LECTURE NOTES

FLUID DYNAMICS

B.Tech, 4th Semester, CIVIL

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CIPC2005 FLUID DYNAMICS (3-0-0)

Course Objectives:

The course in Fluid Dynamics aims to provide students with a comprehensive understanding of advanced fluid mechanics principles and their practical applications. It seeks to develop deep knowledge of boundary layer theory, momentum equations, fluid machinery, and complex flow systems. Students will explore theoretical concepts and real-world engineering applications, including analysis of pumps, turbines, open channel flows, and fluid dynamic phenomena. The objective is to equip students with sophisticated analytical skills to understand fluid behavior, computational techniques, and engineering design principles related to fluid systems and their dynamic interactions.

Module-I

Boundary Layer Theory: Introduction, thickness of boundary layer, boundary layer along a long thin plate and its characteristics, boundary layer equations, momentum integral equations of the boundary layer, laminar boundary layer, turbulent boundary layer, laminar sub-layer, boundary layer on rough surfaces, separation of boundary layer, methods of controlling the boundary layer.

Drag and Lift: Introduction, Types of Drag, dimensional analysis of drag and lift, drag on a (sphere, cylinder, flat plate and air foil), effect of free surface on drag, effect of compressibility on drag, development of lift on immersed body, induced drag on an air foil, of finite length, polar diagram for lift and drag of an air foil.

Module-II

Momentum equation and its applications: Introduction, impulse momentum equation, momentum correction factor, application of impulse momentum equation, force on a pipe bed, jet propulsion (orifice tank, ship), momentum theory of propellers, angular momentum principle

Impact of free jets: Introduction, force exerted by fluid jets on (stationary flat plate, moving flat plate, stationary curved vane, moving curved vane), Torque exerted on a wheel with radial curved vane

Module-III

Reciprocating Pump: Introduction, main components, types, work done (single acting and double acting), coefficient of discharge, slip, percentage slip and negative slip, effects of acceleration of piston on velocity and pressure in suction and delivery pipes, indicator diagram, operating characteristic curves

Centrifugal Pump: Introduction, advantages, component parts, working, types, work done by the impeller, head, losses and efficiencies, minimum starting speed, loss of head due to reduced or increased flow, diameter of impeller and pipes, specific speed, characteristic curves, cavitation, priming devices, troubles and remedies

Turbines: Introduction, elements of hydraulic power plant, head and efficiencies of hydraulic turbine, classification.

Pelton wheel: work done and efficiencies, working proportions, design of runner, multiple jet wheel.

Radial flow impulse turbine: reaction turbine, Francis turbine, work done and efficiencies, working proportions, design of runner, draft tube theory, Kaplan turbine, working proportions.

Expression for specific speed in terms of known coefficients for different turbines, performance characteristic curves.

Classification, reaction, impulse, outward flow, inward flow & mixed flow turbines, Francis& Kaplan turbines, Pelton Wheel, Physical description and principle of operation, Governing of turbine.

Module-IV

Uniform flow in open channels: Introduction, types, geometrical properties, velocity distribution, uniform flow, most economical section, computation of uniform flow, specific energy and critical depth, specific force, critical flow and its computation, application of specific energy to channel transitions

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Non-uniform flow in open channel: Introduction, gradually varied flow, classification of cannel bottom slopes, classification of surface profiles, characteristics of surface profiles, integration of varied flow equations, hydraulic jump, location of hydraulic jump, surges in open channel

Flow over notches and weirs: Introduction, classification, sharp-crested weir, rectangular weir, triangular weir, trapezoidal weir, broad-crested weir.

Measurement of depth of flow: point gauge, hook gauge, float gauge

Course Outcomes:

1. To adopt the dimensional analysis and study of viscous incompressible flow
2. To understand the boundary layer growth and its application in drag and lift phenomena
3. To study momentum equation and its application in impact of jet
4. To analyse velocity triangles for different pumps and turbine
5. To understand the basics of open channel flow and detail flow profiles

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REFERENCES

FLUID DYNAMICS

B.Tech, 4th Semester, CIVIL

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Digital References:

MODULE-I

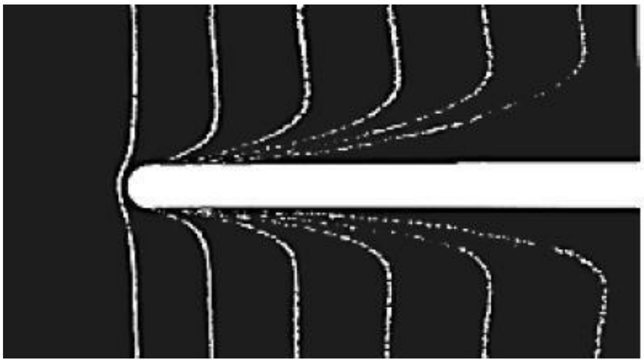
Boundary Layer

If the movement of fluid is not affected by its viscosity, it could be treated as the flow of ideal fluid , therefore its analysis would be easier. The flow around a solid, however ,cannot be treated in such a manner because of viscous friction. Nevertheless ,only the very thin region near the wall is affected by this friction. Prandtl identified this phenomenon and had the idea to divide the flow into two regions. They are:

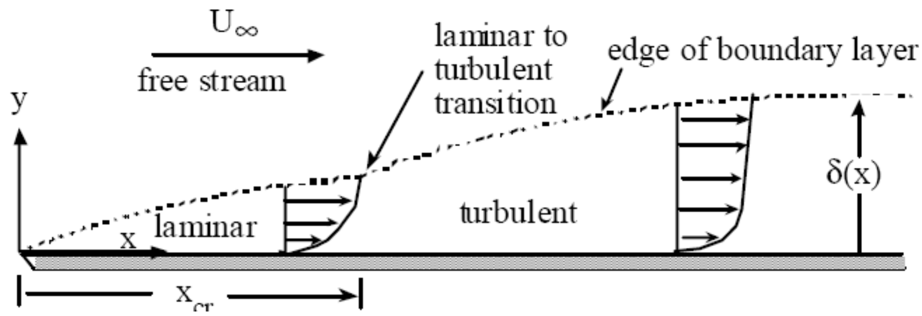
1. the region near the wall where the movement of flow is controlled by the frictional resistance .
2. the other region outside the above not affected by the friction and,

Development of boundary layer

The distance from the body surface when the velocity reaches 99% of the velocity of the main flow is defined as the boundary layer thickness δ. The boundary layer continuously thickens with the distance over which it flows. This process is visualized as shown in the below Figure.



However , viscous flow boundary layer characteristics for external flows are significantly different as shown below for flow over a flat plate:



The most important fluid flow parameter is the local Reynolds number defined as :

$$Re_x = \frac{\rho U_\infty x}{\mu} = \frac{U_\infty x}{\upsilon}$$

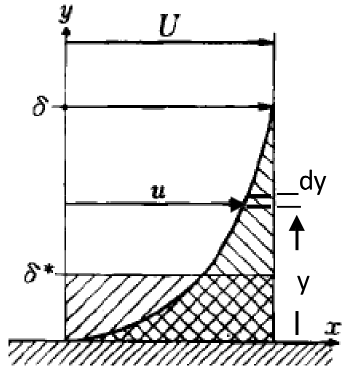
Transition from laminar to turbulent flow typically occurs at the local transition Reynolds number which for flat plate flows can be in the range of

$$500{,}000 \leq Re_{cr} \geq 3{,}000{,}000$$

When the flow distribution and the drag are considered, it is useful to use the following displacement thickness **δ*** and momentum thickness **θ** instead of **δ** .

Displacement thickness δ*

δ^* = distance the solid surface would have to be displaced to maintain the same mass flow rate as for non-viscous flow.



$$U\delta^* = \int_0^\infty (U-u)\,dy$$

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U_\infty}\right) dy$$

Therefore, with an expression for the local velocity profile we can obtain δ* = f(δ)

Example:

Given: $\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ determine an expression for $\delta^* = f(\delta)$

Note that for this assumed form for the velocity profile:
1. At y = 0, u = 0 correct for no slip condition
2. At y = δ, u = U∞ correct for edge of boundary layer

let $\eta = y/\delta$, at $y = 0$, $\eta = 0$; at $y = \delta$, $\eta = 1$; $dy = \delta\, d\eta$

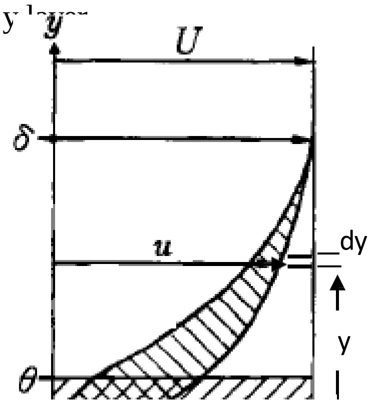
Therefore: $\frac{u}{U_\infty} = 2\eta - \eta^2$

Substituting: $$\delta^* = \int_0^1 \left(1 - 2\eta + \eta^2\right)\delta\, d\eta = \delta\left\{\eta - \frac{2\eta^2}{2} + \frac{\eta^3}{3}\right\}_0^1$$

which yields $\delta^* = \frac{1}{3}\delta$. This closely approximates flow for a flat plate.

Momentum Thickness θ:

The concept is similar to that of displacement thickness in that **θ** is related to the loss of momentum due to viscous effects in the boundary layer.



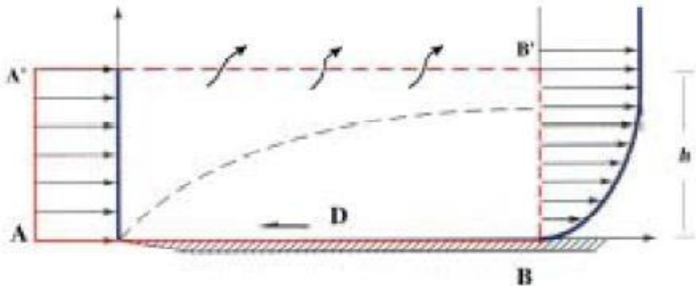
$$\rho U^2\theta = \rho\int_0^\infty u(U-u)\,dy$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right) dy$$

The momentum thickness θ equates the momentum decrease per unit time due to the existence of the body wall to the momentum per unit time which passes at velocity ***U*** through a height of thickness **θ.** The momentum decrease is equivalent to the force acting on the body according to
the law of momentum conservation. Therefore the drag **on** a body generated by the viscosity can be obtained by using the momentum thickness

Drage on a flat plate

Consider the viscous flow regions shown in the adjacent figure. Define a control volume as shown and integrate around the control volume to obtain the net change in momentum for the
control volume.



If D = drag force on the plate due to viscous flow, we can write

- D = Σ (momentum leaving c.v.) - Σ (momentum entering c.v.)

The drag force on the plate is given by the following momentum integral across the exit plane

$$D(x) = \rho b \int_0^{\delta(x)} u(U - u)\, dy$$, where *b* is the plate width into the paper .

The above Equation was derived in 1921 by Kármán , who wrote it in the convenient form
of the ***momentum thickness* θ** :

$$D(x) = \rho b U^2 \theta \qquad \theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Momentum thickness is thus a measure of total plate drag. Kármán then noted that the drag also equals the integrated wall shear stress along the plate

$$D(x) = b \int_0^{x} \tau_w(x)\, dx$$

$$\frac{dD}{dx} = b\tau_w \qquad (1)$$

$$\frac{dD}{dx} = \rho b U^2 \frac{d\theta}{dx}$$

By comparing the above equations , Kármán arrived at what is now called ***the momentum integral relation*** for flat-plate boundary-layer flow

$$\tau_w = \rho U^2 \frac{d\theta}{dx} \qquad \ldots\ldots\ldots\ldots \qquad (1)$$

It is valid for either laminar or turbulent flat-plate flow

Laminar flow

To get a numerical result for laminar flow, Kármán assumed that the velocity profiles had an approximately parabolic shape

$$u(x, y) \approx U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) \qquad 0 \leq y \leq \delta(x)$$

which makes it possible to estimate both momentum thickness and wall shear

$$\theta = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy \approx \frac{2}{15}\delta$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \approx \frac{2\mu U}{\delta} \qquad \ldots\ldots\ldots\ldots \qquad (2)$$

By substituting (2) into (1) and integrating from 0 to *x*, assuming that δ = 0 at *x* = 0, the leading edge

$$\frac{\delta}{x} \approx 5.5\left(\frac{\nu}{Ux}\right)^{1/2} = \frac{5.5}{\mathrm{Re}_x^{1/2}} \qquad \ldots\ldots\ldots\ldots \qquad (3)$$

This is the desired thickness estimate. We define the boundary layer thickness δ as the locus of points where the velocity *u* parallel to the plate reaches 99% of the external velocity *so that the* above value of δ is represented about 10% higher than the known exact solution for laminar flow, the accepted formulas for flat-plate flow

$$\frac{\delta}{x} \approx \frac{5.0}{\mathrm{Re}_x^{1/2}} \qquad \text{Blasius (1908)}$$

By combining this Equation and (2), we also obtain a shear-stress estimate along the plate

$$c_f = \frac{2\tau_w}{\rho U^2} \approx \left(\frac{\frac{8}{15}}{\mathrm{Re}_x}\right)^{1/2} = \frac{0.73}{\mathrm{Re}_x^{1/2}} \qquad \ldots\ldots\ldots\ldots \qquad (4)$$

The dimensionless quantity *cf*, called the ***skin-friction coefficient***, is analogous to the friction factor *f* in ducts. Again this estimate, in spite of the crudeness of the profile above assumption is only 10% higher than of exact solution . So the known exact laminar-plate-flow solution *of* ***cf***
as the following :

$$c_f = \frac{0.664}{\mathrm{Re}_x^{1/2}}$$

With the profile known, Blasius, of course, could also compute the wall shear and displacement
thickness

$$\delta^* \approx \frac{1}{3}\delta \qquad \frac{\delta^*}{x} \approx \frac{1.83}{\mathrm{Re}_x^{1/2}} \qquad \text{. . . . .} \quad (5)$$

These estimates are only 6 percent away from the exact solutions for laminar flat-pl $\frac{\delta^*}{x} = \frac{1.721}{\mathrm{Re}_x^{1/2}}$ (Exact solution of the displacement thicknesss)

Notice how close these are to our integral estimates, Eqs. (3), (4), and (5). we have

$$\tau_w(x) = \frac{0.332\rho^{1/2}\mu^{1/2}U^{1.5}}{x^{1/2}}$$

$$\tau_w(x) = \frac{0.332\rho\, U^{2}}{\mathrm{Re}_L^{1/2}}$$

we compute the total drag force

$$D(x) = b\int_0^x \tau_w(x)\,dx = 0.664b\rho^{1/2}\mu^{1/2}U^{1.5}x^{1/2} \qquad \text{------------} \quad (1)$$

The drag increases only as the square root of the plate length. The nondimensional *drag coefficient* is defined as

$$C_D = \frac{2D(L)}{\rho U^2 bL} = \frac{1.328}{\mathrm{Re}_L^{1/2}}$$

Thus the drag on one side in the airflow is

$$D = C_D \tfrac{1}{2}\rho U^2 bL$$

EXAMPLE

A sharp flat plate with *L*= 1 m and *b*= 3 m is immersed parallel to a stream of velocity 2 m/s.

