{D2}(s) - \Delta P_{D1}(s)}{s^2 + \left(\frac{f}{2RH}\right)s + \left(\frac{2\pi T f}{H}\right)}$$

The denominator tells us several important factors

(1) The denominator being of the form

$$s^2 + 2\alpha s + \omega^2 = (s + \alpha)^2 + \omega^2 - \alpha^2$$

The system is stable and damped.

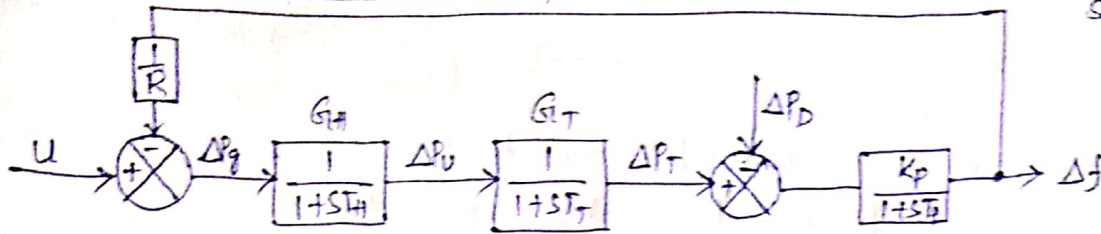
(2) The system will oscillate at the damped angular frequency

$$\omega_0 = \sqrt{\omega^2 - \alpha^2} = \sqrt{\frac{2\pi T f}{H} - \left(\frac{f}{4RH}\right)^2}$$

$$2\alpha = \frac{f}{2RH} \Rightarrow \alpha = \frac{f}{4RH}$$

(3) The system's damping strongly depends on α parameters. Since H & f are constant, the damping will be a function of R parameter. Lower R value will give strong damping. Higher R value gives weak damping. The system will perform undamped oscillations of frequency $\omega_0 = \omega$ where $R = \infty$ i.e. speed governor is nonexistent.

State variable model of single area system:- (Dynamic model in state variable form)



$$\Delta P_v(s) = \left[U(s) - \frac{1}{R} \Delta f(s) \right] \frac{1}{1+sT_H}$$

$$\Delta P_T(s) = \Delta P_v(s) \frac{1}{1+sT_T}$$

$$\Delta f(s) = [\Delta P_T(s) - \Delta P_D(s)] \frac{K_p}{1+sT_p}$$

if we will express in time domain

$$\Delta P_v + T_H \frac{d(\Delta P_v)}{dt} = U - \frac{1}{R} \Delta f$$

$$\Delta P_T + T_T \frac{d(\Delta P_T)}{dt} = \Delta P_v$$

$$\Delta f + T_p \frac{d(\Delta f)}{dt} = (\Delta P_T - \Delta P_D) K_p$$

Introducing three state variables x_1, x_2, x_3 forming the state vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \triangleq \begin{bmatrix} \Delta P_v \\ \Delta P_T \\ \Delta f \end{bmatrix}$$

The disturbance force, $p \triangleq \Delta P_D$

$$\dot{\Delta P_v} = -\frac{\Delta P_v}{T_H} + \frac{U}{T_H} - \frac{1}{RT_H} \Delta f$$

$$\dot{\Delta P_T} = -\frac{\Delta P_T}{T_T} + \frac{\Delta P_v}{T_T}$$

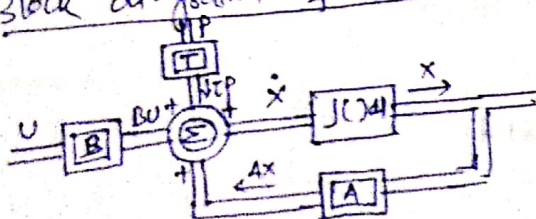
$$\dot{\Delta f} = -\frac{\Delta f}{T_p} + \frac{K_p}{T_p} \Delta P_T - \frac{K_p}{T_p} \Delta P_D$$

$$\begin{bmatrix} \dot{\Delta P_v} \\ \dot{\Delta P_T} \\ \dot{\Delta f} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_H} & 0 & -\frac{1}{RT_H} \\ \frac{1}{T_T} & -\frac{1}{T_T} & 0 \\ 0 & \frac{K_p}{T_p} & -\frac{1}{T_p} \end{bmatrix} \begin{bmatrix} \Delta P_v \\ \Delta P_T \\ \Delta f \end{bmatrix} + \begin{bmatrix} \frac{1}{T_H} \\ 0 \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ -\frac{K_p}{T_p} \end{bmatrix} p$$

$$\dot{x} = Ax + Bu + Cp$$

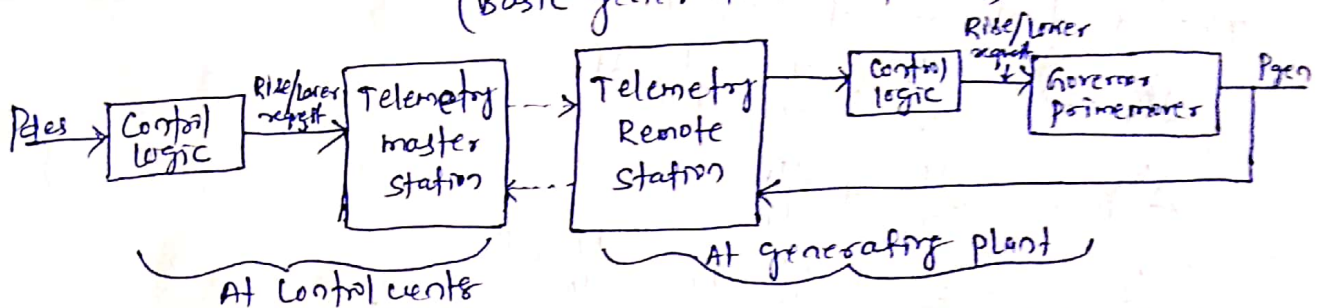
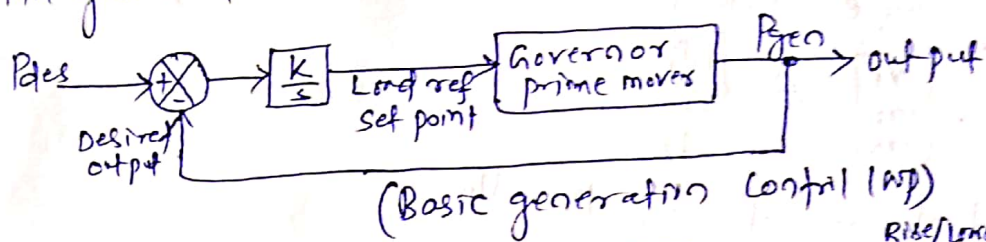
$$\dot{x}_1 = \frac{dx_1}{dt} \quad \Delta P_v = \frac{dx_1}{dt} \quad \dot{x} = Ax + Bu$$

Block diagram of linear state model:-

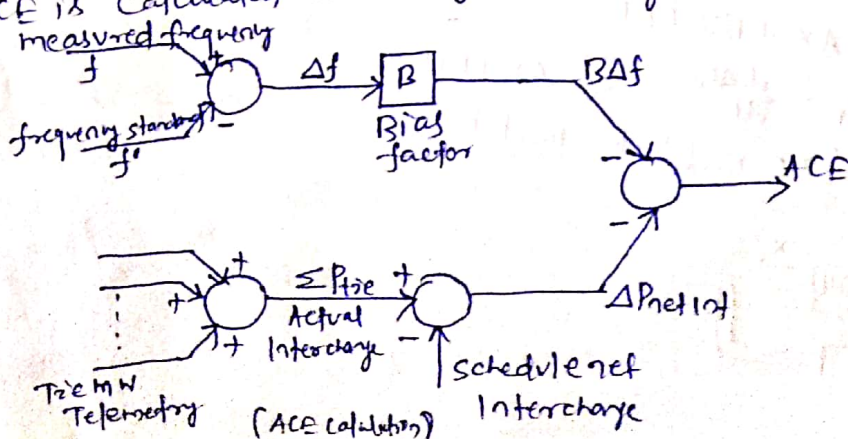


Automatic Generation Control (AGC) Implementation:-

- To implement an AGC system, one would require the following information at Control Center.
 1. Unit MW output for each unit committed unit.
 2. MW flow over each tie-line to neighbouring system.
 3. System frequency.
- The output of the execution of an AGC program must be transmitted to each of generating units. present practice is to transmit raised or lower pulses of varying lengths to the unit.
- Control equipment then changes the unit's load reference set point up or down in proportion to the pulse length. The length of the control pulse may be encoded in the bit of a digital word that is transmitted over a digital telemetry channel.
- The basic reset control loop for a unit consists of an integrator with gain K as shown in fig. below. The control loop implementation is also shown.



- The overall control scheme we are going to develop starts with ACE, which is a measure of the error in total generation from total desired generation.
- ACE is calculated according to the figure shown below.



- ACE serves to indicate when total generation must be raised or lowered in a control area. (19)
- However, ACE is not the only error signal ^{that must} to drive our controller. The AGC control logic must also be driven by the errors in unit output so as to force the units to obey the economic dispatch. To do this, the sum of unit output errors is added to ACE to form a composite error signal that drives the entire control system.

General Criteria for a good AGC:-

1. The ACE signal should ideally be kept from becoming too large. Since ACE is directly influenced by random load variations, this criterion can be treated statistically by saying that the standard deviation of ACE should be small.
2. ACE should not be allowed to drift. This means that the integral of ACE over an appropriate time should be small. Drift in ACE has the effect of creating system time errors or what are termed inadvertent interchange errors.
3. The amount of control action called for by the AGC should be kept to a minimum.

LFC With Generation Rate Constraints (GRCs)

The LFC problem does not consider the effect of the restrictions on the rate of change of power generation. In power systems having steam plants, power generation can change only at a specified maximum rate. The generation rate for reheat units is quite low. Most of the reheat units have a generation rate around 3%/min. Some have a generation rate between 5 to 10%/min. If these constraints are not considered, system is likely to chase large momentary disturbances. This results in undue wear and tear of the controller. Even when GRC is considered, the system dynamic model becomes non-linear and linear control techniques cannot be applied for the optimization of the controller setting.

Considering GRCs for both areas is to add limiters to the governors i.e. the max^m rate of valve opening or closing speed is restricted by the limiters.

The GRCs results in larger deviations in ACEs as the rate at which generation can change in the area is constrained by the limits imposed. Therefore the duration for which the power needs to be imported increases -

considerably as compared to the case where generation rate is not (20) constrained. With GRCs, R should be selected with care so as to give the best dynamic response.