Find the drag on one side of the plate, and at the trailing edge find the thicknesses δ, δ^*, and θ for (*a*) air, $\rho = 1.23$ kg/m^3 and $\nu = 1.46 \times 10^{-5}$ m^2/s, and (*b*) water, $\rho = 1000$ kg/m^3 and
$\nu = 1.02 \times 10^{-6}$ m^2/s

Part (a)

The airflow Reynolds number is

$$\frac{VL}{\nu} = \frac{(2.0 \text{ m/s})(1.0 \text{ m})}{1.46 \times 10^{-5} \text{ m}^2/\text{s}} = 137{,}000$$

Since this is less than 3×10^6, the boundary layer is laminar. The drag coefficient is :

$$C_D = \frac{1.328}{(137{,}000)^{1/2}} = 0.00359$$

Thus the drag on one side in the airflow is

$$D = C_D \tfrac{1}{2}\rho U^2 bL = 0.00359(\tfrac{1}{2})(1.23)(2.0)^2(3.0)(1.0) = 0.0265 \text{ N}$$

We find the other two thicknesses simply by ratios:

$$\delta^* = \frac{1.721}{5.0}\,\delta = 4.65 \text{ mm} \qquad \theta = \frac{\delta^*}{2.59} = 1.79 \text{ mm}$$

Part (b)

The water Reynolds number is

$$\text{Re}_L = \frac{2.0(1.0)}{1.02 \times 10^{-6}} = 1.96 \times 10^6$$

This is rather close to the critical value of 3×10^6, so that a rough surface or noisy free stream
might trigger transition to turbulence; but let us assume that the flow is laminar. The water drag
coefficient is

$$C_D = \frac{1.328}{(1.96 \times 10^6)^{1/2}} = 0.000949$$

$$D = 0.000949(\tfrac{1}{2})(1000)(2.0)^2(3.0)(1.0) = 5.70 \text{ N}$$

The drag is 215 times more for water in spite of the higher Reynolds number and lower drag
coefficient because water is 57 times more viscous and 813 times denser than air. From Eq.(1), in laminar flow, it should have $(57)^{1/2}(813)^{1/2} = 7.53\,(28.5) = 215$ times more drag. The boundary-layer thickness is given by :

$$\frac{\delta}{L} = \frac{5.0}{(1.96 \times 10^6)^{1/2}} = 0.00357$$

$$\delta = 0.00357(1000 \text{ mm}) = 3.57 \text{ mm}$$

$$\delta^* = \frac{1.721}{5.0}\,\delta = 1.23 \text{ mm} \qquad \theta = \frac{\delta^*}{2.59} = 0.48 \text{ mm}$$

The water layer is 3.8 times thinner than the air layer, which reflects the square root of the 14.3
ratio of air to water kinematic viscosity.

Turbulent flow

A Prandtl, is pointed out that the turbulent profiles can be approximated by a one-seventh-power law

$$\left(\frac{u}{U}\right)_{\text{turb}} \approx \left(\frac{y}{\delta}\right)^{1/7}$$

There is no exact theory for turbulent flat-plate flow, the most widely accepted result is simply an integral analysis similar to our study of the laminar-profile approximation . We begin with the same equation , which is valid for laminar or turbulent flow :

$$\tau_w(x) = \rho U^2 \,\frac{d\theta}{dx}$$

With this simple approximation, the momentum thickness can easily be evaluated:

$$\theta \approx \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7}\left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy = \frac{7}{72}\,\delta$$

$$c_f = \frac{2\tau_w}{\rho U^2}$$

From the definition of c_f , it can be rewritten as :

$$c_f = 2\,\frac{d\theta}{dx}$$

A Prandtl is simplified the friction law as the following a suggestion of power-law approximation

$$c_f = 0.02\ \mathrm{Re}_\delta^{-1/6} = 2\,\frac{d}{dx}\left(\frac{7}{72}\,\delta\right)$$

$$\mathrm{Re}_\delta^{-1/6} = 9.72\,\frac{d\delta}{dx} = 9.72\,\frac{d(\mathrm{Re}\,\delta)}{d(\mathrm{Re}_x)}$$

Separate the variables and integrate, assuming $\delta = 0$ at $x = 0$:

$$\mathrm{Re}_\delta \approx 0.16\,\mathrm{Re}_x^{6/7} \qquad \text{or} \qquad \frac{\delta}{x} \approx \frac{0.16}{\mathrm{Re}_x^{1/7}}$$

we obtain the friction variation

$$c_f \approx \frac{0.027}{\mathrm{Re}_x^{1/7}}$$

Writing this out in dimensional form, we have

$$\tau_{w,\mathrm{turb}} \approx \frac{0.0135\mu^{1/7}\rho^{6/7}U^{13/7}}{x^{1/7}}$$

Turbulent plate friction drops slowly with x, increases nearly as ρ and U^2, and is rather
insensitive to viscosity. We can evaluate the drag coefficient

$$C_D = \frac{0.031}{\mathrm{Re}_L^{1/7}}$$

Then the drag on both sides of the plate :

$$D = 2C_D(\tfrac{1}{2}\rho U^2)bL$$

EXAMPLE

A hydrofoil 0.366 m long and 1.82 m wide is placed in a water flow of 12.9 m/s , with $\rho = 1025.3\ \mathrm{kg/m^3}$ and $\nu = 1.02\times 10^{-6}\ \mathrm{m^2/s}$. (*a*) Estimate the boundary-layer thickness at the end of the plate. Estimate the friction drag for (*b*) turbulent smooth-wall flow from the leading edge, (*c*) laminar turbulent flow with Re_{tren} =

$$\mathrm{Re}_L = \frac{UL}{\nu} = 4.36 \times 10^6$$

Thus the trailing-edge flow is certainly turbulent. The maximum boundary-layer thickness would
occur for turbulent flow starting at the leading edge.

$$\frac{\delta}{x} \approx \frac{0.16}{\mathrm{Re}_x^{1/7}} \qquad \delta = 0.0065\ \mathrm{m}$$

For fully turbulent smooth-wall flow, the drag coefficient on one side of the plate is,

$$C_D = \frac{0.031}{\mathrm{Re}_L^{1/7}} \qquad C_D = \frac{0.031}{(4.36 \times 10^6)^{1/7}} = 0.00349$$

Then the drag on both sides of the foil is approximately

$$D = 2C_D(\tfrac{1}{2}\rho U^2)bL \qquad D = 55.84 \text{ N}$$

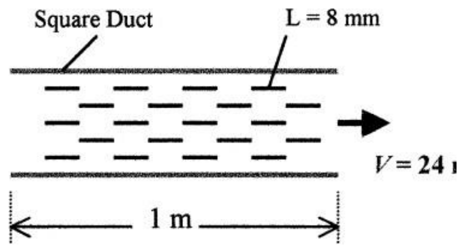
Part (c) With a laminar leading edge and $\mathrm{Re}_{trans} = 5 \times 10^5$.

$$C_D = 0.00349 - \frac{1440}{4.36 \times 10^6} = 0.00316$$

The drag can be recomputed for this lower drag coefficient:

$$D = 2C_D(\tfrac{1}{2}\rho U^2)bL \qquad D = 20.25 \text{ N}$$

Example: Consider the smooth square 10 by 10 cm duct in below Figure. The fluid is air at 20°C and 1 atm, flowing at V_{avr} = 24 m/s. It is desired to increase the pressure drop over the 1-m length by adding sharp 8-mm-long flat plates across the duct, as shown. (a) Estimate the pressure drop if there are no plates. (b) Estimate how many plates are needed to generate an additional 10(For air, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8$E-5 kg/m·s.



$$\mathrm{Re}_{Dh} = \frac{VD_h}{\nu} = \frac{(24 \text{ m/s})(0.1 \text{ m})}{0.000015 \text{ m}^2/\text{s}} = 160000 \ (turbulent)$$

$$f_{smooth} = 0.0163$$

$$\Delta p_{Moody} = f\frac{L}{D_h}\frac{\rho V^2}{2} = \mathbf{56\ Pa} \qquad Ans.\ (a)$$

$$\mathrm{Re}_L = \frac{(24)(0.008)}{0.000015} = 12800, \quad C_D = \frac{1.328}{\sqrt{12800}} = 0.0117$$

$$F = C_D\frac{\rho}{2}V^2 bL(2\ \mathbf{sides}) = (0.0117)\frac{1.2}{2}(24)^2(0.1)(0.008)(2) = 0.00649\ N$$

(b) To estimate the plate-induced pressure drop, first calculate the drag on one plate:

$$\Delta p_{extra} = 100\ Pa = \frac{FN_{plates}}{A_{duct}} = \frac{(0.00649\ N)N_{plates}}{(0.1 \text{ m})^2}, \quad or: \quad \mathbf{N_{plates} \approx 154} \quad Ans.\ (b)$$

Since the duct walls must support these plates, the effect is an additional pressure drop:

DRAG and LIFT:-

In aerodynamics, the **lift-to-drag ratio**, or **L/D ratio**, is the amount of lift generated by a wing or vehicle, divided by the aerodynamic drag it creates by moving through the air. A higher or more favorable L/D ratio is typically one of the major goals in aircraft design; since a particular aircraft's required lift is set by its weight, delivering that lift with lower drag leads directly to better fuel economy in aircraft, climb performance, and glide ratio.

The term is calculated for any particular airspeed by measuring the lift generated, then dividing by the drag at that speed. These vary with speed, so the results are typically plotted on a 2D graph. In almost all cases the graph forms a U-shape, due to the two main components of drag.

Lift and Drag for Flow About a Rotating Cylinder

The pressure at large distances from the cylinder is uniform and given by p_0.

Deploying Bernoulli's equation between the points at infinity and on the boundary of the cylinder,

$$p_b = \rho g\left[\frac{U_0^2}{2g} + \frac{p_0}{\rho g} - \frac{U_b^2}{2g}\right] \tag{23.9}$$

Hence,

$$U_b = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -2U_0 \sin\theta - \frac{r}{2\pi}\left[\frac{U_0}{\chi}\right]^{1/2} \tag{23.10}$$

From Eqs (23.9) and (23.10) we can write

$$p_b = \rho g\left[\frac{U_0^2}{2g} + \frac{p_0}{\rho g}\right] - \left[\frac{-2U_0 \sin\theta - \frac{\tau}{2\pi}\left(\frac{U_0}{\chi}\right)^{1/2}}{2g}\right]^2 \tag{23.11}$$

The lift may calculated as

$$L = -\int_0^{2\pi} p_b \sin\theta\left[\frac{\chi}{U_0}\right]^{1/2} d\theta$$

or,

$$L = -\int_0^{2\pi} \left[\frac{\rho U_0^2}{2} \left(\frac{\chi}{U_0} \right)^{1/2} \sin\theta + p_0 \left(\frac{\chi}{U_0} \right)^{1/2} \sin\theta - \frac{\rho}{2} \left\{ 4U_0^2 \sin^2\theta + \frac{4U_0 r \sin\theta}{2\pi} \left(\frac{U_0}{\chi} \right)^{1/2} + \frac{\Gamma^2}{4\pi^2} \left[\frac{U}{\chi} \right. \right. \right.$$

$$L = -\int_0^{2\pi} \left[\frac{\rho U_0^2}{2} \left(\frac{\chi}{U_0} \right)^{1/2} \sin\theta + p_0 \left(\frac{\chi}{U_0} \right)^{1/2} \sin\theta - 2\rho U_0^2 \sin^3\theta \left(\frac{\chi}{U_0} \right)^{1/2} - \frac{\rho U_0 \Gamma}{\pi} \sin^2\theta - \frac{\rho \Gamma^2}{8\pi^2} \left(\frac{\chi}{U_0} \right. \right.$$

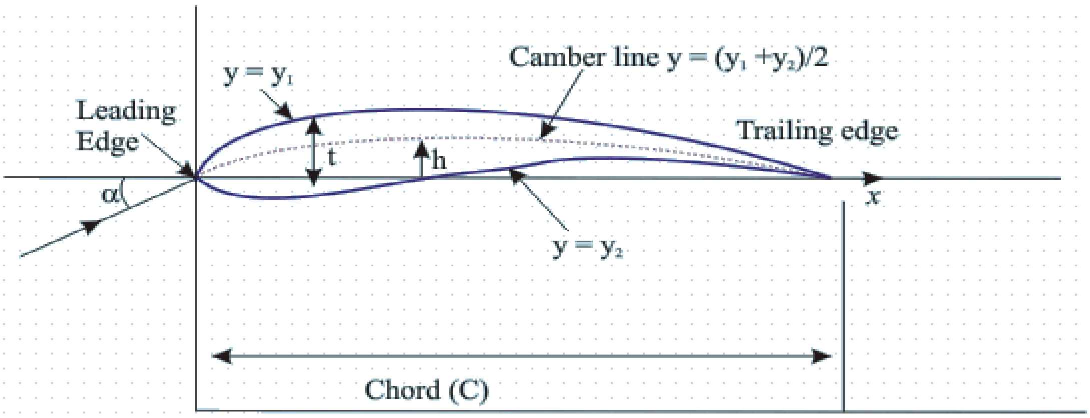
$$L = \rho U_0 \Gamma \qquad (23.12)$$

The drag force , which includes the multiplication by cosθ (and integration over 2π) is zero.

- Thus the inviscid flow also demonstrates lift.
- lift becomes a simple formula involving only the density of the medium, free stream velocity and circulation.
- in two dimensional incompressible steady flow about a boundary of any shape, the lift is always a product of these three quantities -------------------- **Kutta- Joukowski theorem**

Aerofoil Theory

Aerofoils are streamline shaped wings which are used in airplanes and turbo machinery. These shapes are such that the drag force is a very small fraction of the lift. The following nomenclatures are used for defining an aerofoil



- The **chord** (C) is the distance between the leading edge and trailing edge.
- The length of an aerofoil, normal to the cross-section (i.e., normal to the plane of a paper) is called the **span** of a aerofoil.
- The **camber line** represents the mean profile of the aerofoil. Some important geometrical parameters for an aerofoil are the ratio of maximum thickness to chord (t/C) and the ratio of maximum camber to chord (h/C). When these ratios are small, an aerofoil can be considered to be thin. For the analysis of flow, a thin aerofoil is represented by its camber.

The theory of thick cambered aerofoils uses a complex-variable mapping which transforms the inviscid flow across a rotating cylinder into the flow about an aerofoil shape with circulation.

Flow Around a Thin Aerofoil

- Thin aerofoil theory is based upon the superposition of uniform flow at infinity and a continuous distribution of clockwise free vortex on the camber line having circulation density $\gamma(s)$ per unit length .
- The circulation density $\gamma(s)$ should be such that the resultant flow is tangent to the camber line at every

$$\gamma(s)ds = \gamma(\eta)d\eta$$

point.

- Since the slope of the camber line is assumed to be small, . The total circulation around the profile is given by

$$\Gamma = \int_0^C \gamma(\eta)\, d\eta \tag{23.13}$$

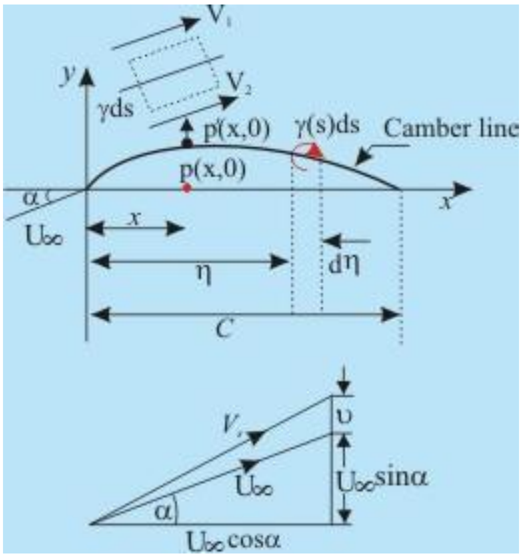


Fig 23.5 Flow Around Thin Aerofoil

A vortical motion of strength $\gamma d\eta$ at x= η develops a velocity at the point p which may be expressed as

$$d\upsilon = \frac{\gamma(\eta)d\eta}{2\pi(\eta - x)}\, acting\ upwards$$

The total induced velocity in the upward direction at **point p** due to the entire vortex distribution along the camber line is

$$\upsilon(x) = \frac{1}{2\pi}\int_0^C \frac{\gamma(\eta)d\eta}{(\eta - x)} \tag{23.14}$$

For a small camber (having small α), this expression is identically valid for the induced velocity at **point p'** due to the vortex sheet of variable strength $\gamma(s)$ on the camber line. The resultant velocity due to U_∞ and v(x) must be tangential to the camber line so that the slope of a camber line may be expressed as

$$\frac{dy}{dx} = \frac{U_\infty \sin\alpha + \upsilon}{U_\infty \cos\alpha} = \tan\alpha + \frac{\upsilon}{U_\infty \cos\alpha}$$

$$\frac{dy}{dx} = \alpha + \frac{\nu}{U_\infty}\left[\sin ce\ \alpha\ is\ verysmall\right] \tag{23.15}$$

From Eqs (23.14) and (23.15) we can write

$$\frac{dy}{dx} = \alpha + \frac{1}{2\pi U_\infty}\int_0^c \frac{\gamma(\eta)d\eta}{\eta - x}$$

Consider an element ds on the camber line. Consider a small rectangle (drawn with dotted line) around ds. The upper and lower sides of the rectangle are very close to each other and these are parallel to the camber line. The other two sides are normal to the camber line. The circulation along the rectangle is measured in clockwise direction as

$V_1 ds - V_2 ds = \gamma ds$ [normal component of velocity at the camber line should be zero]

or $V_1 - V_2 = \gamma$

If the mean velocity in the tangential direction at the camber line is given by $Vs = (V_1 + V_2)/2$, it can be rewritten as

$V_1 = Vs + \frac{\gamma}{2}$ and $V_2 = V_s - \frac{\gamma}{2}$

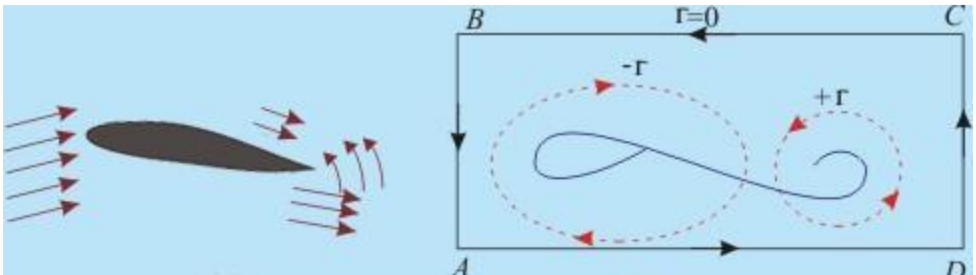
if v is very small $[v << U_\infty], V_s$ becomes equal to U_∞. The difference in velocity across the camber line brought about by the vortex sheet of variable strength $\gamma(s)$ causes pressure difference and generates lift force.

Generation of Vortices Around a Wing

- **The lift around an aerofoil is generated following Kutta-Joukowski theorem** . Lift is a product of ρ , U_∞ and the circulation Γ.

$$Lift = \rho U_\infty \Gamma$$

- When the motion of a wing starts from rest, vortices are formed at the trailing edge.
- At the start, there is a velocity discontinuity at the trailing edge. This is eventual because near the trailing edge, the velocity at the bottom surface is higher than that at the top surface. This discrepancy in velocity culminates in the formation of vortices at the trailing edge.
- Figure 23.6(a) depicts the formation of starting vortex by impulsively moving aerofoil. However, the starting vortices induce a counter circulation as shown in Figure 23.6(b). The circulation around a path (ABCD) enclosing the wing and just shed (starting) vortex must be zero. Here we refer to **Kelvin's theorem** once again.



(a) (b)

Fig 23.6 Vortices Generated when an Aerofoil Just Begins to Move

- Initially, the flow starts with the zero circulation around the closed path. Thereafter, due to the change in angle of attack or flow velocity, if a fresh starting vortex is shed, the circulation around the wing will adjust itself so that a net zero vorticity is set around the closed path.
- Real wings have finite span or finite aspect ratio (AR) λ , defined as

$$\lambda = \frac{b^2}{A_s} \qquad (23.16)$$

where b is the span length, A_s is the plan form area as seen from the top..

- For a wing of finite span, the end conditions affect both the lift and the drag. In the leading edge region, pressure at the bottom surface of a wing is higher than that at the top surface. The

  longitudinal vortices are generated at the edges of finite wing owing to pressure differences between the bottom surface directly facing the flow and the top surface.

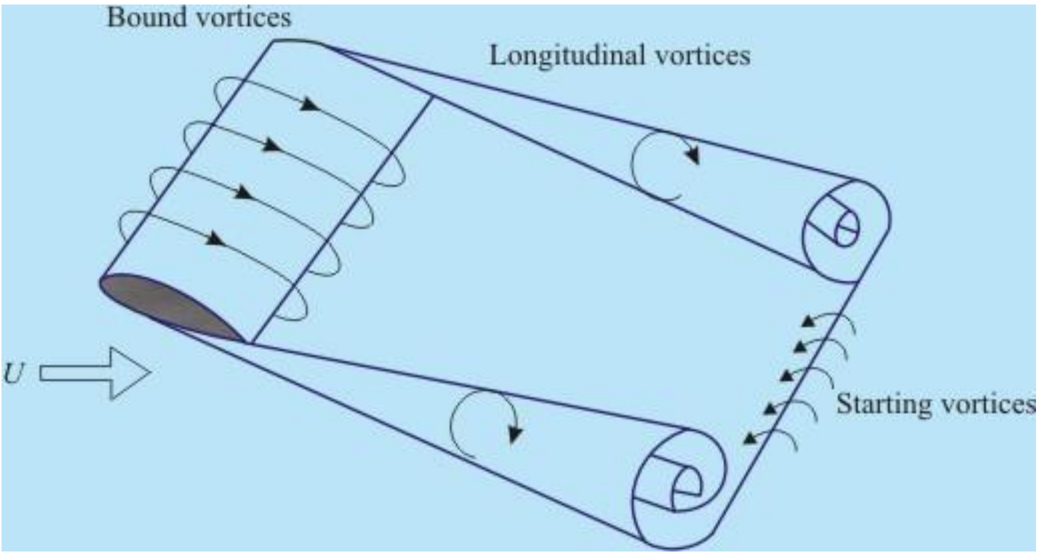
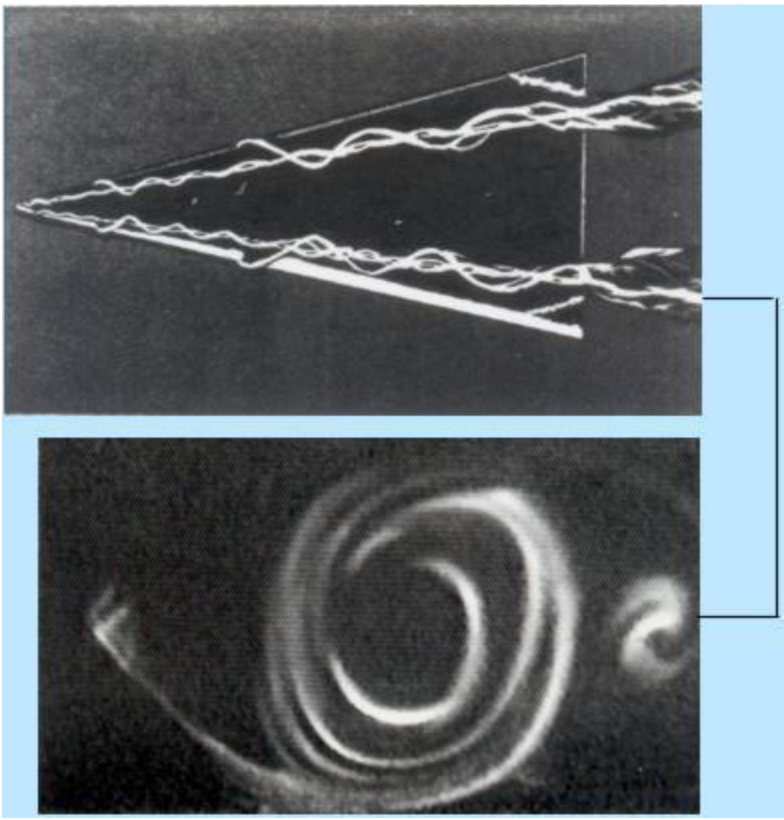


Fig 23.7 Vortices Around a Finite Wing



MODULE-II

INTRODUCTION:

AnalysisandDesignofHydraulicMachines(TurbinesandPumps)isessentiallybasedontheknowledgeoffor cesexerted on orbythe movingfluids.

ImpactofJets

The jet is a stream of liquid comes out from nozzlewith a high velocity under constant pressure.When the jet impinges on plates or vanes, its momentum is changed and a hydrodynamic force isexerted.Vaneis aflator curvedplatefixed to the rimofthe wheel.

1. ***Forceexertedbythejetonastationaryplate***
   a) Plateisverticaltothejet
   b) Plateisinclinedtothejet
   c) Plateiscurved
2. ***Forceexertedbythejetonamovingplate***
   a) Plateisverticaltothejet
   b) Plateisinclinedtothejet
   c) Plateiscurved

Impulse-MomentumPrinciple

FromNewton's2nd Law,it canbewrittenthat:

$$F = mXa = mX\left(\frac{V_2 - V_1}{t}\right)$$

$$\Rightarrow FX\,t = mX(V_2 - V_1)$$

$$\Rightarrow FXt = mV_2 - mV_1$$

$$\Rightarrow FXt = \text{Finalmomentum} - \text{Initalmomentum}$$

Therefore,Impulse of aforceis givenbythe changeinmomentumcausedbytheforceonthebody.

Hence,basedontheconceptonofimpulse,Forceexertedbyjetofwaterontheplateinthedirectionof jetcan be calculated as:

$$F = \frac{mX(V_2 - V_1)}{t}$$

$$F = \frac{m}{t} X(V_2 - V_1)$$

$$F = \frac{m}{t} X\text{Finalvelocity} - \text{Initalvelocity}$$

$$F=\rho X Q X(V_2-V_1)$$

$$F=\rho X(aXV)X \qquad (V_2 - V_1)$$

Based on the above equation stated, the force exerted by the jet of water on different types of plates /vanes / buckets in both stationary and moving cases are discussed in the following sections. Followingaredifferenttypes of cases:

Case: I
Whenthevanes / platesarestationary

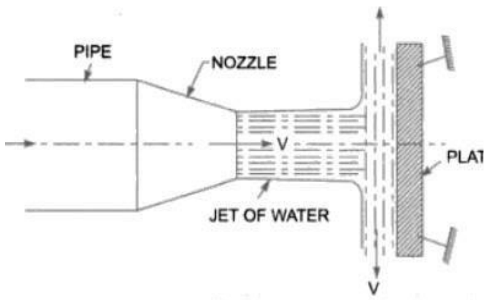
- Theplate/vaneisflatandperpendicularto thedirection ofjet
- Theplate/vaneisflatand inclinedto anangle θtojetof water
- Theplate/ vaneiscurvedandthejet ofwaterisstrikingat itscentreexactly
- Theplate/ vaneiscurved andthejet of wateris strikingthe vaneatitsoneend ofthetip.

Case:II
Whenthevanes/ platesaremoving

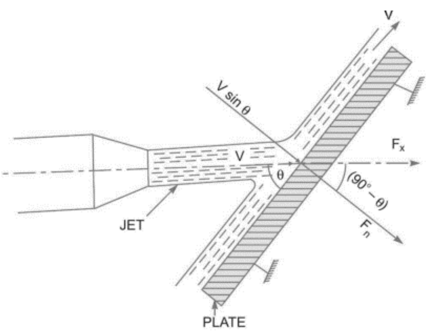
- Theplate/vaneisflatandperpendicularto thedirection ofjet
- Theplate/vaneisflatand inclinedto an angle θtojetof water
- Theplate/ vaneiscurvedandthejet ofwaterisstrikingat itscentreexactly
- Theplate/ vaneiscurved andthejet of wateris strikingthe vaneatitsoneend ofthetip.

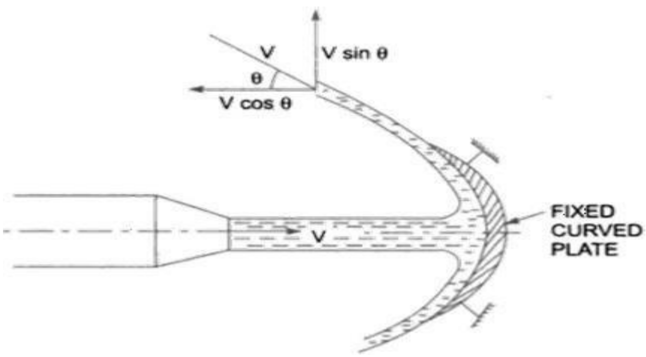
Case: III
Whenthejet ofwater strikesonaseriesflat plate/ vanemounted onperipheryofwheel

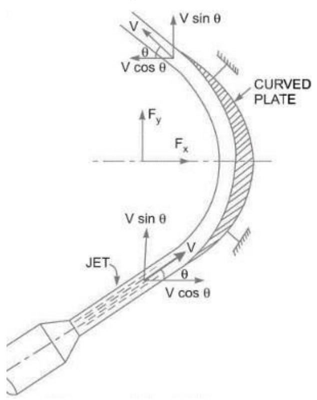
Case:IV
Whenthejet ofwater strikesonaseriescurvedplate/vane mountedonperipheryofwheel



Force exerted by jet on vertical plate.







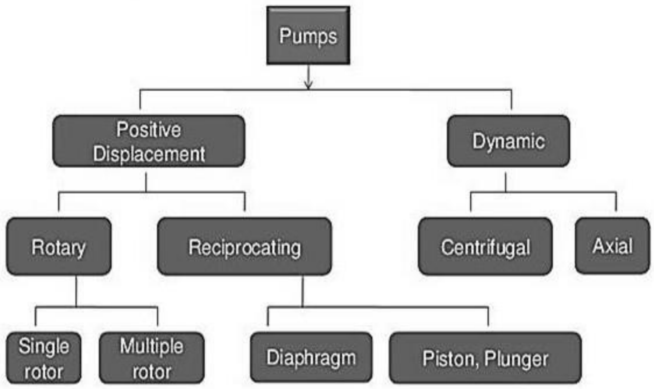
Forceexertedbythejetonastationaryflatandcurvedplateandthejetisstrikingtheplateperpendicularly:

MODULE-III

Hydraulic Pump

A hydraulic pump is a mechanical source of power that converts mechanical power into hydraulic energy. It generates flow with enough power to overcome pressure induced by the load at the pump outlet. When a hydraulic pump operates, it creates a vacuum at the pump inlet, which forces liquid from the reservoir into the inlet line to the pump and by mechanical action delivers this liquid to the pump outlet and forces it into the hydraulic system.