Speed Governor Dead Band and its effect on AGC:-

The Governor dead-band is defined as the total magnitude of sustained speed change within which there is no change in valve position. The limiting value of dead band is specified as 0.06%.

- The effect of governor dead-band is to increase the apparent steady-state speed regulation R .
- The presence of governor dead band makes the dynamic response oscillatory.

- ① A 100 MW generator is operated onto an infinite network. It has a regulation parameter R of 4% (0.04 pu). By how much will the turbine power increase if the frequency drops by 0.1 Hz with the reference unchanged?

Solⁿ Drop in frequency = 4% of 60 Hz = 2.4 Hz

$$R = \frac{2.4}{100} = 0.024 \text{ Hz/MW}$$

$$\Delta P_{T,0} = -\frac{1}{R} \Delta f_0 = -\frac{1}{0.024} (-0.1) = 4.17 \text{ MW} \quad (\Delta P_{ref} = 0)$$

- (ii) If the frequency drops by 0.1 Hz but the turbine power must remain unchanged, by how much should the reference setting be change?

$$\Delta P_{T,0} = 0$$

$$\Delta P_{T,0} = \frac{1}{R} \Delta f_0 = \frac{1}{0.024} (-0.1) = -4.17 \text{ MW}$$

We command the speed change to lower by 4.17 MW.

- ② Two generators are supplying power to a system. Their ratings are 50 and 500 MW respectively. The frequency is 60 Hz and each generator is half loaded. The system load increases by 10 MW and as a result the frequency drops to 59.5 Hz. What must the individual regulators be if the two generators should increase their turbine powers in proportion to their ratings.

Solⁿ The two gen. should pick up 10 and 100 MW respectively.

$$R = -\frac{-0.5}{10} = 0.05 \text{ Hz/MW}$$

$$R = -\frac{-0.5}{100} = 0.005 \text{ Hz/MW}$$

③ Determine the primary ALFC loop parameters for a control area having the following data.

Total rated capacity $P = 2000 \text{ MW}$.

Normal operating load $P_D = 1000 \text{ MW}$.

Inertia Constant $H = 5.0 \text{ sec}$.

Regulation $R = 2.40 \text{ Hz/pu MW}$ (all area generators)

Given that Assume load frequency dependency to be linear.

Solⁿ let Increase in load 1% for 1% frequency increase.

$$1\% \text{ of } 1000 \text{ MW} = 10 \text{ MW}$$

$$1\% \text{ of } 60 \text{ Hz} = 0.6 \text{ Hz}$$

$$D = \frac{\Delta P_D}{\Delta f} = \frac{10}{0.6} = 16.67 \text{ MW/Hz}$$

$$D \text{ in pu} = \frac{16.67}{2000} = 8.33 \times 10^{-3} \text{ pu MW/Hz}$$

$$K_p = \frac{1}{D} = \frac{1}{8.33 \times 10^{-3}} = 120 \text{ Hz/pu MW}$$

$$T_p = \frac{2H}{f^0 D} = \frac{2 \times 5}{60 \times 8.33 \times 10^{-3}} = 20 \text{ sec}$$

⑥ Find the static frequency drop for the 2 GW system following 1% load increase

$$\Delta P_D = m = 20 \text{ MW} = \frac{20 \times 10^6}{2 \times 10^9} = 0.01 \text{ pu MW}$$

$$\Delta f = -\frac{\Delta P_D}{\beta} = -\frac{\Delta P_D}{D + \frac{1}{R}} = -\frac{0.01}{8.33 \times 10^{-3} + \frac{1}{2.4}} = -\frac{0.01}{0.425} = -0.0235 \text{ Hz}$$

⑦ what would the frequency drop if the speed governor loop were non-existent or open.

Solⁿ $R = \infty$

$$\Delta f = -\frac{\Delta P_D}{\beta} = -\frac{\Delta P_D}{D + \frac{1}{\infty}} = -\frac{\Delta P_D}{D} = -\frac{0.01}{8.33 \times 10^{-3}} = -1.2 \text{ Hz}$$

$$2\% \text{ of } 60 \text{ Hz rated frequency } \left(\frac{1.2}{60} = 0.02 \text{ Hz} \right)$$

③ A Subgrid has total rated Capacity 3000 MW. It encounters a
 (a) load increase of 40 MW when the normal operating level is 2000 MW.
 Assume inertia Constant (H) to be 5 sec and Regulation of generator in the system as 3 Hz/pu MW.

Find (i) ALFC loop parameters (T_p, K_p)

(ii) Static frequency drop

(iii) Transient response of ALFC loop.

Given that Assume load frequency dependency to be linear.

Solⁿ
 Normal operating level = 2000 MW.

(i) So 40 MW is $\frac{40}{2000} = 0.02$ i.e. 2% of 2000 MW
 $\therefore$ 2% of 50 Hz is 1 Hz.

$$K_p = \frac{1}{D}$$

$$D = \frac{\partial P_D}{\partial f} = \frac{40}{1} = 40 \text{ MW/Hz}$$

$$D \text{ in pu} = \frac{40}{3000} = 13.33 \times 10^{-3} \text{ pu MW/Hz}$$

$$K_p = \frac{1}{D} = \frac{1}{13.33 \times 10^{-3}} = 75 \text{ Hz/pu MW}$$

$$T_p = \frac{2H}{f^0 D} = \frac{2 \times 5}{50 \times 13.33 \times 10^{-3}} = 15 \text{ sec.}$$

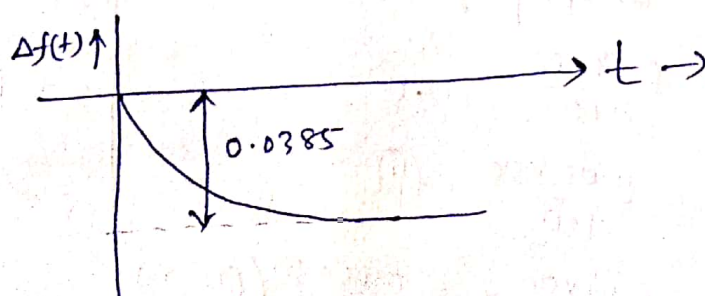
$$(ii) \Delta f = \frac{-\Delta P_D}{\beta} = \frac{-\Delta P_D}{D + \frac{1}{R}} = -\frac{\frac{40}{3000}}{13.33 \times 10^{-3} + \frac{1}{3}} = -0.0384 \text{ Hz}$$

i.e. 0.0384% of 50 Hz rated frequency ($\frac{0.0384}{50} = 0.000768$)

$$(iii) \Delta f(t) = \frac{-\Delta P_D R K_p}{R + K_p} \left(1 - e^{-\left(\frac{R + K_p}{R T_p}\right)t} \right)$$

$$= \frac{-\frac{40}{3000} \times 3 \times 75}{3 + 75} \left(1 - e^{-\left(\frac{3 + 75}{3 \times 15}\right)t} \right)$$

$$= -0.0385 (1 - e^{-1.3333t}) \text{ Hz.}$$



⑥ If the regulation is to be reduced from 3 Hz/p.u.mw to 2 Hz/p.u.mw. what is the effect on system response time on static frequency drop.

when $R = 3 \text{ Hz/p.u.mw}$

$$\Delta f(t) = -0.0385 (1 - e^{-1.7333t})$$

The time response of the loop $= \frac{1}{1.7333} = 0.577 \text{ sec.}$

and static drop is 0.0385

If $R_{\text{new}} = 2 \text{ Hz/p.u.mw}$

$$\Delta f(t) = \Delta f = \frac{-\Delta P_D}{R + \frac{1}{R}} = \frac{-\frac{40}{3000}}{(13.33 \times 10^3 + \frac{1}{2})} = -0.026 \text{ Hz.}$$

Static frequency drop reduces on reducing R

for time response,

$$\Delta f(t) = \frac{-\Delta P_D}{R + \frac{1}{R}} (1 - e^{-(\frac{R+K_D}{RT_P})t})$$

$$= -0.026 (1 - e^{-2.57t})$$

$$\text{Time response} = \frac{1}{2.57} = 0.3896 \text{ sec.}$$

on reducing R time response reduces.

④ Two generators rated 200 MW and 400 MW are operating in parallel. The droop characteristics of their governors are 4% and 5% respectively from no load to full load. Assuming that the generators are operating at 50 Hz at no load, how would a load of 600 MW be shared between them. What will be the system frequency at this load.

Solⁿ Since generators are in parallel, they will operate at the same frequency at steady load.

Let Load on ^{200 MW} generator = $x \text{ MW}$

and Load on 400 MW generator = $(600 - x) \text{ MW.}$

Reduction in frequency = Δf

$$\text{Now } \frac{\Delta f}{x} = \frac{0.04 \times 50}{200} \quad \text{--- (I)}$$

$$\frac{\Delta f}{600 - x} = \frac{0.05 \times 50}{400} \quad \text{--- (II)}$$

$$\text{equally } \Delta f \text{ in eqn (I) \& (II), } \frac{0.04 \times 50}{200} x = \frac{0.05 \times 50}{400} (600 - x)$$

$$0.01x = 0.0625 \frac{600 - x}{160}$$

$$1.6x = 600 - x \quad \Rightarrow \quad \frac{2.6x}{1} = \frac{600}{1} \quad \Rightarrow \quad x = \frac{600}{2.6} = 231 \text{ MW load on 200 MW gen.}$$

$$600 - x = 600 - 231 = 369 \text{ MW load on 400 MW gen.}$$

System frequency

$$= 50 - \frac{0.04 \times 50}{200} \times 231 = 49.69 \text{ Hz.}$$

⑤ Consider two power areas which may be interconnected by the line for pool operation. Assume no loss in the system.

Area-1 has following specifications

Rated Capacity = 3000 MW
 Operating power = 2000 MW
 Inertia Constant = 5 sec
 Regulation $R = 3 \text{ Hz/pu MW}$

Area-2

Rated Capacity = 9000 MW
 Operating power = 2500 MW
 Inertia Constant = 8 sec
 $R = 2.5 \text{ Hz/pu MW}$

- ② Assume 2% operating load increase of each area separately obtain individual static frequency drop if the two areas are not interconnected.
- ③ Assume the load increase on area 1 only but the two areas remains interconnected.
- ④ Assume the load increase on area 2 only but the two areas are interconnected. Assuming the load frequency dependency to be linear.