<u>Classifications of Pump</u>



Centrifugal Pump

The main components of a centrifugal pump are:

i) Impeller
ii) Casing
iii) Suction pipe
iv) Foot valve with strainer,
v) Delivery pipe
vi) Delivery valve.

Impeller is the rotating component of the pump. It is made up of a series of curved vanes. The impeller is mounted on the shaft connecting an electric motor.

Casing is an air tight chamber surrounding the impeller. The shape of the casing is designed in such a way that the kinetic energy of the impeller is gradually changed to potential energy. This is achieved by gradually increasing the area of cross section in the direction of flow.

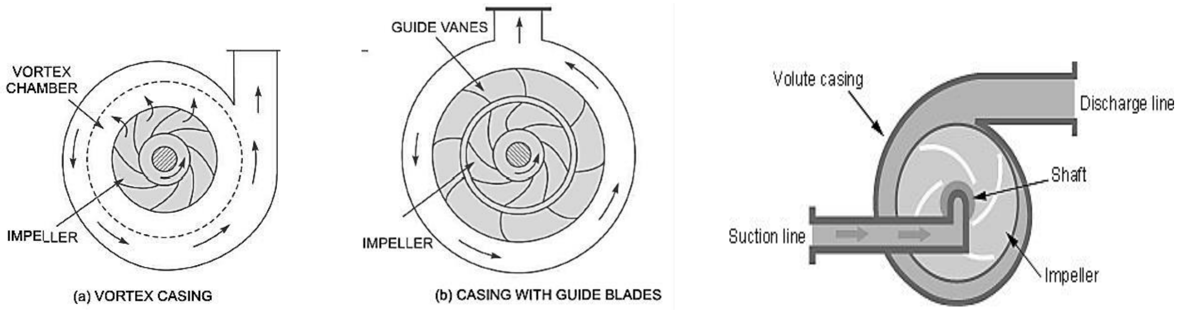


Fig. 1 Types of Casing

Suction pipe: It is the pipe connecting the pump to the sump, from where the liquid has to be lifted up.

Foot valve with strainer: The foot valve is a non-return valve which permits the flow of the liquid from the other words the foot valve opens only in the upward direction. The strainer is a mesh surrounding the valve, it p debris and silt into the pump.

Delivery pipe is a pipe connected to the pump to the overhead tank. Delivery valve is a valve which can regulate the pump.

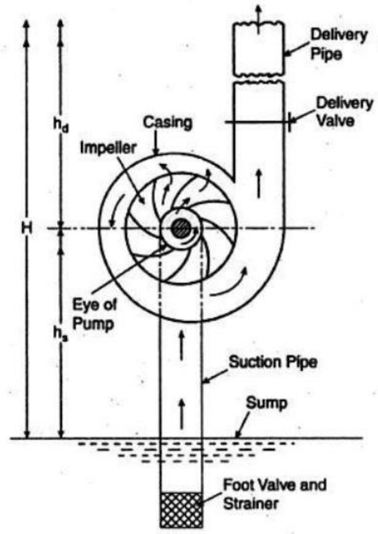


Fig. 2 Main parts of a centrifugal pump

Working

A centrifugal pump works on the principle that when a certain mass of fluid is rotated by an external source, it is thrown away from the central axis of rotation and a centrifugal head is impressed which enables it to rise to a higher level.

Working operation of a centrifugal pump is explained in the following steps:

1. Close the delivery valve and prime the pump.
2. Start the motor connected to the pump shaft, this causes an increase in the impeller pressure.
3. Open the delivery valve gradually, so that the liquid starts flowing into the deliver pipe.
4. A partial vacuum is created at the eye of the centrifugal action, the liquid rushed from the sump to the pump due to pressure difference at the two ends of the suction pipe.
5. As the impeller continues to run, move & more liquid are made available to the pump at its eye. Therefore impeller increases the energy of the liquid and delivers it to the reservoir.
6. While stopping the pump, the delivery valve should be closed first; otherwise there may be back flow from the reservoir.

It may be noted that a uniform velocity of flow is maintained in the delivery pipe. This is due to the special design of the casing. As the flow proceeds from the tongue of the casing to the delivery pipe, the area of the casing increases. There is a corresponding change in the quantity of the liquid from the impeller. Thus a uniform flow occurs in the delivery pipe.

Centrifugal pump converts rotational energy, often from a motor, to energy in a moving fluid. A portion of the energy goes into kinetic energy of the fluid. Fluid enters axially through eye of the casing, is caught up in the impeller blades, and is whirled tangentially and radially outward until it leaves through all circumferential parts of the impeller into the diffuser part of the casing. The fluid gains both velocity and pressure while passing through the impeller. The doughnut-shaped diffuser, or scroll, section of the casing decelerates the flow and further increases the pressure. The negative pressure at the eye of the impeller helps to maintain the flow in the system. If no water is present initially, the negative pressure developed by the rotating air, at the eye will be negligibly small to suck fresh stream of water. As a result the impeller will rotate without sucking and discharging any water content. So the pump should be initially filled with water before starting it. This process is known as priming.

Use of the Casing

From the illustrations of the pump so far, one speciality of the casing is clear. It has an increasing area along the flow direction. Such increasing area will help to accommodate newly added water stream, and will also help to reduce the exit flow velocity. Reduction in the flow velocity will result in increase in the static pressure, which is required to overcome the resistance of pumping system.

NPSH - Overcoming the problem of Cavitation

If pressure at the suction side of impeller goes below vapour pressure of the water, a dangerous phenomenon could happen. Water will start to boil forming vapour bubbles. These bubbles will move along with the flow and will break in a high pressure region. Upon breaking the bubbles will send high impulsive shock waves and spoil impeller material overtime. This phenomenon is known as cavitation. More the suction head, lesser should be the pressure at suction side to lift the water. This fact puts a limit to the maximum suction head a pump can have. However Cavitation can be completely avoided by careful pump selection. The term NPSH (Net Positive Suction Head) helps the designer to choose the right pump which will completely avoid Cavitation. NPSH is defined as follows:

$$NPSH = \left(\frac{P}{\rho g} + \frac{V^2}{2g}\right)_{Suction} - \frac{P_v}{\rho g}$$

Where P_v is vapour pressure of water
V is speed of water at suction side

Work done by the centrifugal pump (or by impeller) on water Velocity triangles at inlet and outlet

Let,

D_1 : *Diameter of impeller at inlet* $= 2 \times R_1$

D_2 : *Diameter of impeller at outlet* $= 2 \times R_2$

N : *Speed of impeller in rpm*

u_1 : *Tangential blade velocity at inlet* $= wR_1 = (\frac{2\pi N}{60})R_1$

u_2 : *Tangential blade velocity at outlet* $= wR_2 = (\frac{2\pi N}{60})R_2$

V : *Absolute velocity*

V_r : *Relative velocity*

V_f : *Velocity of flow*

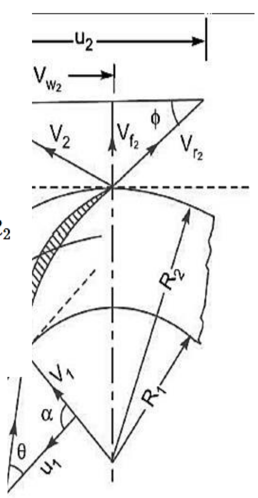
V_w; *Velocity of whirl*

α_1 : *Angle mode by absolute velocity V_1 at inlet*

θ : *Inlet angle of vane*

ϕ : *Outlet angle of vane*

β : *Discharge angle of absolute velocity at outlet*



$$Angular\ momentum = mass \times tangentialvelocity \times Radius$$

$$Angular\ momentum\ entering\ the\ impeller\ per\ sec = m.V_{w1}.R_1$$

$$Angular\ momentum\ leaving\ the\ impeller\ per\ sec = m.V_{w2}.R_2$$

$$Torque\ transmitted = rate\ of\ change\ of\ angular\ momentum$$
$$= m.V_{w2}.R_2 - m.V_{w1}.R_1$$
$$= \frac{w}{g}(V_{w2}.R_2 - V_{w1}.R_1)$$

Since the work done in unit time is given by the product of torque and angular velocity

W.D per sec = Torque x W

$$= \frac{w}{g}(V_{w2}.R_2w - V_{w1}.R_1w)$$

But $R_2w = u_2$ and $R_1w = u_1$

W.D per sec = $\frac{w}{g}(V_{w2}u_2.V_{w1}u_1)$

Work done by impeller per N weight of liquid per sec,

W.D = $\frac{1}{g}(V_{w2}u_2 - V_{w1}u_1)$

But $V_{w1} = 0$ since entry is radial

W.D per N weight per sec = $\frac{V_{w2}.u_2}{g}$

Definitions of Heads and Efficiencies of a centrifugal pump

1. Suction Head (h_s). It is the vertical height of the centre line of the centrifugal pump above the water surface in the tank or pump from which water is to be lifted as shown in Fig. This height is also called suction lift and is denoted by 'h_s'.

2. Delivery Head (h_d). The vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is known as delivery head. This is denoted by 'h_d'.

3. Static Head (H_s). The sum of suction head and delivery head is known as static head. This is represented by 'H_s' and is written as

$$H_s = h_s + h_d.$$

4. Manometric Head (H_m). The manometric head is defined as the head against which a centrifugal pump has to work. It is denoted by 'H_m'. It is given by the following expressions :

(*a*) H_m = Head imparted by the impeller to the water – Loss of head in the pump

$$= \frac{V_{w_2}u_2}{g} - \text{Loss of head in impeller and casing}$$

$$= \frac{V_{w_2}u_2}{g} \quad \text{...if loss of pump is zero}$$

(*b*) H_m = Total head at outlet of the pump – Total head at the inlet of the pump

$$= \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right) \quad \text{...(}$$

(c) $$H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}$$

where h_s = Suction head, h_d = Delivery head,
h_{f_s} = Frictional head loss in suction pipe, h_{f_d} = Frictional head loss in delivery pipe,
V_d = Velocity of water in delivery pipe.

(a) **Manometric Efficiency (η_{man}).**

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$$

$$= \frac{H_m}{\left(\frac{V_{w_2} u_2}{g}\right)} = \frac{gH_m}{V_{w_2} u_2}$$

The power at the impeller of the pump is more than the power given to the water at outlet of the pump. The ratio of the power given to water at outlet of the pump to the power available at the impeller, is known as manometric efficiency.

(b) **Mechanical Efficiency (η_m).**

$$\eta_m = \frac{\text{Power at the impeller}}{\text{Power at the shaft}}$$

The power at the impeller in kW $= \frac{\text{Work done by impeller per second}}{1000}$

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000}$$

ıe pump to the power input to

$$\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} u_2}{1000}\right)}{\text{S.P.}}$$

$\frac{H_m}{00}$

where S.P. = Shaft power.

or

= S.P. of the pump.

$\therefore$ $$\eta_o = \frac{\left(\frac{WH_m}{1000}\right)}{\text{S.P.}}$$

Also $\eta_o = \eta_{man} \times \eta_m$.

PRIMING OF A CENTRIFUGAL PUMP

Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe upto the delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump. Thus the air from these parts of the pump is removed and these parts are filled with the liquid to be pumped.

CAVITATION

Cavitation includes formation of vapour bubbles of the flowing liquid and collapsing of the vapour bubbles. Formation of vapour bubbles of the flowing liquid take place only whenever the pressure in any region falls below vapour pressure. When the pressure of the flowing liquid is less than its vapour pressure, the liquid starts boiling and vapour bubbles are formed. These vapour bubbles are carried along with the flowing liquid to higher pressure zones where these vapours condense and bubbles collapse. Due to sudden collapsing of the bubbles on the metallic surface, high pressure is produced and metallic surfaces are subjected to high local stresses. Thus the surfaces are damaged.

Cavitation in Centrifugal Pumps. In centrifugal pumps the cavitation may occur at the inlet of the impeller of the pump, or at the suction side of the pumps, where the pressure is considerably reduced. Hence if the pressure at the suction side of the pump drops below the vapour pressure of the liquid then the cavitation may occur. The cavitation in a pump can be noted by a sudden drop in efficiency and head. In order to determine whether cavitation will occur in any portion of the suction side of the pump, the critical value of Thoma's cavitation factor (σ) is calculated.

Precaution Against Cavitation.

(*i*) The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. If the flowing liquid is water, then the absolute pressure head should not be below 2.5 m of water.

(*ii*) The special materials or coatings such as aluminium-bronze and stainless steel, which are cavitation resistant materials, should be used.

Effects of Cavitation.

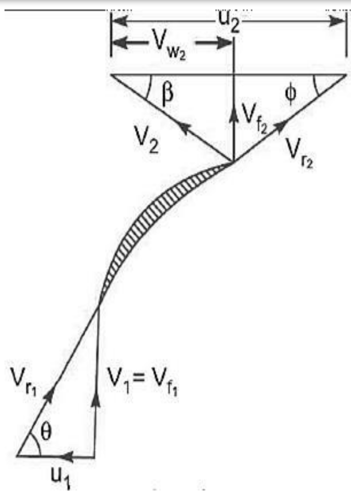
(*i*) The metallic surfaces are damaged and cavities are formed on the surfaces.

(*ii*) Due to sudden collapse of vapour bubble, considerable noise and vibrations are produced.

(*iii*) The efficiency of a turbine decreases due to cavitation. Due to pitting action, the surface of the turbine blades becomes rough and the force exerted by water on the turbine blades decreases. Hence, the work done by water or output horse power becomes less and thus efficiency decreases.

Example *The internal and external diameters of the impeller of a centrifugal pump are 200 and 400 mm respectively. The pump is running at 1200 rpm. The vane angles of the impeller at inlet and outlet are 20 and 30 respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.*

Given:



Internal diameter of impeller, $D_1 = 200$ mm $= 0.20$ m

External diameter of impeller, $D_2 = 400$ mm $= 0.40$ m

Speed, $N = 1200$ r.p.m.

Vane angle at inlet, $\theta = 20°$

Vane angle at outlet, $\phi = 30°$

Water enters radially* means, $\alpha = 90°$ and $V_{w_1} = 0$

Velocity of flow, $V_{f_1} = V_{f_2}$

Tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.20 \times 1200}{60} = 12.56 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13 \text{ m/s.}$$

From inlet velocity triangle, $\tan\theta = \frac{V_{f_1}}{u_1} = \frac{V_{f_1}}{12.56}$

$\therefore$ $V_{f_1} = 12.56 \tan\theta = 12.56 \times \tan 20° = 4.57$ m/s

$\therefore$ $V_{f_2} = V_{f_1} = 4.57$ m/s.