Solⁿ

② Area-1

2% of 2000 MW = 40 MW

2% of 50 Hz = 1 Hz

$$D_1 = \frac{\partial P_D}{\partial f} = \frac{40}{1} = 40 \text{ MW/Hz}$$

$$D_1 \text{ in pu} = \frac{40}{2000} = \frac{1}{50} \text{ pu MW/Hz}$$

$$K_{P1} = \frac{1}{D_1} = \frac{1}{1/50} = 50 \text{ Hz/pu MW}$$

$$\beta_1 = D_1 + \frac{1}{K_{P1}} = \frac{1}{50} + \frac{1}{50} = 0.04 \text{ pu MW/Hz}$$

$$\Delta f_1 = \frac{-\Delta P_{D1}}{\beta_1} = \frac{-1/50}{0.04} = -0.0385 \text{ Hz}$$

$$T_{P1} = \frac{2H}{f_0 D_1} = \frac{2 \times 5}{50 \times (1/50)} = 10 \text{ sec}$$

③ $\Delta P_{D1} = m_1 = \frac{40}{2000} = 0.02 \text{ pu MW}$

$$\beta_1 = 0.04 \text{ pu MW/Hz}$$

$$\beta_2 = 0.0417 \times \frac{9000}{2000} = 1.875 \text{ pu MW/Hz}$$

on area-1 power base

$$m_2 = 0$$

$$\Delta f = \frac{-(m_1 + m_2)}{\beta_1 + \beta_2} = \frac{-0.02}{0.04 + 1.875} = -0.0105 \text{ Hz}$$

$$\Delta P_{12} = \frac{\beta_2 m_1 - \beta_1 m_2}{\beta_1 + \beta_2} = \frac{-1.875 \times 0.02}{0.04 + 1.875} = -0.02 \text{ pu MW}$$

Area-2 2% of 2500 MW = 50 MW

$$D_2 = \frac{\partial P_D}{\partial f} = \frac{50}{1} = 50 \text{ MW/Hz}$$

$$D_2 \text{ in pu} = \frac{50}{2500} = \frac{1}{50} \text{ pu MW/Hz}$$

$$K_{P2} = \frac{1}{D_2} = \frac{1}{1/50} = 50 \text{ Hz/pu MW}$$

$$T_{P2} = \frac{2H}{f_0 D_2} = \frac{2 \times 8}{50 \times (1/50)} = 16 \text{ sec}$$

$$\beta_2 = D_2 + \frac{1}{K_{P2}} = \frac{1}{50} + \frac{1}{50} = 0.0417 \text{ pu MW/Hz}$$

$$\Delta f_2 = \frac{-\Delta P_{D2}}{\beta_2} = \frac{-1/50}{0.0417} = -0.04 \text{ Hz}$$

$$③ \Delta P_{D2} = m_2 = \frac{150}{9000} = 0.01667 \text{ pu} \cdot \text{MW}$$

$$\beta_2 = 0.417 \text{ pu} \cdot \text{MW} / \text{Hz}$$

$$\beta_1 = 0.317 \times \frac{3000}{9000} = 0.1157 \text{ (on area 2 power base)}$$

$$\Delta f = \frac{-(m_1 + m_2)}{\beta_1 + \beta_2} = \frac{-0.01667}{0.1157 + 0.417} = -0.0313 \text{ Hz}$$

$$\Delta P_2 = \frac{\beta_1 m_2 - \beta_2 m_1}{\beta_1 + \beta_2} = \frac{0.1157 \times 0.01667}{0.1157 + 0.417} = 32.57 \text{ MW}$$

It is observed that in case ③ and ④ though power transfer through tie line is nearly same, the frequency drop improvement of area 1 is remarkable while that of area 2 is marginal. This is because of the fact that while area-1 encounters load increases, distance of area 2 is of greater effects because of its higher capacity. Thus interconnection of any area with higher capacity area is always advantages.

⑥ Two area systems are interconnected by a tie line with the following characteristics.

Area 1: $R = 0.01 \text{ pu}$, $D = 0.8 \text{ pu}$, Base $\text{MW} = 500$

Area 2: $R = 0.02 \text{ pu}$, $D = 1.0 \text{ pu}$, Base $\text{MW} = 500$

A load change of 100 MW (0.2 pu) occurs in area 1. What is the new steady state frequency and what is the change in tie flow. Assume both areas were at nominal frequency 60 Hz to begin.

Solⁿ

$$\Delta P_{D1} = m_1 = \frac{100}{500} = 0.2 \text{ pu} \cdot \text{MW}$$

$$\beta_1 = D_1 + \frac{1}{R_1} = 0.8 + \frac{1}{0.01} = 100.8 \text{ pu}$$

$$\beta_2 = D_2 + \frac{1}{R_2} = 1 + \frac{1}{0.02} = 51 \text{ pu}$$

$$\Delta f = \frac{-(m_1 + m_2)}{\beta_1 + \beta_2} = \frac{-0.2}{100.8 + 51} = -0.00131752 \text{ pu}$$

$$f_{\text{new}} = 60 - (0.00132 \times 60) = 60 - 0.079 = 59.92 \text{ Hz}$$

$$\begin{aligned} \Delta P_1 &= \frac{-\beta_2 m_1}{\beta_1 + \beta_2} = \frac{-51 \times 0.2}{100.8 + 51} = -0.0672 \text{ pu} \\ &= -0.0672 \times 500 \\ &= -33.6 \text{ MW} \end{aligned}$$

Small Signal Stability Analysis:-

Dynamics of a synchronous machine:-

The kinetic energy of the rotor of synchronous speed is

$$KE = \frac{1}{2} J \omega_{sm}^2 \times 10^6 \text{ MJ (megajoules)}$$

where J = Rotor moment of inertia in kg-m^2

ω_{sm} = Synchron. speed in rad (mech)/sec

But $\omega_s = (\frac{p}{2}) \omega_{sm}$ = Rotor speed in rad (elect)/sec.

where p = number of poles in the m/c.

$$KE = \frac{1}{2} [J (\frac{2}{p})^2 \omega_s \times 10^6] \omega_s$$

$$= \frac{1}{2} m \omega_s$$

where $m = J (\frac{2}{p})^2 \omega_s \times 10^6$ = moment of inertia in MJ-s/elect.rad

We shall define the inertia constant H such that

$$GH = KE = \frac{1}{2} m \omega_s, \text{ MJ}$$

where G = machine rating (base) in MVA (3- ϕ)

H = inertia constant in MJ/MVA or MW-s/MVA.

It immediately follows that

$$m = \frac{2GH}{\omega_s} = \frac{GH}{\pi f} \text{ MJ-s/elect.rad.} \\ = \frac{GH}{180f} \text{ MJ-s/elect.degree} \quad \text{--- (1)}$$

where m is called the inertia constant

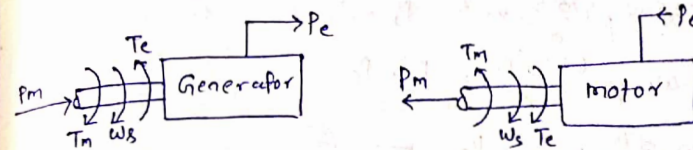
Taking G as base, the inertia constant in per unit (pu) is

$$M(\text{pu}) = \frac{H}{\pi f} \text{ s}^2/\text{elect-rad} \\ = \frac{H}{180f} \text{ s}^2/\text{elect.degree} \quad \text{--- (2)}$$

The inertia constant H has a characteristic value or a range of values for each class of machines.

Dilip Khamari

The Swing Equation:-



(Flow of mech. and elect. power in a synch m/c)

The above figure shows the torque, speed and flow of mechanical and electrical powers in a synch. It is assumed that the windage, friction and iron loss torque is negligible.

The differential eqn governing the rotor dynamic can be written as, $J \frac{d^2 \theta_m}{dt^2} = T_m - T_e$ N-m --- (3)

where θ_m = angle in radian (mech)

T_m = Turbine torque in N-m (it acquires -ve value for a motoring m/c.)

T_e = Electromagnetic torque developed, N-m. While the rotor undergoes dynamics as per eqn (3), the rotor speed changes by significant magnitude for the time period of interest 1 sec. The eqn (3) can then be converted into its more convenient power form by assuming the rotor speed to remain constant at its synchronous speed (ω_{sm}).

Multiplying both sides of the eqn by ω_{sm} , we can write

$$J \omega_{sm} \frac{d^2 \theta_m}{dt^2} \times 10^6 = T_m \omega_{sm} - T_e \omega_{sm} \\ = P_m - P_e \text{ MW} \quad \text{--- (4)}$$

where P_m = mech power I/p in MW

P_e = electrical power o/p in MW

Stator cu-loss is assumed to be negligible.

$$\text{Some } J \omega_{sm} \frac{d^2 \theta_m}{dt^2} \times 10^6 = P_m - P_e \quad \text{--- (5)}$$

$$\omega_s = \frac{p}{2} \omega_{sm} \Rightarrow \omega_{sm} = \frac{2}{p} \omega_s$$

$$\text{Also } \theta_e = \frac{p}{2} \theta_m \Rightarrow \frac{d\theta_e}{dt} = \frac{p}{2} \frac{d\theta_m}{dt}$$

$$\Rightarrow \frac{d^2 \theta_e}{dt^2} = \left(\frac{p}{2}\right) \frac{d^2 \theta_m}{dt^2}$$

$$\Rightarrow \frac{d^2 \theta_m}{dt^2} = \left(\frac{2}{p}\right) \frac{d^2 \theta_e}{dt^2} \quad \text{--- (6)}$$

Using eqn (5) & (6) we get

$$J\left(\frac{2}{p}\right)\omega_s \cdot \left(\frac{2}{p}\right) \frac{d^2\theta_e}{dt^2} \times 10^{-6} = P_m - P_e \quad (6)$$

$$\Rightarrow \left[J\left(\frac{2}{p}\right)^2 \omega_s \times 10^{-6} \right] \frac{d^2\theta_e}{dt^2} = P_m - P_e$$

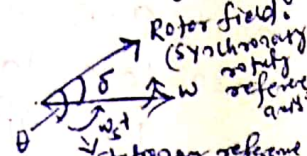
$$\Rightarrow m \frac{d^2\theta_e}{dt^2} = P_m - P_e \quad \text{--- (7)}$$

where $m = J\left(\frac{2}{p}\right)^2 \omega_s \times 10^{-6} = m \cdot I$ of the m/c.

It is more convenient to measure the angular position of the rotor w.r.t a synchronously rotating reference frame.

$$\text{Let } \delta = \theta_e - \omega_s t \quad \because \theta_e = \delta + \omega_s t$$

= Rotor angular disp. from synchronously rotating ref. frame called as torque angle or rotor angle



$$\frac{d^2\delta}{dt^2} = \frac{d^2\theta_e}{dt^2} \quad \begin{cases} \frac{d\delta}{dt} = \frac{d}{dt}(\theta_e - \omega_s t) = \frac{d\theta_e}{dt} - \omega_s \\ \frac{d^2\delta}{dt^2} = \frac{d^2\theta_e}{dt^2} - 0 \quad \because \omega_s = 2\pi f \end{cases}$$

Now eqn (7) can be written as

$$m \frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{m.u.} \quad \text{--- (8)}$$

$$\Rightarrow \frac{G_H}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{m.u.} \quad \text{--- (9)} \quad \because m = \frac{G_H}{\pi f}$$

Dividing throughout by G_H , the MVA rating of the m/c

$$\boxed{M(\text{pu}) \frac{d^2\delta}{dt^2} = P_m - P_e} \quad \text{in p.u. of m/c rating as base} \quad \text{--- (10)}$$

$$\text{where } M(\text{pu}) = \frac{H}{\pi f}$$

$$\boxed{\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e, \text{ pu}} \quad \text{--- (11)}$$

The above eqn (10) & (11) is called the swing eqn and it describes the rotor dynamics for a synch. m/c.

It is a 2nd order differential eqn where the damping term (proportional to $d\delta/dt$) is absent because of the assumption of a lossless m/c and the fact that the torque of the damper winding has been ignored. This assumption leads to pessimistic results in transient stability analysis - damping helps to stabilize the system. Damping must of course be considered in a dynamic stability study. Since the electrical power P_e depends upon the sine of angle δ ($\sin\delta$), the swing eqn is a non-linear second order

: Dillip Khamari

Q1 A 50Hz 4 pole turbo generator rated 100 MVA 11kV has an inertia constant of 8.0 MJ/MVA.

- Find the stored energy in the rotor at synch. speed.
- If the mechanical input is suddenly raised to 80 MW for an electrical load of 50 MW, find rotor acceleration neglecting mech & electrical losses.
- If the acceleration calculated in part (b) is maintained for 10 cycles, find the change in torque angle and rotor speed in revolution per minute at the end of this period.

Solⁿ $g = 100 \text{ MVA}$, $H = 8.0 \text{ MJ/MVA}$.

- Energy stored $= GH = 100 \times 8 = 800 \text{ MJ}$
- Accelerating power $P_a = P_m (\text{mech power}) - P_e$
 $= 80 - 50 = 30 \text{ MW}$

$$\text{Accelerating power } P_a = m \frac{d^2\delta}{dt^2}$$

$$\text{Since } m = \frac{GH}{180f} = \frac{800}{180 \times 50} = \frac{4}{45} = 0.088 \text{ (MJ-s/elekt-degree)}$$

$$P_a = m \frac{d^2\delta}{dt^2}$$

$$\Rightarrow \frac{4}{45} \frac{d^2\delta}{dt^2} = 30$$

$$\frac{d^2\delta}{dt^2} = \frac{30 \times 45}{4} = 337.5 \text{ (elekt-degree/sec}^2\text{)}$$

$$\text{Rotor acceleration } \alpha = \frac{d^2\delta}{dt^2} = 337.5 \text{ (elekt-degree/sec}^2\text{)}$$

$$\text{10 cycle} = 0.2 \text{ sec} \quad \left\{ \begin{array}{l} 1 \text{ cycle} = 50 \text{ Hz} \\ = \frac{1}{50} \text{ sec} \\ = 20 \text{ msec} \end{array} \right.$$

$$\text{change in } \delta = \frac{1}{2} \alpha t^2$$

$$= \frac{1}{2} \frac{d^2\delta}{dt^2} t^2$$

$$\Delta\delta = \frac{1}{2} \times 337.5 \times 0.2^2 = 6.75 \text{ elekt-deg.}$$

$$\Delta\delta = \frac{60 \times 6.75}{2 \times 50} = 4.05 \text{ rpm/sec}$$

Rotor speed at the end of 10 cycles

$$= \frac{120 \times 50}{4} + 4.05 \times 0.2$$

$$= 1500 + 0.81 \text{ rpm}$$

$$\left\{ \begin{array}{l} N_s = \frac{120f}{P} \\ \theta = \delta + \omega t \quad (\delta = 0 \text{ at } t = 0) \end{array} \right.$$

$$\alpha = \frac{d^2\delta}{dt^2} = \frac{d}{dt} \left(\frac{d\delta}{dt} \right)$$

$$\int d\alpha = \int \frac{d}{dt} \left(\frac{d\delta}{dt} \right) dt$$

$$= \int \frac{d}{dt} \left(\frac{d\delta}{dt} \right) dt = \frac{d\delta}{dt}$$

$$\alpha t = \frac{d\delta}{dt}$$

$$\int \alpha t dt = \int d\delta$$

$$\frac{\alpha t^2}{2} = \Delta\delta$$

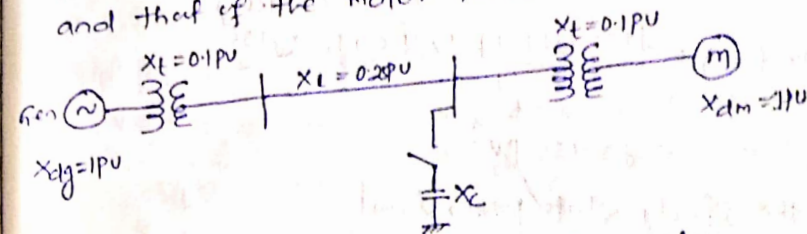
$$\Delta\delta = \frac{1}{2} \alpha t^2$$

Steady state stability:-

The method of improving steady state stability limit of a system are

- To reduce X (Transient reactance of T.L)
- Increase either or both generating voltage $|E|$ & Terminal voltage $|V|$.
- Using two parallel transmission line.
- Series capacitors are used for better voltage regulation and to increase stability limit by decreasing line reactance.
- Higher excitation voltage or quick excitation system.

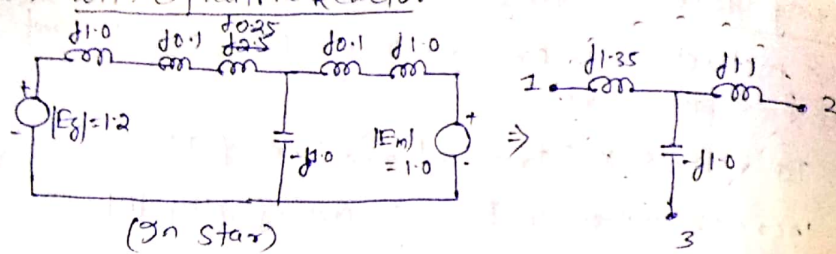
Q2 In the system shown in fig. below, a 5- ϕ static capacitive reactor of reactance 1 pu/phase is connected through a switch at motor bus bar. Calculate the limit of steady state power with and without reactor switch closed. Recalculate the power limit with capacitive reactor replaced by inductive reactor of the same value. Assume the internal voltage of the generator to be 1.2 pu and that of the motor to be 1.0 pu.



Solⁿ Steady state power limit without reactor

$$= \frac{|E_g| |E_m|}{X_{total}} = \frac{1.2 \times 1}{(1 + 0.1 + 0.25 + 0.1 + 1)} = 0.49 \text{ pu}$$

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Converting from star to delta,

$$jX_{T_{\text{transf}}} = \frac{j1.35 \times j1.1 + j1.1 \times (-j1.0) + (-j1.0) \times j1.35}{-j1.0}$$

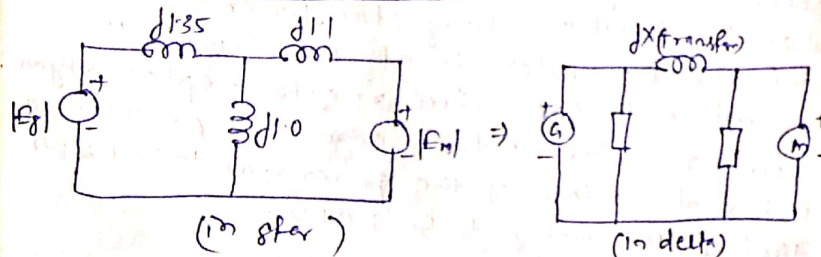
$$= j0.965$$

$$\textcircled{c} \quad j1.35 \parallel j1.1 + \frac{j1.35 \times j1.1}{-j1.0}$$

steady state power limit

$$P_e = \frac{|E_g| |E_m|}{X_{T_{\text{transf}}}} = \frac{1.2 \times 1.0}{0.965} = 1.244 \text{ p.u.}$$

(iii) with inductive reactance



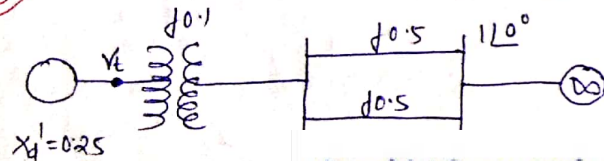
$$jX_{T_{\text{transform}}} = \frac{j1.35 \times j1.1 + j1.1 \times j1.0 + j1.0 \times j1.35}{j1.0}$$

$$= j3.935 \text{ p.u.}$$

Now the steady state power limit

$$P_e = \frac{1.2 \times 1.0}{3.935} = 0.304 \text{ p.u.}$$

Q3:-



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The generator of the above figure-2 is delivering 1.0 pu power to the infinite bus ($|V| = 1.0 \text{ pu}$), with the generator terminal voltage of $|V_t| = 1.0 \text{ pu}$. Calculate the generator emf behind transient reactance. Find the max power that can be transferred under the following conditions.

(a) system healthy.

(b) one line shorted (3-phase) in the middle.