From outlet velocity triangle, $\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{4.57}{25.13 - V_{w_2}}$

$$25.13 - V_{w_2} = \frac{4.57}{\tan\phi} = \frac{4.57}{\tan 30°} = 7.915$$

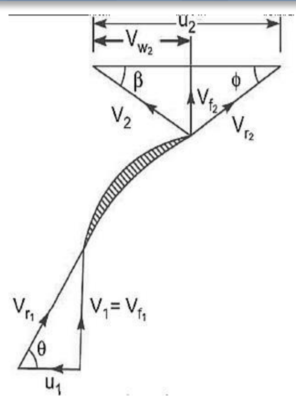
$\therefore$ $V_{w_2} = 25.13 - 7.915 = 17.215$ m/s.

The work done by impeller per kg of water per second is given by equation (

$$= \frac{1}{g} V_{w_2} u_2 = \frac{17.215 \times 25.13}{9.81} = \mathbf{44.1\ Nm/N.}$$

Example *A centrifugal pump is to discharge 0.118 m³/s at a speed of 1450 rpm against a head of 25m. the impeller diameter is 250 mm, its width at outlet is 50 mm and manometric efficiency is 75%. Determine the vane angle at the outer periphery of the impeller.*

Given:



Discharge, $Q = 0.118$ m^3/s

Speed, $N = 1450$ r.p.m.

Head, $H_m = 25$ m

Diameter at outlet, $D_2 = 250$ mm $= 0.25$ m

Width at outlet, $B_2 = 50$ mm $= 0.05$ m

Manometric efficiency, $\eta_{man} = 75\% = 0.75$.

Let vane angle at outlet $= \phi$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1450}{60} = 18.98 \text{ m/s}$$

Discharge is given by $\quad Q = \pi D_2 B_2 \times V_{f_2}$

$$\therefore \quad V_{f_2} = \frac{Q}{\pi D_2 B_2} = \frac{0.118}{\pi \times 0.25 \times .05} = 3.0 \text{ m/s.}$$

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 25}{V_{w_2} \times 18.98}$$

$$V_{w_2} = \frac{9.81 \times 25}{\eta_{man} \times 18.98} = \frac{9.81 \times 25}{0.75 \times 18.98} = 17.23.$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{\left(u_2 - V_{w_2}\right)} = \frac{3.0}{(18.98 - 17.23)} = 1.7143$$

$$\therefore \quad \phi = \tan^{-1} 1.7143 = 59.74° \text{ or } \mathbf{59° \; 44'}. \textbf{ Ans.}$$

Example *A centrifugal pump delivers water against a net head of 14.5 m and a design speed of 1000 rpm. The vanes are curved back at an angle of 30° with the periphery. The impeller diameter is 300 mm and outlet width is 50 mm. determine the discharge of the pump if manometric efficiency is 95%.*

Given:

Net head, $\quad H_m = 14.5$ m

Speed, $\quad N = 1000$ r.p.m.

Vane angle at outlet, $\quad \phi = 30°$

Impeller diameter means the diameter of the impeller at outlet

$\therefore$ Diameter, $\quad D_2 = 300$ mm $= 0.30$ m

Outlet width, $\quad B_2 = 50$ mm $= 0.05$ m

Manometric efficiency, $\quad \eta_{man} = 95\% = 0.95$

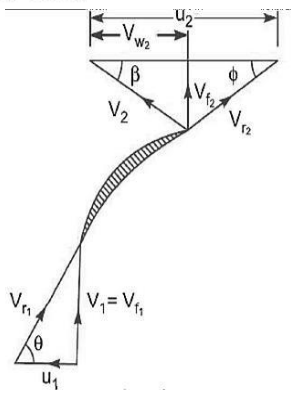
Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1000}{60} = 15.70 \text{ m/s.}$$

$$\eta_{man} = \frac{gH_m}{V_{w_2} \times u_2}$$

$$0.95 = \frac{9.81 \times 14.5}{V_{w_2} \times 15.70}$$

$$V_{w_2} = \frac{0.95 \times 14.5}{0.95 \times 15.70} = 9.54 \text{ m/s.}$$



From outlet velocity triangle, we have

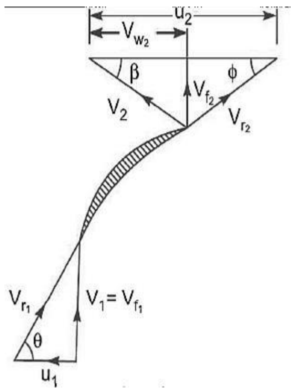
$$\tan\phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \text{ or } \tan 30° = \frac{V_{f_2}}{(15.70 - 9.54)} = \frac{V_{f_2}}{6.16}$$

$$V_{f_2} = 6.16 \times \tan 30° = 3.556 \text{ m/s.}$$

$$Q = \pi D_2 B_2 \times V_{f_2}$$

$$= \pi \times 0.30 \times 0.05 \times 3.556 \text{ m}^3\text{/s} = \mathbf{0.1675\ m^3/s.\ Ans.}$$

Example *A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 rpm works against a total head of 40 m. the velocity of flow through the impeller is constant and equal to 2.5 m/s. the vanes are set back at an angle of 40° at*



Speed, $N = 1000$ r.p.m.

Head, $H_m = 40$ m

Velocity of flow, $V_{f_1} = V_{f_2} = 2.5$ m/s

Vane angle at outlet, $\phi = 40°$

Outer dia. of impeller, $D_2 = 500$ mm $= 0.50$ m

Inner dia. of impeller, $D_1 = \frac{D_2}{2} = \frac{0.50}{2} = 0.25$ m

Width at outlet, $B_2 = 50$ mm $= 0.05$ m

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s}$$

and $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 1000}{60} = 26.18 \text{ m/s.}$$

Discharge is given by, $Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.50 \times .05 \times 2.5 = 0.1963$ m^3/s.

(*i*) **Vane angle at inlet (θ).**

From inlet velocity triangle $\tan\theta = \frac{V_{f_1}}{u_1} = \frac{2.5}{13.09} = 0.191$

$\therefore$ $$\theta = \tan^{-1} .191 = 10.81° \text{ or } \mathbf{10°\ 48'}.$$

(*ii*) **Work done by impeller on water per second** is given by equation

$$= \frac{W}{g} \times V_{w_2} u_2 = \frac{\rho \times g \times Q}{g} \times V_{w_2} \times u_2$$

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times V_{w_2} \times 26.18$$

But from outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.5}{(26.18 - V_{w_2})}$$

$\therefore$ $$26.18 - V_{w_2} = \frac{2.5}{\tan\phi} = \frac{2.5}{\tan 40°} = 2.979$$

$\therefore$ $$V_{w_2} = 26.18 - 2.979 = 23.2 \text{ m/s.}$$

Substituting this value of V_{w_2} in equation (*i*), we get the work done by impeller as

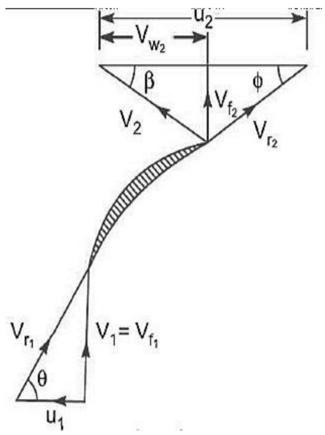
$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times 23.2 \times 26.18$$

$$= \mathbf{119227.9\ Nm/s.} \quad \textbf{Ans.}$$

(*iii*) **Manometric efficiency (η_{man}).** Using equation (19.8), we have

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = \mathbf{64.4\%}.$$

Example *The outer diameter of an impeller of a centrifugal pump is 400 mm and outlet width is 50 mm. the pump is running at 800 rpm and is working against a total head of 15 m. the vanes angle at outlet is 40° and manometric efficiency is 75%. Determine: i) Velocity of flow at outlet. ii) velocity of water leaving the vane. iii) angle made by the*



Outer diameter, $D_2 = 400$ mm$= 0.4$ m

Width at outlet, $B_2 = 50$ mm $= 0.05$ m

Speed, $N = 800$ r.p.m.

Head, $H_m = 15$ m

Vane angle at outlet, $\phi = 40°$

Manometric efficiency, $\eta_{man} = 75\% = 0.75$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 800}{60} = 16.75 \text{ m/s.}$$

$$\eta_{man} = \frac{gH_m}{V_{w_2}\ u_2}$$

$$0.75 = \frac{9.81 \times 15}{V_{w_2} \times 16.75}$$

$$V_{w_2} = \frac{9.81 \times 15}{0.75 \times 16.75} = 11.71 \text{ m/s.}$$

From the outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{V_{f_2}}{(16.75 - 11.71)} = \frac{V_{f_2}}{5.04}$$

(*i*) ∴ $V_{f_2} = 5.04 \tan\phi = 5.04 \times \tan 40° = \mathbf{4.23\ m/s.}$

(*ii*) **Velocity of water leaving the vane (V_2).**

$$V_2 = \sqrt{V_{f_2}^2 + V_{w_2}^2} = \sqrt{4.23^2 + 11.71^2}$$

$$= \sqrt{17.89 + 137.12} = \mathbf{12.45\ m/s.}$$

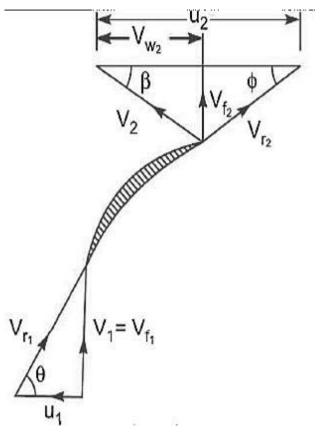
(*iii*) **Angle made by absolute velocity at outlet** (β),

$$\tan\beta = \frac{V_{f_2}}{V_{w_2}} = \frac{4.23}{11.71} = 0.36$$

∴ $\beta = \tan^{-1} 0.36 = 19.80°$ or $\mathbf{19° 48'}$. .23 = $\mathbf{0.265\ m^3/s.}$

Example *The internal diameter and external diameter of an impeller of a centrifugal pump which is running at 1000 rpm are 200 and 40 mm respectively. The discharge through pump is 0.04 m3/s and velocity of flow is constant and equal to 2.0 m/s. the diameter of the suction and delivery pipes are 150 and 100 mm respectively and suction and delivery heads are 6 m (abs.) and 30 m (abs.) of water respectively. If the outlet vane angle is 45° and power required to drive the pump is 16.168 kW, determine: i) Vane angle of the impeller at inlet, ii) the overall efficiency of the pump and iii) manometric efficiency of the pump*

Given:



Speed,	$N = 1000$ r.p.m.
Internal dia.,	$D_1 = 200$ mm $= 0.2$ m
External dia.,	$D_2 = 400$ mm $= 0.4$ m
Discharge,	$Q = 0.04$ m^3/s
Velocity of flow,	$V_{f_1} = V_{f_2} = 2.0$ m/s
Dia. of suction pipe,	$D_s = 150$ mm $= 0.15$ m
Dia. of delivery pipe,	$D_d = 100$ mm $= 0.10$ m
Suction head,	$h_s = 6$ m (abs.)
Delivery head,	$h_d = 30$ m (abs.)
Outlet vane angle,	$\phi = 45°$
Power required to drive the pump,	$P = 16.186/$ kW

From inlet velocity, we have $\tan\theta = \dfrac{V_{f_1}}{u_1} = \dfrac{2.0}{u_1}$, where $u_1 = \dfrac{\pi D_1 N}{60} = \dfrac{\pi \times 0.2 \times 1000}{60} = 10.47$ m/s

$$\therefore \qquad \tan\theta = \frac{2.0}{10.47} = 0.191 \text{ or } \theta = \tan^{-1} .191 = \mathbf{10° \ 48'.\ Ans.}$$

(*ii*) **Overall efficiency of the pump (η_o).**

Using equation (19.10), we have $\eta_o = \dfrac{\left(\dfrac{WH_m}{1000}\right)}{\text{S.P.}}$

where S.P. = Power required to drive the pump and equal to P here.

$$\eta_o = \frac{\left(\dfrac{\rho \times g \times Q \times H_m}{1000}\right)}{P} = \frac{\rho g \times Q \times H_m}{1000 \times P}$$

$$= \frac{1000 \times 9.81 \times .04 \times H_m}{1000 \times 16.186} = 0.02424\ H_m \qquad \text{...}(i)$$

Now H_m is given by equation (19.6) as

$$H_m = \left(\frac{p_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right) \qquad \text{...}(ii)$$

$$H_m = \left(30 + \frac{V_d^2}{2g}\right) - \left(6 + \frac{V_s^2}{2g}\right) \qquad \text{...}(iii)$$

$$V_d = \frac{\text{Discharge}}{\text{Area of delivery pipe}} = \frac{0.04}{\dfrac{\pi}{4}(D_d)^2} = \frac{.04}{\dfrac{\pi}{4} \times .1^2} = 5.09 \text{ m/s}$$

$$V_s = \frac{.04}{\text{Area of suction pipe}} = \frac{.04}{\dfrac{\pi}{4} D_s^2} = \frac{.04}{\dfrac{\pi}{4} \times .15^2} = 2.26 \text{ m/s.}$$

$$H_m = \left(30 + \frac{5.09^2}{2 \times 9.81}\right) - \left(6 + \frac{2.26^2}{2 \times 9.81}\right)$$

$$= (30 + 1.32) - (6 + .26) = 31.32 - 6.26 = 25.06 \text{ m.}$$

Substituting the value of 'H_m' in equation (i), we get

$$\eta_o = .02424 \times 25.06 = 0.6074 = \mathbf{60.74\%}.$$

(iii) **Manometric efficiency of the pump (η_{man}).**

Tangential velocity at outlet is given by

$$u_2 = \frac{\pi D_2 \times N}{60} = \frac{\pi \times 0.4 \times 1000}{60} = 20.94 \text{ m/s.}$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.0}{20.94 - V_{w_2}}$$

$$\therefore \quad 20.94 - V_{w_2} = \frac{2.0}{\tan \phi} = \frac{2.0}{\tan 45} = 2.0$$

$$\therefore \quad V_{w_2} = 20.94 - 2.0 = 18.94.$$

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 25.06}{18.94 \times 20.94} = 0.6198 = \mathbf{61.98\%}.$$

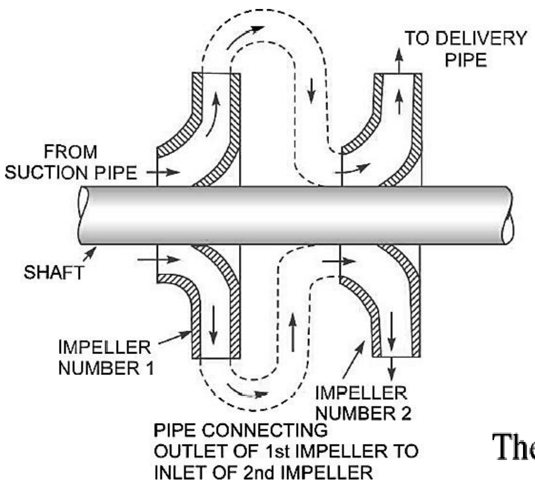
MULTISTAGE CENTRIFUGAL PUMPS

If a centrifugal pump consists of two or more impellers, the pump is called a multistage centrifugal pump. The impellers may be mounted on the same shaft or on different shafts. A multistage pump is having the following two important functions :

1. To produce a high head, and
2. To discharge a large quantity of liquid.

If a high head is to be developed, the impellers are connected in series (or on the same shaft) while for discharging large quantity of liquid, the impellers (or pumps) are connected in parallel.