(c) one line open.

plot all the three power angle curves.

$$\text{sol}^n \quad \text{Let } V_t = |V_t| \angle \alpha = 1 \angle \alpha$$

From the power angle eqn

$$P_e = \frac{|V_t| |V|}{X} \sin \alpha$$

$$\Rightarrow P_e = \frac{1 \times 1}{0.25 + 0.1} \sin \alpha$$

$$\Rightarrow 1 = \frac{1 \times 1}{0.35} \sin \alpha$$

$$\therefore 0.5 \parallel 0.5 = 0.25$$

$$\text{So } \alpha = 20.5^\circ$$

Current into infinite bus

$$I = \frac{|V_t| \angle \alpha - |V| \angle 0}{jX}$$

$$= \frac{1 \angle 20.5^\circ - 1 \angle 0^\circ}{j0.35}$$

$$= 1 + j0.18 = 1.016 \angle 10.3^\circ$$

$$\therefore X = j0.1 + (j0.5 \parallel j0.5) = j0.1 + \frac{j0.5 \times j0.5}{2 \times j0.5} = j0.1 + j0.25 = j0.35$$

$$= j0.1 + j0.25 = j0.35$$

voltage behind transient reactance,

$$E' = 1 \angle 0 + j0.6 \times I$$

$$= 1 \angle 0 + j0.6 \times 1.016 \angle 10.3^\circ$$

$$= 1.075 \angle 33.9^\circ$$

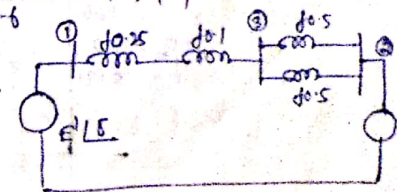
$$jX = j0.25 + j0.25 + j0.1 = j0.6$$

(a) system healthy

$$P_{\text{max}} = \frac{|V| |E'|}{X_{12}} = \frac{1 \times 1.075}{0.6} = 1.79 \text{ pu.}$$

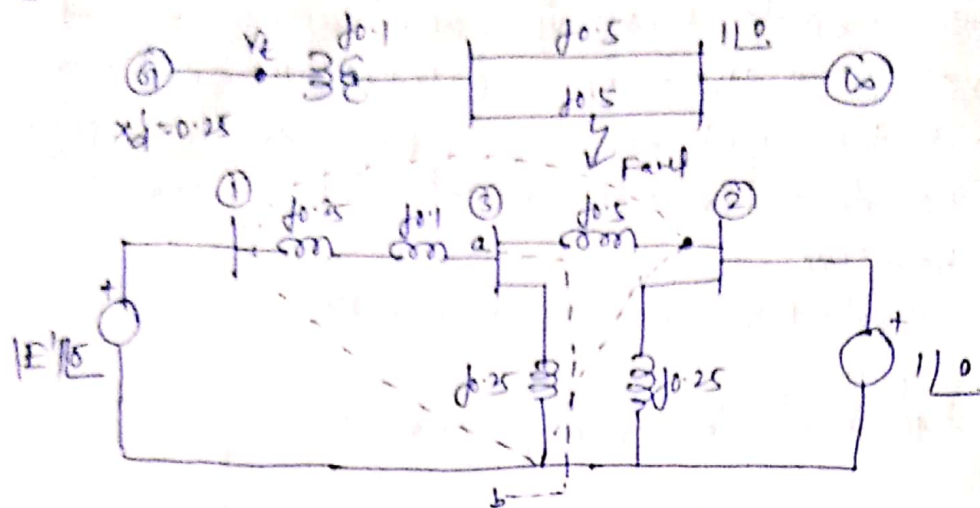
$$P_e = 1.79 \sin \delta$$

$$\therefore X_{12} = j0.25 + j0.1 + j0.25 = j0.6$$



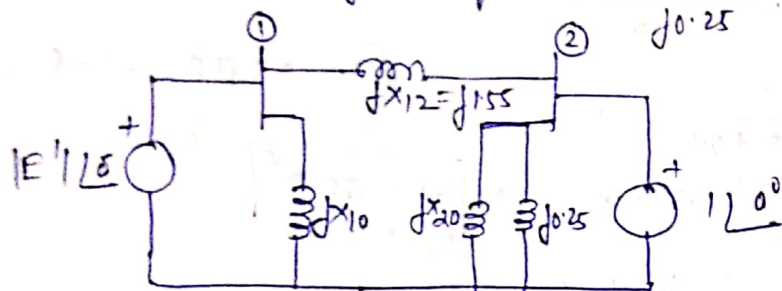
A simple system with its reactive diagram.

(b) one line shorted in the middle:



By Star-Delta Conversion

$$X_{12} = j0.35 + j0.5 + \frac{j0.35 \times j0.5}{j0.25} = j1.55$$



$$P_{max} = \frac{1 \times 1.075}{1.55} = 0.694 \text{ pu}$$

$$P_e = P_{max} \sin \delta$$

$$\Rightarrow 1 = 0.694 \sin \delta$$

$$\delta =$$

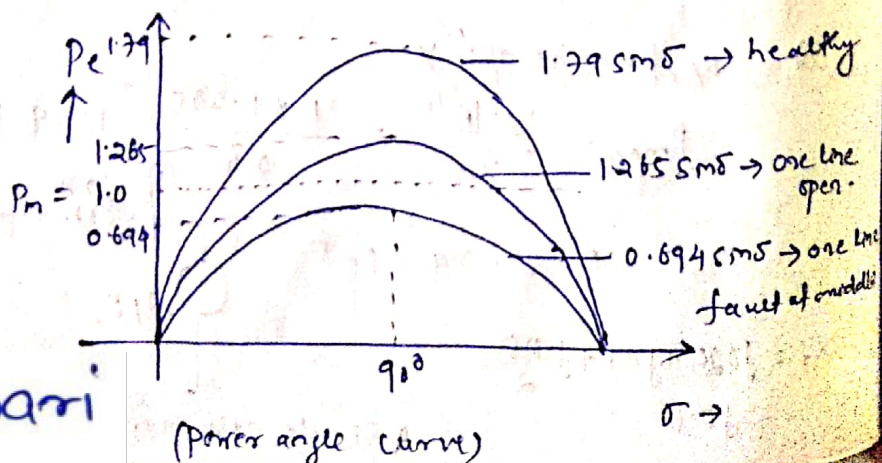
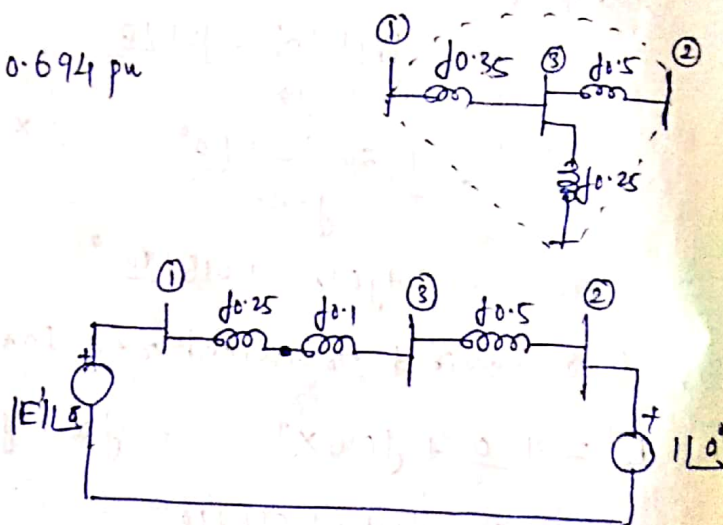
(c) one line open

$$X_{12} = j0.25 + j0.1 + j0.5 = j0.85$$

$$P_{max} = \frac{1 \times 1.075}{0.85} = 1.265 \text{ pu}$$

$$P_e = 1.265 \sin \delta$$

$$\delta =$$



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Steady State Stability:-

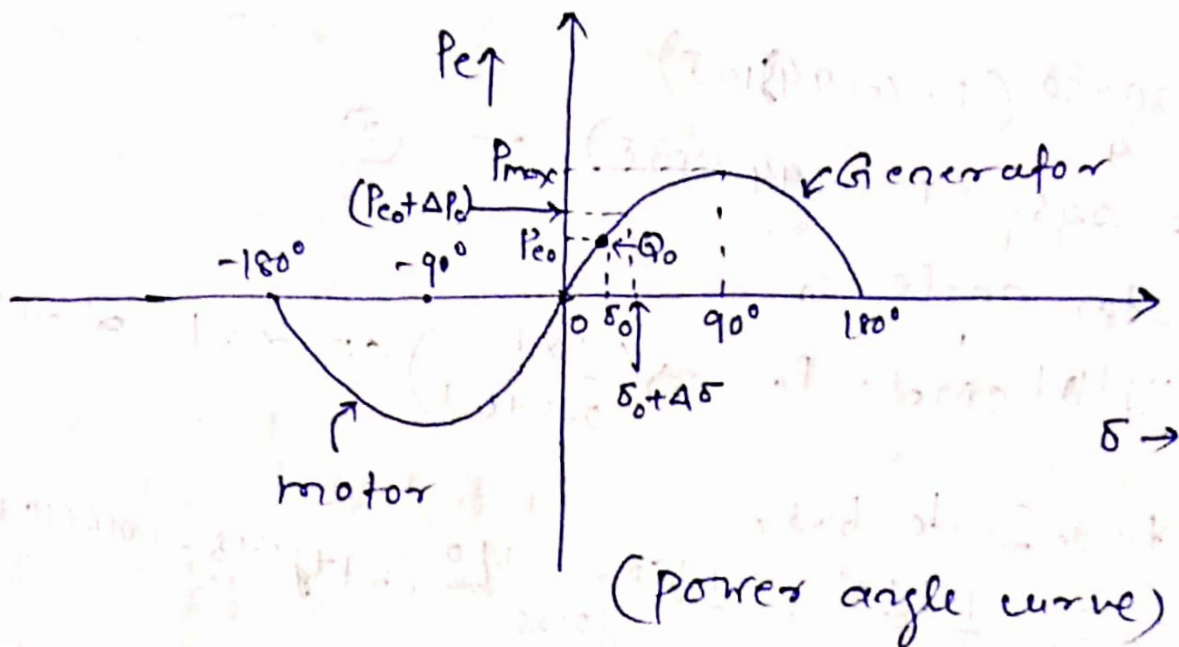
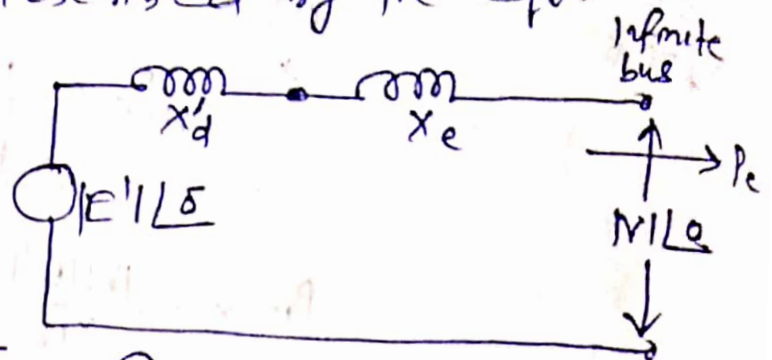
It is defined as the max^m power that can be transmitted to the receiving end without loss of synchronism. (16)

Consider a simple system (m/c) connected to infinite bus whose dynamics is described by the eqn

$$m \frac{d^2\delta}{dt^2} = (P_m - P_e); \text{ m.u.} \quad \text{--- (1)}$$

$m = \frac{H}{\pi f}$ in pu system

$$\text{and } P_e = \frac{|E| |V|}{X_d} \sin \delta = P_{\max} \sin \delta \quad \text{--- (2)}$$



(25)

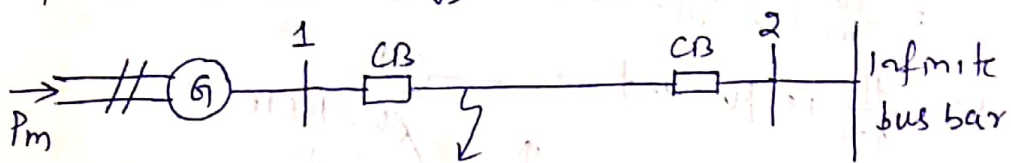
Transient stability:- Single synch m/c is connected to infinite bus bars. The non-linear differential eqnⁿ can be written as

$$m \frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{where } P_e = P_{\max} \sin\delta$$

$$m \frac{d^2\delta}{dt^2} = P_m - P_{\max} \sin\delta \quad \text{--- (1)}$$

The above eqnⁿ (1) is known as swing eqnⁿ.

No generalized criteria are available for determining system stability with large disturbances (called transient stability).



upon occurrence of a severe disturbance, say a short circuit, the power transfer between the machines is greatly reduced, causing the machine torque angles to swing relatively. The circuit breakers near the fault disconnect the unhealthy part of the system so that power transfer can be partially restored, improving the chances of the system remaining stable. The shorter the time to breaker operating called clearing time, the higher the probability of the system being stable. Most of the line faults are transient in nature and get cleared on opening the line. Therefore it is common practice now to employ autoreclose breakers which automatically close rapidly after each of the two sequential openings. If the fault still persists, the circuit breakers open and lock permanently till cleared manually. Since in the majority of faults the first reclosure will be successful, the chances of system stability are greatly enhanced by using autoreclose breakers.

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* Sudden disturbances in transient stability can be solved by using Equal Area Criteria.

Equal Area Criteria:-

In a system where one machine is swinging with respect to an infinite bus, it is possible to study transient stability by means of a simple criterion without resorting to the numerical solution of a swing equation.

Consider the swing eqn

$$\frac{d^2\delta}{dt^2} = \frac{1}{m} (P_m - P_e)$$

$$= \frac{P_a}{m} ; P_a = \text{accelerating power.}$$

$$m = \frac{H}{\pi f} \text{ in pu system.} \quad \text{--- (1)}$$

If the system is unstable δ continuously to increase indefinitely with time and the machine loses synchronism.

If the system is stable it performs oscillations (non-sinusoidal) whose amplitude decreases in actual practice because of damping terms.

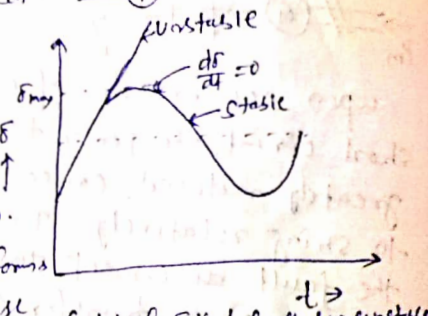
For a stable system, indication of stability will be given by observation of the first swing where δ will go to a max and will start to reduce. This fact can be stated as a stability criterion, that the system is stable if at some time

$$\frac{d\delta}{dt} = 0 \quad \text{--- (2)}$$

and is unstable if $\frac{d\delta}{dt} > 0$ --- (3)

For a sufficiently long time (more than 1s will generally be)

The stability criterion for power system can be converted into a simple and easily applicable form for a single machine infinite bus system.



$$\frac{d^2\delta}{dt^2} = \frac{P_a}{m} \quad \text{--- (4)}$$

multiply both side by of the swing eqn by $2 \frac{d\delta}{dt}$ we get,

$$2 \frac{d\delta}{dt} \frac{d^2\delta}{dt^2} = \frac{2 P_a}{m} \frac{d\delta}{dt} \quad \text{--- (5)}$$

integrate both side

$$\int 2 \frac{d\delta}{dt} \frac{d^2\delta}{dt^2} = \int \frac{2 P_a}{m} \frac{d\delta}{dt}$$

$$\Rightarrow \left(\frac{d\delta}{dt} \right)^2 = \frac{2}{m} \int_{\delta_0}^{\delta} P_a d\delta$$

$$\frac{d\delta}{dt} = \left[\frac{2}{m} \int_{\delta_0}^{\delta} P_a d\delta \right]^{1/2} \quad \text{--- (6)}$$

where δ_0 is the initial rotor angle before it begins to swing due to disturbance.

Condition for stability is $\left(\frac{d\delta}{dt} = 0 \right)$

$$\left[\frac{2}{m} \int_{\delta_0}^{\delta} P_a d\delta \right]^{1/2} = 0$$

$$\Rightarrow \int_{\delta_0}^{\delta} P_a d\delta = 0 \quad \text{--- (7)}$$

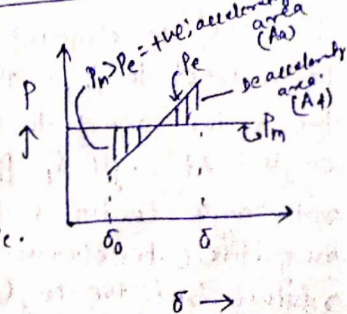
The system is stable if the area under $P_a - \delta$ curve reduces to zero at some value of δ .

On other hand, the positive (accelerating) area under $P_a - \delta$ curve must be equal the negative (decelerating) area and hence the name equal area criterion of stability.

Accelerating area (A_a) = Decelerating area (A_d)

$$A_a = A_d \quad \text{equal area condition}$$

If $A_a = A_d$ the system is stable.

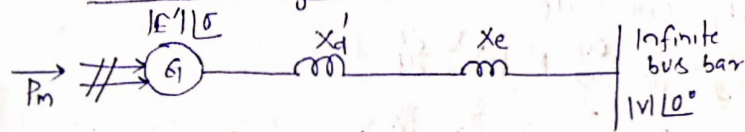


$$P_m - P_e = P_a$$

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For equal area criterion of stability, we now consider several types of disturbances that may occur in a single machine infinite busbar system.

(a) Sudden change in mechanical input:-

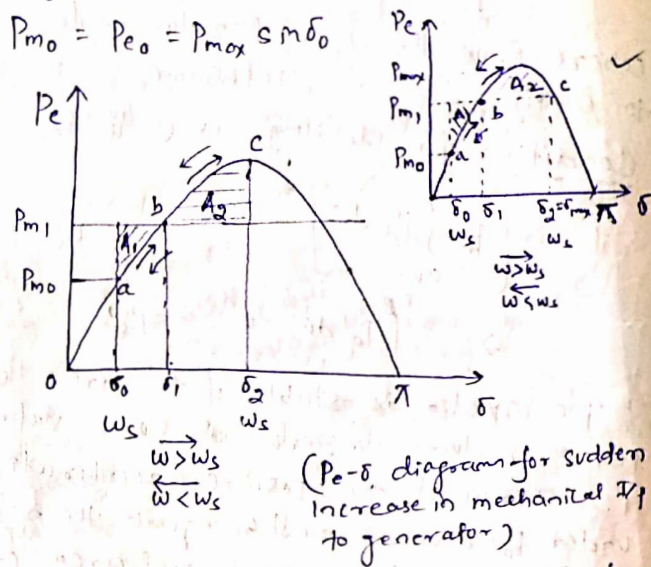


The electrical power transmitted is,

$$P_e = \frac{|E'| |V|}{X_d' + X_e} \sin \delta = P_{max} \sin \delta$$

Under steady state operating condition

$$P_{m0} = P_{e0} = P_{max} \sin \delta_0$$



Let the mechanical input to the rotor be suddenly increased to P_{m1} . The accelerating power $P_a = P_{m1} - P_e$ causes the rotor speed to increase ($\omega > \omega_s$) and so does the rotor angle. At angle δ_1 , $P_a = P_{m1} - P_e = 0$ (State point at b) but the rotor angle continues to increase as $\omega > \omega_s$. P_a now becomes negative (decelerating), the rotor speed begins to reduce but the angle continues to increase till at angle δ_2 , $\omega = \omega_s$ once again (State point at c). At c the decelerating area A_2 equals the accelerating area A_1 ,

$$\text{or } \int_{\delta_0}^{\delta_2} P_a d\delta = 0$$

Since the rotor is decelerating, the speed reduces below ω_s and the rotor angle begins to reduce. The state point now traverses the P_e - δ curve in opposite direction. Then the system oscillates about the new steady state point b ($\delta = \delta_1$) with angle excursion up to δ_0 and δ_2 on the two sides and the system settles to the new steady state where $P_{m1} = P_e = P_{max} \sin \delta_1$.

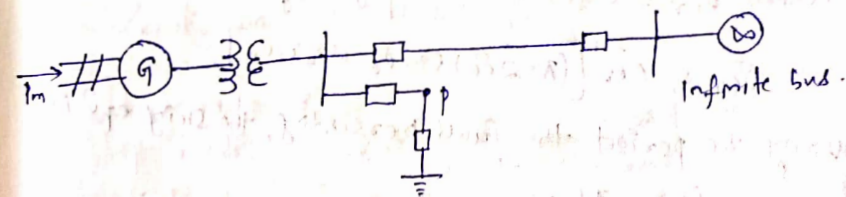
$$\text{Area } A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta \quad \text{and} \quad A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m1}) d\delta$$

For the system to be stable, it should be possible to find out angle δ_2 such that $A_1 = A_2$

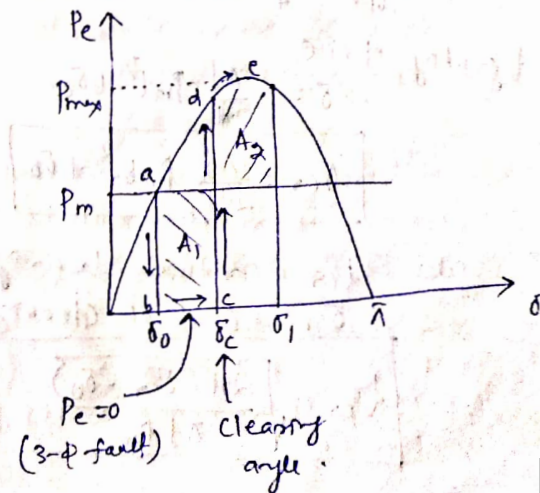
As P_{m1} is increased, a limiting condition is finally reached when A_1 equals the area above the P_{m1} line. Under this condition, δ_2 acquires the max value such that

$$\delta_2 = \delta_{max} = \pi - \delta_1 = \pi - \sin^{-1} \left(\frac{P_{m1}}{P_{max}} \right)$$

(b) Effect of clearing time on stability:-



δ_c = clearing angle.
 t_c = clearing time.