Multistage Centrifugal Pumps for High Heads.



Then total head developed

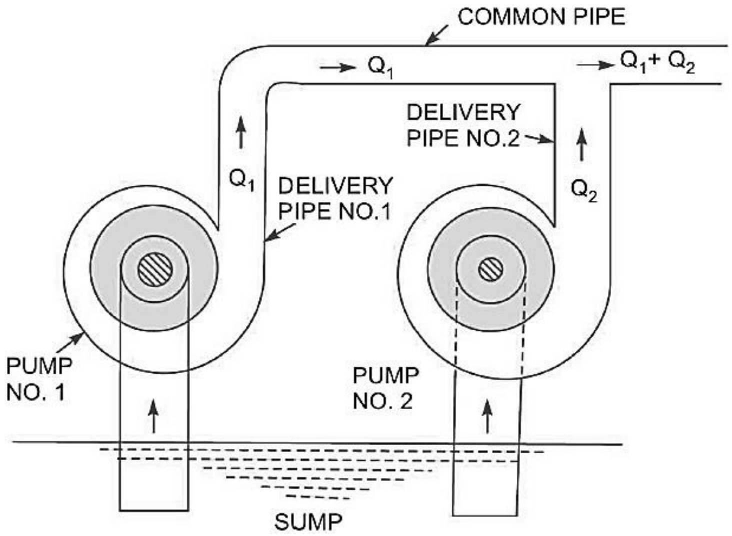
$$= n \times H_m$$

The discharge passing through each impeller is same

n = Number of identical impellers mounted on the same shaft,

H_m = Head developed by each impeller.

Multistage Centrifugal Pumps for High Discharge.



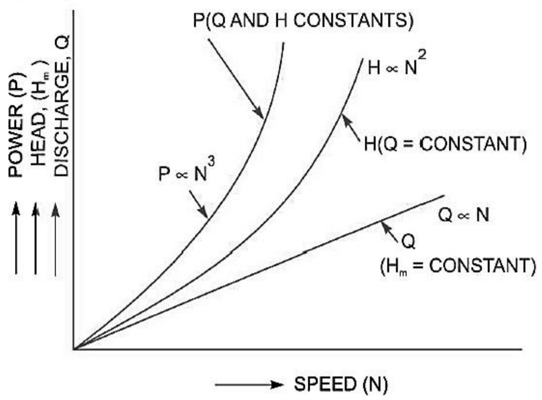
Let n = Number of identical pumps arranged in parallel.

Q = Discharge from one pump.

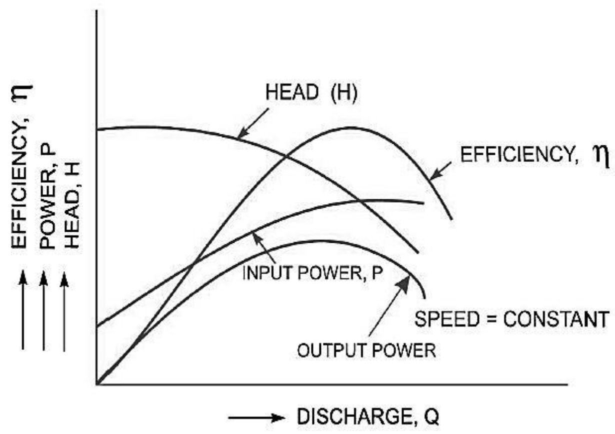
$\therefore$ Total discharge $= n \times Q$

CHARACTERISTIC CURVES OF CENTRIFUGAL PUMPS

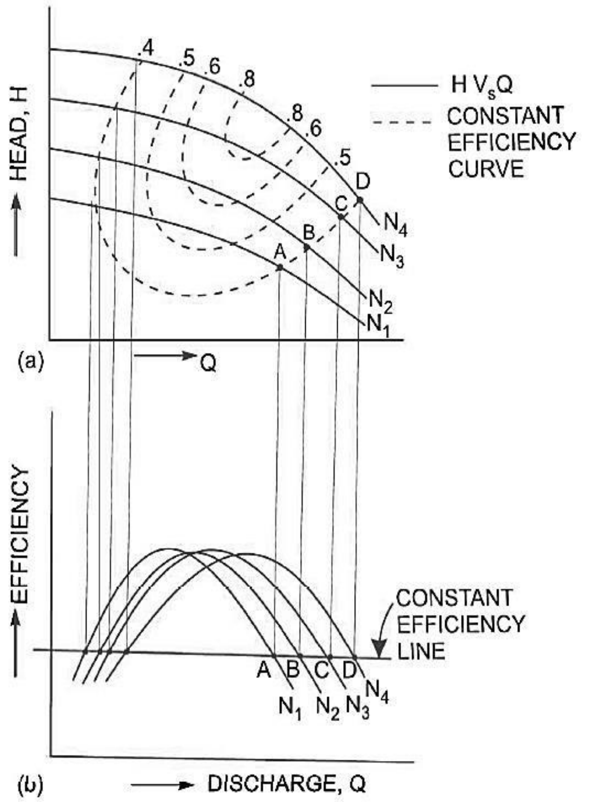
Main Characteristic Curves.



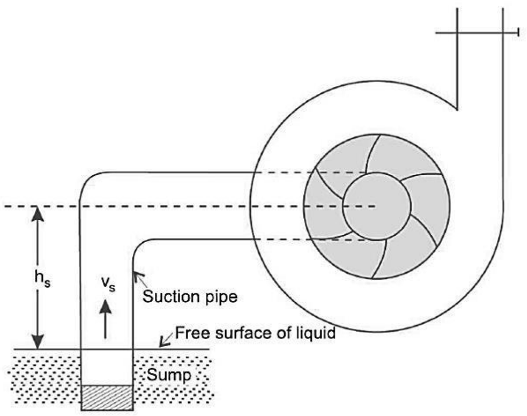
Operating Characteristic Curves.



Constant Efficiency Curves.



MAXIMUM SUCTION LIFT (or SUCTION HEIGHT)



Applying Bernoulli's equation at the free surface of liquid in the sump and section 1 in the suction pipe just at the inlet of the pump and taking the free surface of liquid as datum line, we get

$$\frac{p_a}{\rho g} + \frac{V_a^2}{2g} + Z_a = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + h_L \qquad ...(i)$$

$$\frac{p_a}{\rho g} + 0 + 0 = \frac{p_1}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right) \qquad ...(ii)$$

For finding the maximum suction lift, the pressure at the inlet of the pump should not be less than the vapour pressure of the liquid. Hence for the limiting case, taking the pressure at the inlet of pump equal to vapour pressure of the liquid, we get

$p_1 = p_v$, where p_v = vapour pressure of the liquid in absolute units.

Now the equation (*ii*) becomes as

$$\frac{p_v}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

$$\frac{p_a}{\rho g} = \frac{p_v}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s} \qquad (\because \quad p_1 = p_v) \ ...(iii)$$

$$\frac{p_a}{\rho g} = \text{Atmospheric pressure head } = H_a \text{ (meter of liquid)}$$

$$\frac{p_v}{\rho g} = \text{Vapour pressure head} = H_v \text{ (meter of liquid)}$$

Now, equation (*iii*) becomes as

$$H_a = H_v + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

$$h_s = H_a - H_v - \frac{v_s^2}{2g} - h_{f_s}$$

Equation (19.31) gives the value of maximum suction lift (or maximum suction height) for a centrifugal pump. Hence, the suction height of any pump should not be more than that given by equation (19.31). If the suction height of the pump is more, then vaporization of liquid at inlet of pump will take place and there will be a possibility of cavitation.

NET POSITIVE SUCTION HEAD (NPSH)

The term NPSH (Net Positive Suction Head) is very commonly used in the pump industry. Actually the minimum suction conditions are more frequently specified in terms of NPSH.

The net positive suction head (NPSH) is defined as the *absolute* pressure head at the inlet to the pump, minus the vapour pressure head (in absolute units) plus the velocity head.

$\therefore$ NPSH = Absolute pressure head at inlet of the pump – vapour pressure head (absolute units) + velocity head

$$= \frac{p_1}{\rho g} - \frac{p_v}{\rho g} + \frac{v_s^2}{2g} \qquad (\because \text{ Absolute pressure at inlet of pump} = p_1)\ .$$

the absolute pressure head at inlet of the pump is given by as

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

$$\text{NPSH} = \left[\frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)\right] - \frac{p_v}{\rho g} + \frac{v_s^2}{2g}$$

$$= \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{f_s}$$

$$= H_a - H_v - h_s - h_{f_s}$$

RECIPROCATING PUMP

If the mechanical energy is converted into hydraulic energy by sucking the liquid into a cylinder in which a piston is reciprocating, which exerts the thrust on the liquid and increases its hydraulic energy is known as reciprocating pump. A reciprocating pump is a positive displacement pump. It is often used where relatively small quantity of liquid is to be handled and where delivery pressure is quite large.

Reciprocating pump consists of following parts.

1. A cylinder with a piston	5. suction pipe
2. piston rod	6. delivery pipe
3. connecting rod	7. suction valve
4. crank	8. delivery valve

WORKING OF A SINGLE-ACTING RECIPROCATING PUMP

Single acting reciprocating pump:-

A single acting reciprocating pump, which consists of a piston which moves forwards and backwards in a close fitting cylinder. The movement of the piston is obtained by connecting the piston rod to crank by means of a connecting rod. The crank is rotated by means of an electric motor. Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder. The suction and delivery valves are one way valves or non- return valves, which allow the water to flow in one direction only. Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe.

The rotation of the crank brings about an outward and inward movement of the piston in the cylinder. During the suction stroke the piston is moving towards right in the cylinder, this movement of piston causes vacuum in the cylinder. The pressure of the atmosphere acting on the sump water surface forces the water up in the suction pipe. The forced water opens the suction valve and the water enters the cylinder. The piston from its extreme right position starts moving towards left in the cylinder. The movement of the piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure. Hence suction valve closes and delivery valve opens. The liquid is forced into the delivery pipe and is raised to a required height.

For one revolution of the crank, the quantity of water raised up in the delivery pipe is equal to the stroke volume in the cylinder in the single acting pump and twice this volume in the double acting pump. Discharge through a single acting reciprocating pump.

D = diameter of the cylinder
A = cross section are of the piston or cylinder
r = radius of crank
N = r.p.m of the crank
L = Length of the stroke = 2 x r
h_s = Suction head or height of axis of the cylinder from water surface in sump.
h_d = Delivery head or height of the delivery outlet above the cylinder axis.
Discharge of water in one revolution = Area x Length of stroke
= A x L
Number of revolution per second = N/60
Discharge of the pump per second
Q = Discharge in one revolution x No.of revolution per second

$$= A \times L \times \frac{N}{60} = \frac{ALN}{60} \text{ m}^3\text{/sec}$$

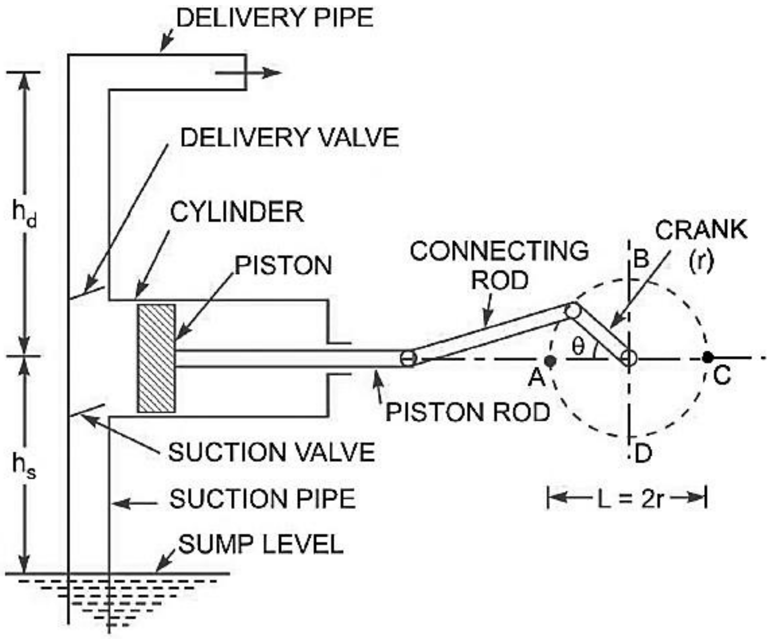
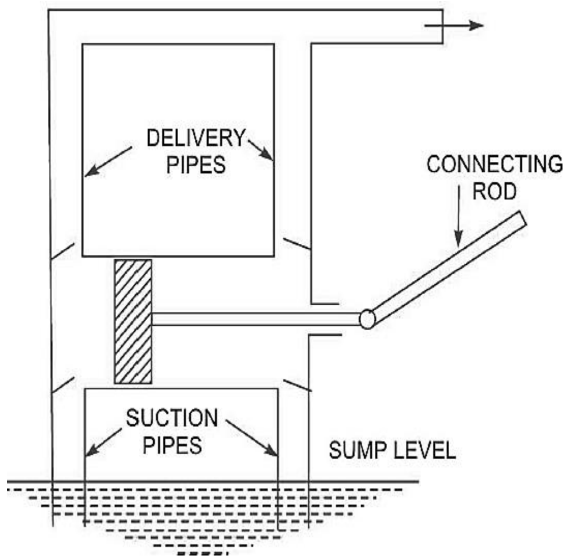


Fig.3

<u>Double acting reciprocating pump</u>



<u>Fig.4</u>

Discharge Through a Reciprocating Pump

Let D = Diameter of the cylinder

A = Cross-sectional area of the piston or cylinder

$$= \frac{\pi}{4} D^2$$

r = Radius of crank

N = r.p.m. of the crank

L = Length of the stroke = $2 \times r$

h_s = Height of the axis of the cylinder from water surface in sump.

h_d = Height of delivery outlet above the cylinder axis (also called delivery head)

Volume of water delivered in one revolution or discharge of water in one revolution

$$= \text{Area} \times \text{Length of stroke} = A \times L$$

Number of revolution per second, $= \dfrac{N}{60}$

$\therefore$ Discharge of the pump per second,

$$Q = \text{Discharge in one revolution} \times \text{No. of revolution per second}$$

$$= A \times L \times \frac{N}{60} = \frac{ALN}{60} \qquad ...(20.1)$$

Weight of water delivered per second,

$$W = \rho \times g \times Q = \frac{\rho g ALN}{60}.$$

Work done by Reciprocating Pump.

Work done per second = Weight of water lifted per second × Total height through which water is lifted

$$= W \times (h_s + h_d) \qquad ...(i)$$

where $(h_s + h_d)$ = Total height through which water is lifted.

From equation (20.2), Weight, W, is given by

$$W = \frac{\rho g \times ALN}{60}.$$

Substituting the value of W in equation (i), we get

$$\text{Work done per second} = \frac{\rho g \times ALN}{60} \times (h_s + h_d) \qquad ...(20.3)$$

$\therefore$ Power required to drive the pump, in kW

$$P = \frac{\text{Work done per second}}{1000} = \frac{\rho g \times ALN \times (h_s + h_d)}{60 \times 1000}$$

$$= \frac{\rho g \times ALN \times (h_s + h_d)}{60{,}000} \text{ kW} \qquad ...(20.4)$$

Discharge, Work done and Power Required to Drive a Double-acting Pump.

Let D = Diameter of the piston,

d = Diameter of the piston rod

$\therefore$ Area on one side of the piston,

$$A = \frac{\pi}{4} D^2$$

Area on the other side of the piston, where piston rod is connected to the piston,

$$A_1 = \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2 = \frac{\pi}{4} (D^2 - d^2).$$

$\therefore$ Volume of water delivered in one revolution of crank

$$= A \times \text{Length of stroke} + A_1 \times \text{Length of stroke}$$

$$= AL + A_1L = (A + A_1)L = \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}\left(D^2 - d^2\right)\right] \times L$$

$\therefore$ Discharge of pump per second

= Volume of water delivered in one revolution $\times$ No. of revolution per second

$$= \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}\left(D^2 - d^2\right)\right] \times L \times \frac{N}{60}$$

If 'd' the diameter of the piston rod is very small as compared to the diameter of the piston, then it can be neglected and discharge of pump per second,

$$Q = \left(\frac{\pi}{4}D^2 + \frac{\pi}{4}D^2\right) \times \frac{L \times N}{60} = 2 \times \frac{\pi}{4}D^2 \times \frac{L \times N}{60} = \frac{2ALN}{60} \quad ...(20.5)$$

Work done by double-acting reciprocating pump

Work done per second = Weight of water delivered $\times$ Total height

$$= \rho g \times \text{Discharge per second} \times \text{Total height}$$

$$= \rho g \times \frac{2ALN}{60} \times (h_s + h_d) = 2\rho g \times \frac{ALN}{60} \times (h_s + h_d)$$

$\therefore$ Power required to drive the double-acting pump in kW,

$$P = \frac{\text{Work done per second}}{1000} = 2\rho g \times \frac{ALN}{60} \times \frac{(h_s + h_d)}{1000}$$

$$= \frac{2\rho g \times ALN \times (h_s + h_d)}{60{,}000}$$

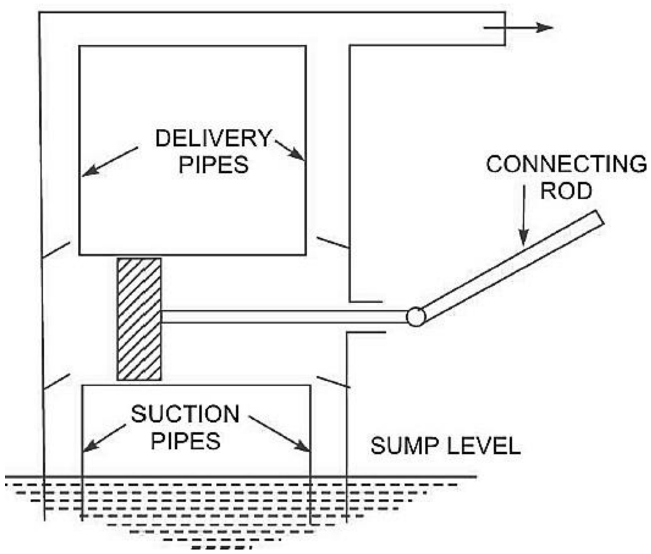


Fig.5

SLIP OF RECIPROCATING PUMP

The actual discharge of the pump is always less than theoretical discharge. The difference between theoretical discharge and actual discharge is known as Slip of the reciprocating pump

$$\text{Slip} = Q_{th} - Q_{act}$$

But slip is mostly expressed as percentage slip which is given by,

$$\text{Percentage slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$= (1 - C_d) \times 100 \qquad \left(\because \frac{Q_{act}}{Q_{th}} = C_d\right)$$

where C_d = Co-efficient of discharge.

Negative Slip of the Reciprocating Pump.

Negative Slip of the Reciprocating Pump. Slip is equal to the difference of theoretical discharge and actual discharge. If actual discharge is more than the theoretical discharge, the slip of the pump will become –ve. In that case, the slip of the pump is known as negative slip.

Negative slip occurs when delivery pipe is short, suction pipe is long and pump is running at high speed.

Example *A single acting reciprocating pump, running at 50 rpm, delivers 0.01m3/s of water. The diameter of the piston is 200 mm and stroke length 400 m. Determine: i) theoretical discharge of the pump ii) Co – efficient of discharge and iii) Slip and the percentage of slip of the pump.*

Given:

Solution. Given :

Speed of the pump, $N = 50$ r.p.m.

Actual discharge, $Q_{act} = .01 \text{ m}^3/\text{s}$

Dia. of piston, $D = 200 \text{ mm} = .20 \text{ m}$

$\therefore$ Area, $A = \frac{\pi}{4}(.2)^2 = .031416 \text{ m}^2$

Stroke, $L = 400 \text{ mm} = 0.40 \text{ m}$.

(*i*) Theoretical discharge for single-acting reciprocating pump is given by equation (20.1) as

$$Q_{th} = \frac{A \times L \times N}{60} = \frac{.031416 \times .40 \times 50}{60} = \mathbf{0.01047\ m^3/s.\ Ans.}$$

(*ii*) Co-efficient of discharge is given by

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{.01047} = \mathbf{0.955.\ Ans.}$$

(*iii*) Using equation (20.8), we get

$$\text{Slip} = Q_{th} - Q_{act} = .01047 - .01 = \mathbf{0.00047\ m^3/s.\ Ans.}$$

And percentage slip
$$= \frac{(Q_{th} - Q_{act})}{Q_{th}} \times 100 = \frac{(.01047 - .01)}{.01047} \times 100$$

$$= \frac{.00047}{.01047} \times 100 = \mathbf{4.489\%.\ Ans.}$$

Example *A double-acting reciprocating pump, running at 40 r.p.m., is discharging 1.0 m^3 of water per minute. The pump has a stroke of 400 mm. The diameter of the piston is 200 mm. The delivery and suction head are 20 m and 5 m respectively. Find the slip of the pump and power required to drive the pump.*

Speed of pump, $N = 40$ r.p.m.

Actual discharge, $Q_{act} = 1.0 \text{ m}^3/\text{min} = \frac{1.0}{60} \text{ m}^3/\text{s} = 0.01666 \text{ m}^3/\text{s}$

Stroke, $L = 400 \text{ mm} = 0.40 \text{ m}$

Diameter of piston, $D = 200 \text{ mm} = 0.20 \text{ m}$

$\therefore$ Area, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (.2)^2 = 0.031416 \text{ m}^2$

Suction head, $h_s = 5$ m

Delivery head, $h_d = 20$ m.

Theoretical discharge for double-acting pump is given by equation (20.5) as,

$$Q_{th} = \frac{2ALN}{60} = \frac{2 \times .031416 \times 0.4 \times 40}{60} = .01675 \text{ m}^3/\text{s}.$$

Using equation (20.8), Slip = $Q_{th} - Q_{act}$ = .01675 – .01666 = **.00009 $\mathbf{m^3/s}$. Ans.**

Power required to drive the double-acting pump is given by equation (20.7) as,

$$P = \frac{2 \times \rho g \times ALN \times (h_s + h_d)}{60{,}000} = \frac{2 \times 1000 \times 9.81 \times .031416 \times .4 \times 40 \times (5 + 20)}{60{,}000}$$

$$= \mathbf{4.109 \text{ kW. Ans.}}$$

INDICATOR DIAGRAM

indicator diagram is a graph between pressure head and stroke length of the piston for one complete revolution. The pressure head is taken as ordinate and stroke length as abscissa.

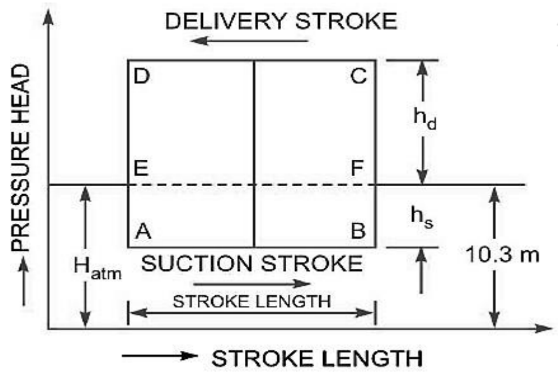


Fig. 6 *Ideal indicator diagram.*

Let H_{atm} = Atmospheric pressure head

= 10.3 m of water,

L = Length of the stroke,

h_s = Suction head, and

h_d = Delivery head.

we know that the work done by the pump per second

$$= \frac{\rho \times g \times ALN}{60} \times (h_s + h_d)$$

$$= K \times L(h_s + h_d) \qquad \left(\text{where } K = \frac{\rho g A N}{60} = \text{Constant}\right)$$

$$\propto L \times (h_s + h_d) \qquad \ldots(i)$$

Work done by pump $\propto$ Area of indicator diagram.

SEPARATION OF LIQUID

If the pressure in the cylinder is below the vapour pressure, dissolved gasses will be liberated from the liquid and cavitation will takes place. The continuous flow of liquid will not exist which means separation of liquid takes place. The pressure at which separation takes place is called separation pressure and head corresponding to the separation pressure is called separation pressure head.

The ways to avoid cavitation in reciprocating pumps:

1. **Design:** Ensure that there are no sharp corners or curvatures of flow in the system while designing the pump.
2. **Material:** Cavitation resistant materials like Bronze or Nickel can be used.
3. **Model Testing:** Before manufacturing, a scaled down model should be tested.
4. **Admission of air:** High pressure air can be injected into the low pressure zones of flowing liquid to prevent bubble formation.

AIR VESSELS

An air vessel is a closed chamber containing compressed air in the top portion and liquid (or water) at the bottom of the chamber. At the base of the chamber there is an opening through which the liquid (or water) may flow into the vessel or out from the vessel. When the liquid enters the air vessel, the air gets compressed further and when the liquid flows out the vessel, the air will expand in the chamber.

An air vessel is fitted to the suction pipe and to the delivery pipe at a point close to the cylinder of a single-acting reciprocating pump :

(*i*) to obtain a continuous supply of liquid at a uniform rate,

(*ii*) to save a considerable amount of work in overcoming **friction in suction and delivery pipes**

(*iii*) to run the pump at a high speed without separation.

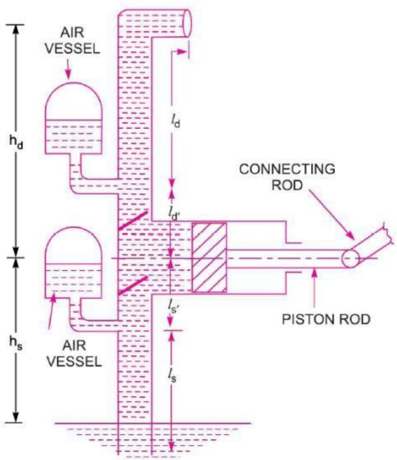


Fig.

Centrifugal pumps	*Reciprocating pumps*
1. The discharge is continuous and smooth.	1. The discharge is fluctuating and pulsating.
2. It can handle large quantity of liquid.	2. It handles small quantity of liquid only.
3. It can be used for lifting highly viscous liquids.	3. It is used only for lifting pure water or less viscous liquids.
4. It is used for large discharge through smaller heads.	4. It is meant for small discharge and high heads.
5. Cost of centrifugal pump is less as compared to reciprocating pump.	5. Cost of reciprocating pump is approximately four times the cost of centrifugal pump.
6. Centrifugal pump runs at high speed. They can be coupled to electric motor.	6. Reciprocating pump runs at low speed. Speed is limited due to consideration of separation and cavitation.
7. The operation of centrifugal pump is smooth and without much noise. The maintenance cost is low.	7. The operation of reciprocating pump is complicated and with much noise. The maintenance cost is high.
8. Centrifugal pump needs smaller floor area and installation cost is low.	8. Reciprocating pump requires large floor area and installation cost is high.
9. Efficiency is high.	9. Efficiency is low.

TURBINES

Hydraulic machines are defined as those machines which convert either hydraulic energy (energy possessed by water) into mechanical energy (which is further converted into electrical energy) or mechanical energy into hydraulic energy. The hydraulic machines, which convert the hydraulic energy into mechanical energy, are called turbines while the hydraulic machines which convert the mechanical energy into hydraulic energy. The study of hydraulic machines consists of turbines and pumps.

Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy. This, mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine. Thus the mechanical energy is converted into electrical energy. The electric power which is obtained from the hydraulic energy (energy of water) is known as Hydroelectric power. At present the generation of hydroelectric power is the cheapest as compared by the power generated by other sources such as oil, coal etc.

General Layout of a Hydroelectric Power Plant

1. A dam constructed across a river to store water.
2. Pipes of large diameters called penstocks, which carry water under pressure from the storage reservoir to the turbines. These pipes are made of steel or reinforced concrete.
3. Turbines having different types of vanes fitted to the wheels.
4. Tail race, which is a channel which carries water away from the turbines after the water has worked on the turbines. The surface of water in the tail race channel is also known as tail race.

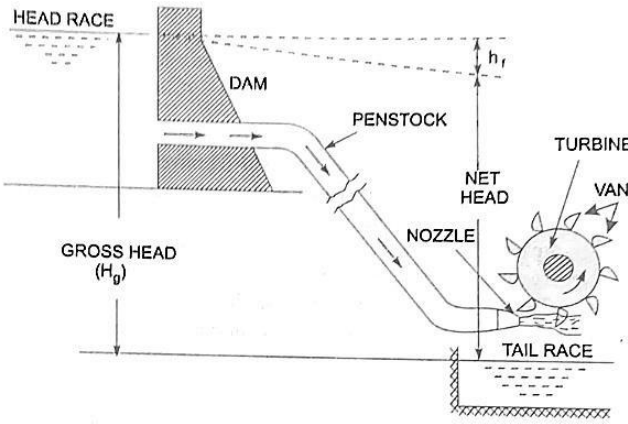


Fig. Layout of hydroelectric power plant

Definitions of Heads and Efficiencies of a Turbine

1. Gross Head. The difference between the head race level and tail race level when no water is flowing is known as Gross Head. It is denoted by 'H_g".
2. Net Head. It is also called effective head and is defined as the head available at the inlet of the turbine, when water is flowing from head race to the turbine, a loss of head due to friction between water and penstock occurs. Though there are other losses also such as loss due to bend, Pipes, fittings, loss at the entrance of penstock etc., yet they are having small magnitude as compared to head loss due to friction. In 'h_f' is the head loss due to friction between penstocks and water then net heat on turbine is given by

$$H = H_g - h_f$$

where H_g = Gross head, $h_f = \dfrac{4 \times f \times L \times V^2}{D \times 2g}$,

in which V = Velocity of flow in penstock,

L = Length of penstock,

D = Diameter of penstock.

Efficiencies of a Turbine.