Under fault condition

$$P_e = 0$$

here,

$$\delta_{max} = \pi - \delta_0$$

$$P_m = P_{max} \sin \delta_0$$

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0) d\delta$$

$$= P_m (\delta_{cr} - \delta_0)$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin \delta - P_m) d\delta$$

$$= P_{max} (\cos \delta_{cr} - \cos \delta_{max}) - P_m (\delta_{max} - \delta_{cr})$$

For the system to be stable, $A_2 = A_1$

$$\cos \delta_{cr} = \frac{P_m}{P_{max}} (\delta_{max} - \delta_0) + \cos \delta_{max}$$

where δ_{cr} = critical clearing angle.

$$\delta_{cr} = \cos^{-1} \left[(\pi - 2\delta_0) \sin \delta_0 + \cos \delta_0 \right]$$

During the period the fault persisting, the swing eqⁿ is,

$$\frac{d^2 \delta}{dt^2} = \frac{\pi f}{H} P_m; \quad P_e = 0$$

Integrating thrice

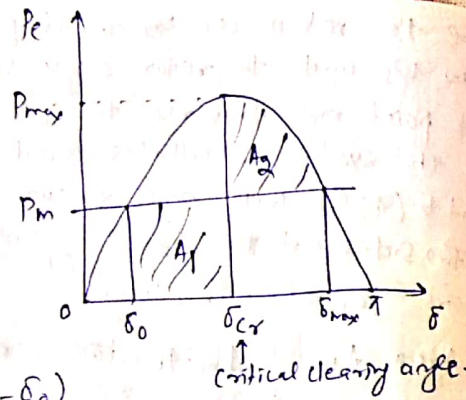
$$\delta = \frac{\pi f}{2H} P_m t^2 + \delta_0$$

$$\delta_{cr} = \frac{\pi f}{2H} P_m t_{cr}^2 + \delta_0$$

where t_{cr} = critical clearing time.

δ_{cr} = critical clearing angle.

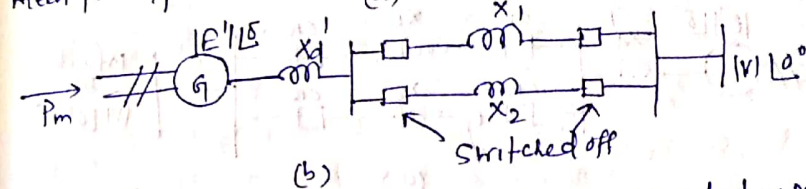
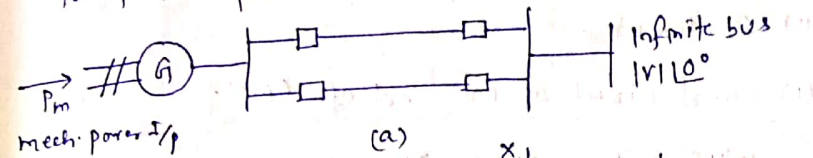
$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m}}$$



(30)

(C) Sudden loss of one parallel line:-

Consider one single machine tied to infinite bus through two parallel lines as shown in fig. below.



(single machine tied to infinite bus through two parallel lines)

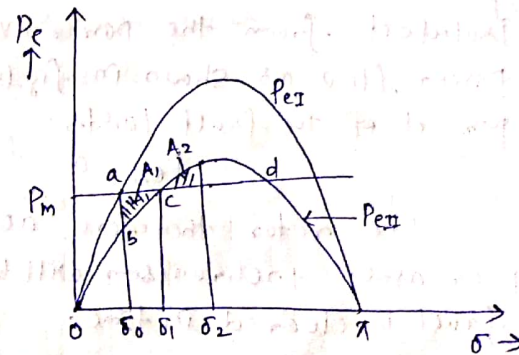
Before one line switched off with the system operating at a steady load.

The power delivered to the load is P_{eI}

$$P_{eI} = \frac{|E'| |V|}{X_d' + (X_1 || X_2)} \sin \delta = P_{maxI} \sin \delta$$

Immediately on switching off line 2, the power delivered to the load is P_{eII}

$$P_{eII} = \frac{|E'| |V|}{X_d' + X_1} \sin \delta = P_{maxII} \sin \delta$$



(Equal area criterion applied to the opening of one of the two lines in parallel)

For the limiting case of stability δ_1

$$\delta_1 = \delta_{max} = \pi - \delta_c$$

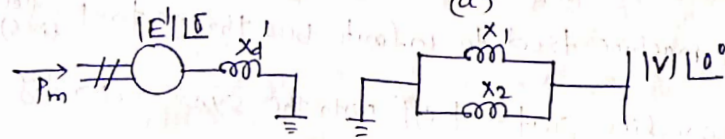
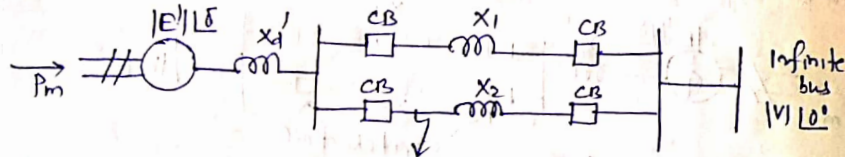
where δ_c = critical clearing angle

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(d) Sudden short circuit on one of parallel lines:-

- Case (i) short circuit at one end of line.
- (ii) Short circuit away from line ends.
- (iii) Rect closure.

Case (i) short circuit at one end of line:-



(Short circuit at one end of the line)

Before the occurrence of a fault, the power angle curve is given by

$$P_{e1} = \frac{|E'| |V|}{X_d' + (X_1 || X_2)} \sin \delta = P_{max} \sin \delta$$

upon the occurrence of a three phase fault at the generator end of line 2, the generator gets isolated from the power system for purposes of power flow as shown in fig (b). Thus during the period of the fault lasts.

$$P_{eII} = 0$$

The rotor therefore accelerates and angle δ increases. Synchronism will be lost unless the fault is cleared in time.

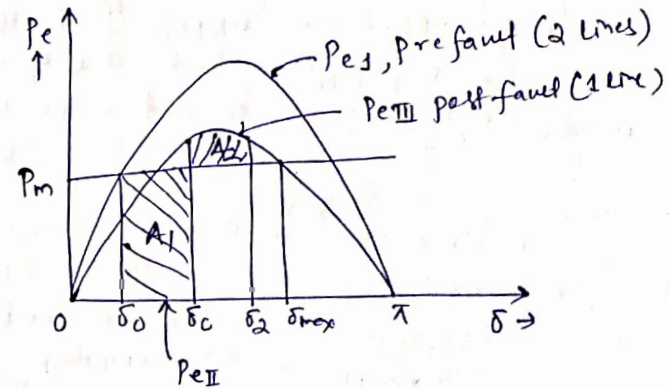
The circuit breakers at the two ends of the faulted line open at time t_c (corresponding to angle δ_c) the clearing time, disconnecting the faulted line.

The power flow is now reestablished via the healthy line (through higher line reactance X_2 in place of $X_1 || X_2$), with power angle curve

$$P_{eIII} = \frac{|E'| |V|}{X_d' + X_2} \sin \delta = P_{maxII} \sin \delta$$

$$\text{Now } P_{maxII} < P_{maxI}$$

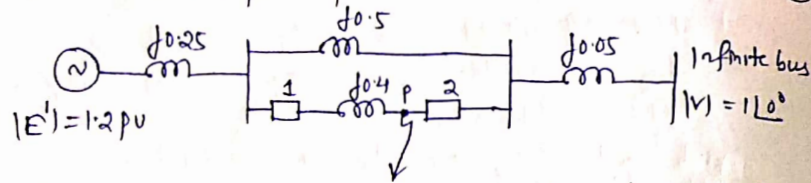
The rotor now starts to decelerate as shown in fig



The system will be stable if a decelerating area can be found equal to accelerating area A_1 before δ reaches the max allowable value δ_{max} .

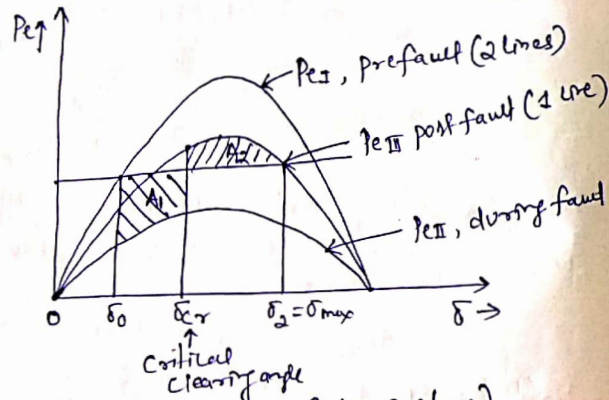
It is to be observed that the equal area criterion helps to determine critical clearing angle and not critical clearing time.

Q Given the system of figure- where a 3- ϕ fault is applied at the point P as shown



Find the critical clearing angle for clearing the fault with simultaneous opening of the breakers 1 and 2. The reactance values of various components are indicated on the diagram. The generator is delivering 1.0 pu power at the instant preceding the fault.

Solⁿ In this case three separate power angle curves are involved.



(Fault on middle of one line of the system)

(i) Normal operation (prefault)

$$X_1 = 0.25 + \frac{0.5 \times 0.4}{0.5 + 0.4} + 0.05$$

$$= 0.522 \text{ pu}$$

$$P_{eI} = \frac{|E'| |V|}{X_1} \sin \delta = \frac{1.2 \times 1}{0.522} \sin \delta$$

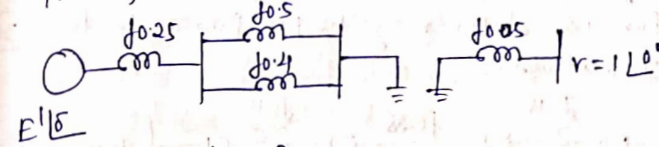
$$= 2.3 \sin \delta$$

prefault operating power angle is given by

$$1.0 = 2.3 \sin \delta_0$$

$$\delta_0 = 25.8^\circ = 0.45 \text{ radian.}$$

(ii) During fault: It is clear that no power is transferred due to the ckt incomplete.



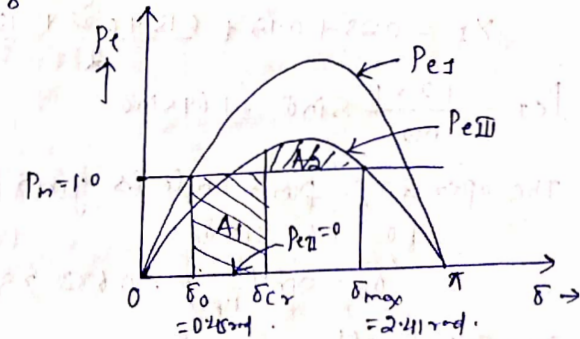
$$\text{or } P_{eII} = 0$$

(iii) post fault operation (fault cleared by opening the faulted line)

$$X_{III} = 0.25 + 0.5 + 0.05$$

$$= 0.8$$

$$P_{eIII} = \frac{1.2 \times 1.0}{0.8} \sin \delta = 1.5 \sin \delta$$



The max^m permissible angle δ_{max} for area $A_1 = A_2$ is given by

$$\delta_{max} = \pi - \sin^{-1} \frac{1}{1.5} = 2.41 \text{ radian.}$$

Applying equal area criterion for critical clearing angle

$$A_1 = P_m (\delta_{cr} - \delta_0) = 1.0 (\delta_{cr} - 0.45) = \delta_{cr} - 0.45$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m) d\delta = \int_{\delta_{cr}}^{2.41} (1.5 \sin \delta - 1) d\delta$$

$$= -1.5 \cos \delta - \delta \Big|_{\delta_{cr}}^{2.41}$$

$$= -1.5 (\cos 2.41 - \cos \delta_{cr}) - (2.41 - \delta_{cr})$$

$$= 1.5 \cos \delta_{cr} + \delta_{cr} - 1.293$$

Setting $A_1 = A_2$ and solving

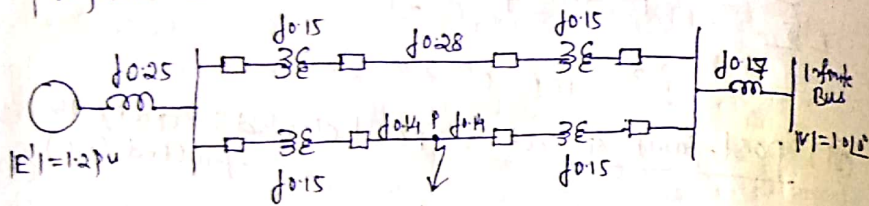
$$\delta_{cr} - 0.45 = 1.5 \cos \delta_{cr} + \delta_{cr} - 1.293$$

$$\Rightarrow \cos \delta_{cr} = \frac{0.843}{1.5} = 0.56$$

$$\Rightarrow \delta_{cr} = 55.8^\circ$$

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Q find the critical clearing angle for the system shown in fig. below for a 3- ϕ fault at a point P. The generator is delivering 1.0 pu power under prefault conditions.



Solⁿ (i) prefault operation

Transfer reactance between generator and infinite bus is

$$X_T = 0.25 + 0.17 + \frac{0.15 + 0.28 + 0.15}{2} = 0.71$$

$$P_{e1} = \frac{1.2 \times 1}{0.71} \sin \delta = 1.69 \sin \delta$$

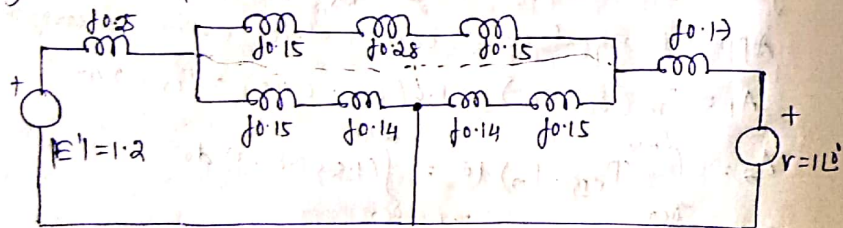
The operating power angle is given by

$$1.0 = 1.69 \sin \delta_0$$

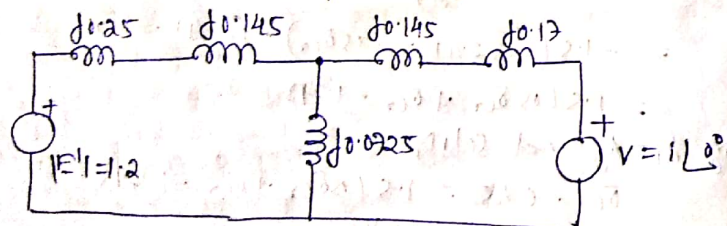
$$\delta_0 = \sin^{-1}\left(\frac{1.0}{1.69}\right) = 0.633 \text{ rad.}$$

ii During fault

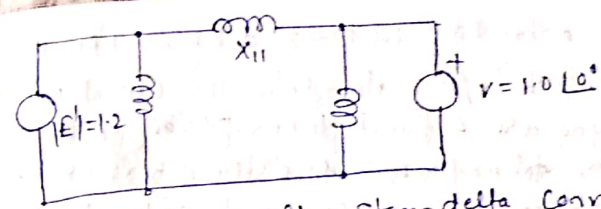
The positive sequence reactance diagram during fault is presented



(a) positive sequence reactance diagram during fault.



(b) Network after delta-star conversion.



(c) Network after star-delta conversion

First converting delta to star, the reactance network is changed. Further upon converting star to delta, we obtain the transfer reactance is given by

$$X_{11} = \frac{(0.25 + 0.145)0.075 + (0.145 + 0.17)0.075 + (0.25 + 0.145)(0.145 + 0.17)}{0.075} = 2.424$$

$$\therefore P_{e11} = \frac{1.2 \times 1}{2.424} \sin \delta = 0.495 \sin \delta$$

(iii) post fault operation (Faulty line switched off)

$$X_{111} = 0.25 + 0.15 + 0.28 + 0.15 + 0.17 = 1.0$$

$$P_{e111} = \frac{1.2 \times 1}{1} \sin \delta = 1.2 \sin \delta$$

$$\text{Here } \delta_{\max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{m_{\max}}}\right)$$

$$= \pi - \sin^{-1}\frac{1}{1.2} = 2.155 \text{ rad}$$

To find the critical clearing angle, area A_1 and A_2 are to be equated.

$$A_1 = 1.0 (\delta_{cr} - 0.633) - \int_{\delta_0}^{\delta_{cr}} 0.495 \sin \delta d\delta$$

$$\text{and } A_2 = \int_{\delta_{cr}}^{\delta_{\max}} 1.2 \sin \delta d\delta - 1.0 (2.155 - \delta_{cr})$$

$$\text{Now } A_1 = A_2, \quad \delta_{cr} = 0.633 - \int_{\delta_0}^{\delta_{cr}} 0.495 \sin \delta d\delta$$

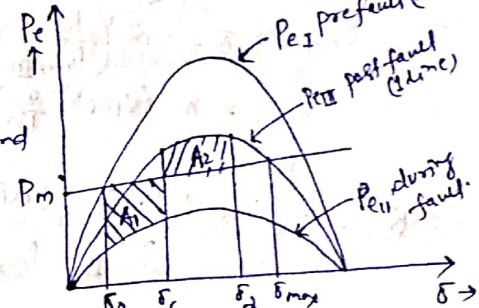
$$= \int_{\delta_{cr}}^{2.155} 1.2 \sin \delta d\delta - 2.155 + \delta_{cr}$$

$$\therefore -0.633 + 0.495 \cos \delta_{cr} = -1.2 \cos \delta \Big|_{\delta_{cr}}^{2.155} - 2.155$$

$$\therefore -0.633 + 0.495 \cos \delta_{cr} - 0.399 = -0.661 + 1.2 \cos \delta_{cr} - 2.155$$

$$\therefore \cos \delta_{cr} = 0.655$$

$$\therefore \delta_{cr} = 49.1^\circ$$



Dilip Khamari

Q. A generator operating at 50Hz delivers 1 pu power to an infinite bus through a transmission circuit in which resistance is ignored. A fault takes place reducing the max^m power transferable to 0.5 pu. where as before the fault, this power was 2.0 pu and after the clearing of the fault, it is 1.5 pu. By the use of equal area criterion, determine the critical clearing angle.

Solⁿ

$$P_{max I} = 2.0 \text{ pu. } P_{max II} = 0.5 \text{ pu.}$$

$$\text{fd } P_{max III} = 1.5 \text{ pu.}$$

$$\text{Initial loading } P_m = 1.0 \text{ pu.}$$

$$P_m = P_{max I} \sin \delta_0$$

$$\sin \delta_0 = \frac{P_m}{P_{max I}}$$

$$\delta_0 = \sin^{-1} \left(\frac{P_m}{P_{max I}} \right) = \sin^{-1} \left(\frac{1}{2} \right) = 0.523 \text{ rad.}$$

$$\begin{aligned} \text{also } \delta_{max} &= \pi - \sin^{-1} \left(\frac{P_m}{P_{max III}} \right) = \pi - \sin^{-1} \left(\frac{1}{1.5} \right) \\ &= \pi - \left(41.8^\circ \times \frac{\pi}{180} \right) = \pi - 0.729 = 3.1415 - 0.729 \\ &= 2.412 \text{ rad.} \end{aligned}$$

$$\cos \delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) - P_{max II} \cos \delta_0 + P_{max III} \cos \delta_{max}}{P_{max III} - P_{max II}}$$

$$= \frac{1(2.41 - 0.523) - 0.5(\cos 0.523) + 1.5 \cos 2.41}{1.5 - 0.5}$$

$$= \frac{1.887 - 0.4331 + 1.5 \times (-0.7411)}{1}$$

$$= \frac{1.887 - 0.4331 - 1.1116}{1}$$

$$\cos \delta_{cr} = 0.333$$

$$\delta_{cr} = 1.224$$

$$= 1.224 \times \frac{180^\circ}{\pi}$$

$$\boxed{\delta_{cr} = 70.133^\circ}$$

Factors influencing transient stability:-

The transient stability of the generator is dependent on the following.

- (i) How heavily the generator is loaded.
- (ii) The generator output during the fault. This depends on the fault location and type.
- (iii) The fault clearing time.
- (iv) The post fault transmission system reactance.
- (v) The generator reactance. A lower reactance increases peak power and reduces initial rotor angle.
- (vi) The generator inertia. The higher the inertia, the slower the rate of change in angle. This reduces the kinetic energy gained during fault; i.e., area A_1 is reduced.
- (vii) The generator internal voltage magnitude (E'). This depends on the field excitation.
- (viii) The infinite bus voltage magnitude E_B .