(*a*) **Hydraulic Efficiency (η_h).**

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}} = \frac{\text{R.P.}}{\text{W.P.}}$$

Power supplied at the inlet of turbine in S.I.units is known as water power. It is given by

$$\text{W.P.} = \frac{\rho \times g \times Q \times H}{1000} \text{ kW}$$

R.P. = Power delivered to runner *i.e.*, runner power

$$= \frac{W}{g}\frac{\left[V_{w_1} \pm V_{w_2}\right] \times u}{1000} \text{ kW} \qquad \text{...for Pelton Turbine}$$

$$= \frac{W}{g}\frac{\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{1000} \text{ kW} \qquad \text{...for a radial flow turbine}$$

(*b*) **Mechanical Efficiency (η_m).**

$$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the runner}} = \frac{\text{S.P.}}{\text{R.P.}}$$

(*c*) **Volumetric Efficiency (η_v).**

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}}$$

(*d*) **Overall Efficiency (η_o)**

$$\eta_o = \frac{\text{Volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}} = \frac{\text{Shaft power}}{\text{Water power}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}}$$

$$= \eta_m \times \eta_h$$

CLASSIFICATION OF HYDRAULIC TURBINES

1. According to the type of energy at inlet :
   (*a*) Impulse turbine, and (*b*) Reaction turbine.
2. According to the direction of flow through runner :
   (*a*) Tangential flow turbine, (*b*) Radial flow turbine,
   (*c*) Axial flow turbine, and (*d*) Mixed flow turbine.
3. According to the head at the inlet of turbine :
   (*a*) High head turbine, (*b*) Medium head turbine, and
   (*c*) Low head turbine.
4. According to the specific speed of the turbine :
   (*a*) Low specific speed turbine, (*b*) Medium specific speed turbine, and
   (*c*) High specific speed turbine.

Impulse Turbine	Reaction Turbine
1. All the available energy of the fluid is converted into kinetic energy by an efficient nozzle that forms a free jet.	1. Only a portion of the fluid energy is transformed into kinetic energy before the fluid enters the turbine runner.
2. The jet is unconfined and at atmospheric pressure throughout the action of water on the runner, and during its subsequent flow to the tail race.	2. Water enters the runner with an excess pressure, and then both the velocity and pressure change as water passes through the runner.
3. Blades are only in action when they are in front of the nozzle.	3. Blades are in action all the time.
4. Water may be allowed to enter a part or whole of the wheel circumference.	4. Water is admitted over the circumference of the wheel.
5. The wheel does not run full and air has free access to the buckets.	5. Water completely fills the vane passages throughout the operation of the turbine.
6. Casing has no hydraulic function to perform; it only serves to prevent splashing and to guide the water to the tail race.	6. Pressure at inlet to the turbine is much higher than the pressure at outlet ; unit has to be sealed from atmospheric conditions and, therefore, casing is absolutely essential.
7. Unit is installed above the tail race.	7. Unit is kept entirely submerged in water below the tail race.
8. Flow regulation is possible without loss.	8. Flow regulation is always accompanied by loss.
9. When water glides over the moving blades, its relative velocity either remains constant or reduces slightly due to friction.	9. Since there is continuous drop in pressure during flow through the blade passages, the relative velocity does increase.

PELTON WHEEL (OR TURBINE)

The Pelton wheel or Pelton turbine is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of the turbine is atmospheric. This turbine is used for high heads and is named after L.A. Pelton, an American Engineer.

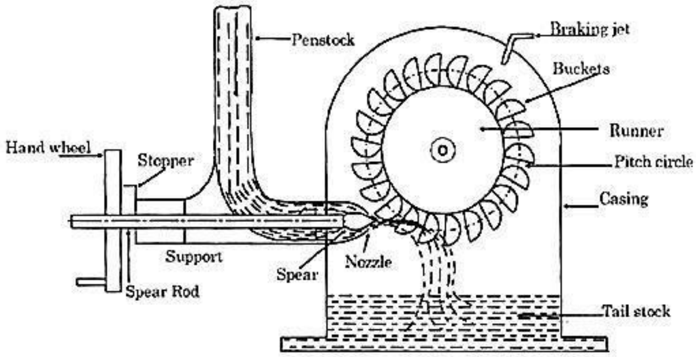
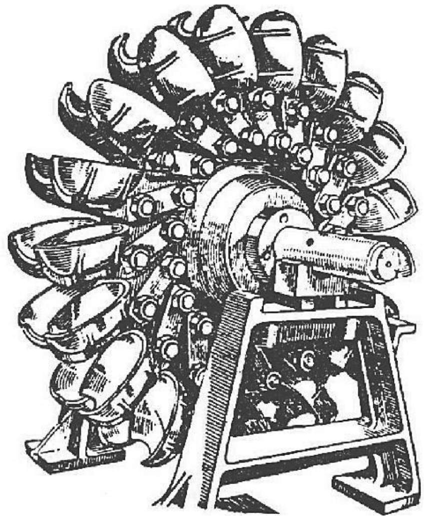


Fig. : Pelton Turbine



Main parts of Pelton Wheel

1. Nozzle and Flow Regulating Arrangement. The amount of water striking the buckets (vanes) of the runner is controlled by providing a spear in the nozzle as shown in Fig. 18.2. The spear is a conical needle which is operated either by a hand wheel or automatically in an axial direction depending upon the size of the unit. When the spear is pushed forward into the nozzle the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.

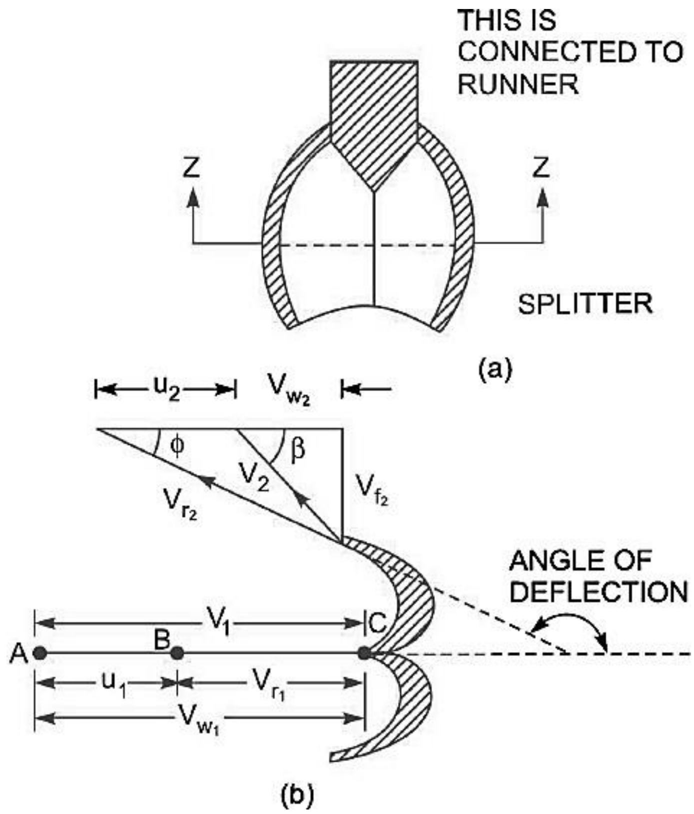
2. Runner with Buckets. Fig. 18.3 shows the runner of a Pelton wheel. It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided into two symmetrical parts by a dividing wall which is known as splitter.

The jet of water strikes on the splitter. The splitter divides the jet into two equal parts and the jet comes out at the outer edge of the bucket. The buckets are shaped in such a way that the jet gets deflected through 160° or 170°. The buckets are made of cast iron, cast steel bronze or stainless steel depending upon the head at the inlet of the turbine.

3. Casing. Fig. 18.4 shows a Pelton turbine with a casing. The function of the casing is to prevent the splashing of the water and to discharge water to tail race. It also acts as safeguard against accidents. It is made of cast iron or fabricated steel plates. The casing of the Pelton wheel does not perform any hydraulic function.

4. Breaking Jet.

Velocity Triangles and Work done for Pelton Wheel.



Let H = Net head acting on the Pelton wheel

$= H_g - h_f$

where H_g = Gross head and $h_f = \dfrac{4fLV^2}{D^* \times 2g}$

where D^* = Dia. of Penstock, N = Speed of the wheel in r.p.m.,

D = Diameter of the wheel, d = Diameter of the jet.

Then V_1 = Velocity of jet at inlet = $\sqrt{2gH}$

$$u = u_1 = u_2 = \frac{\pi DN}{60}.$$

The velocity triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1$$

$$\alpha = 0° \text{ and } \theta = 0°$$

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \text{ and } V_{w_2} = V_{r_2} \cos\phi - u_2.$$

The force exerted by the jet of water in the direction of motion is given by equation

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}]$$

As the angle β is an acute angle, +ve sign should be taken.

$$a = \text{Area of jet} = \frac{\pi}{4}d^2.$$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 \, [V_{w_1} + V_{w_2}] \times u \text{ Nm/s}$$

Power given to the runner by the jet

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{1000} \text{ kW}$$

Work done/s per unit weight of water striking/s

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\text{Weight of water striking/s}}$$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\rho a V_1 \times g} = \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u$$

The energy supplied to the jet at inlet is in the form of kinetic energy and is equal to $\frac{1}{2}mV^2$

$\therefore$ K.E. of jet per second $= \frac{1}{2}\,(\rho a V_1) \times V_1^2$

$\therefore$ **Hydraulic efficiency**, $\eta_h = \dfrac{\text{Work done per second}}{\text{K.E. of jet per second}}$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\frac{1}{2}(\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2}$$

Now $\quad V_{w_1} = V_1,\ V_{r_1} = V_1 - u_1 = (V_1 - u)$

$\therefore \quad V_{r_2} = (V_1 - u)$

and $\quad V_{w_2} = V_{r_2} \cos\phi - u_2 = V_{r_2} \cos\phi - u = (V_1 - u)\cos\phi - u$

Substituting the values of V_{w_1} and V_{w_2} in equation

$$\eta_h = \frac{2\left[V_1 + (V_1 - u)\cos\phi - u\right] \times u}{V_1^2}$$

$$= \frac{2\left[V_1 - u + (V_1 - u)\cos\phi\right] \times u}{V_1^2} = \frac{2(V_1 - u)\,[1 + \cos\phi]\,u}{V_1^2}.$$

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u(V_1 - u)(1 + \cos\phi)}{V_1^2}\right] = 0$$

$$\text{or} \quad \frac{(1 + \cos\phi)}{V_1^2}\frac{d}{du}(2uV_1 - 2u^2) = 0 \quad \text{or} \quad \frac{d}{du}[2uV_1 - 2u^2] = 0 \quad \left(\because \frac{1 + \cos\phi}{V_1^2} \neq 0\right)$$

$$2V_1 - 4u = 0 \quad \text{or} \quad u = \frac{V_1}{2}$$

substituting the value of $u = \dfrac{V_1}{2}$

$$\text{Max. } \eta_h = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1 + \cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \frac{V_1}{2}(1 + \cos\phi)\frac{V_1}{2}}{V_1^2} = \frac{(1 + \cos\phi)}{2}.$$

Points to be Remembered for Pelton Wheel

(*i*) The velocity of the jet at inlet is given by $V_1 = C_v\sqrt{2gH}$

where C_v = Co-efficient of velocity = 0.98 or 0.99

H = Net head on turbine

(*ii*) The velocity of wheel (u) is given by $u = \phi\sqrt{2gH}$

where ϕ = Speed ratio. The value of speed ratio varies from 0.43 to 0.48.

(*iii*) The angle of deflection of the jet through buckets is taken at 165° if no angle of deflection is given.

(*iv*) The mean diameter or the pitch diameter D of the Pelton wheel is given by

$$u = \frac{\pi DN}{60} \quad \text{or} \quad D = \frac{60u}{\pi N}.$$

(*v*) **Jet Ratio.** It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d). It is denoted by 'm' and is given as

$$m = \frac{D}{d} \quad (= 12 \text{ for most cases})$$

(*vi*) Number of buckets on a runner is given by

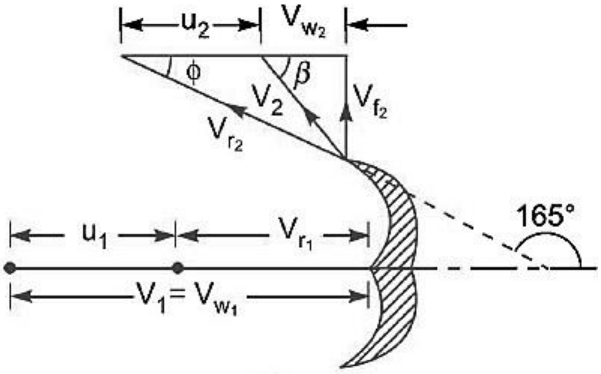
$$Z = 15 + \frac{D}{2d} = 15 + 0.5\text{ m} \qquad \text{...(18.17)}$$

where m = Jet ratio

(*vii*) **Number of Jets.** It is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet.

Example *A Pelton wheel has a mean bucket speed of 10 metres per second with a jet of water flowing at the rate of 700 litres/s under a head of 30 metres. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.*

Speed of bucket,	$u = u_1 = u_2 = 10$ m/s
Discharge,	$Q = 700$ litres/s $= 0.7$ m^3/s, Head of water, $H = 30$ m
Angle of deflection	$= 160°$
∴ Angle,	$\phi = 180° - 160° = 20°$
Co-efficient of velocity,	$C_v = 0.98$.
The velocity of jet,	$V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/s



$$\therefore \quad V_{r_1} = V_1 - u_1 = 23.77 - 10$$

$$= 13.77 \text{ m/s}$$

$$V_{w_1} = V_1 = 23.77 \text{ m/s}$$

From outlet velocity triangle,

$$V_{r_2} = V_{r_1} = 13.77 \text{ m/s}$$

$$V_{w_2} = V_{r_2} \cos \phi - u_2$$

$$= 13.77 \cos 20° - 10.0 = 2.94 \text{ m/s}$$

Work done by the jet per second on the runner is given by equation (18.9) as

$$= \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u$$

$$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10 \qquad (\because \quad aV_1 = Q = 0.7 \text{ m}^3\text{/s})$$

$$= 186970 \text{ Nm/s}$$

∴ Power given to turbine $= \dfrac{186970}{1000} =$ **186.97 kW. Ans.**

The hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$

$= \mathbf{0.9454}$ or **94.54%. Ans.**

Example *A Pelton wheel is to be designed for the following specifications : Shaft power = 11,772 kW ; Head = 380 metres ; Speed = 750 r.p.m. ; Overall efficiency = 86% ; Jet diameter is not to exceed one-sixth of the wheel diameter. Determine :*

(i) The wheel diameter, *(ii) The number of jets required, and*

(iii) Diameter of the jet.

Take $K_{v_1} = 0.985$ *and* $K_{u_1} = 0.45$

Shaft power, S.P. = 11,772 kW

Head , $H = 380$ m

Speed, $N = 750$ r.p.m.

Overall efficiency, $\eta_0 = 86\%$ or 0.86

Ratio of jet dia. to wheel dia. $= \dfrac{d}{D} = \dfrac{1}{6}$

Co-efficient of velocity, $K_{v_1} = C_v = 0.985$

Speed ratio, $K_{u_1} = 0.45$

The velocity of wheel, $u = u_1 = u_2$

$$= \text{Speed ratio} \times \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85 \text{ m/s}$$

$$u = \frac{\pi DN}{60} \quad \therefore \quad 38.85 = \frac{\pi DN}{60}$$

$$D = \frac{60 \times 38.85}{\pi \times N} = \frac{60 \times 38.85}{\pi \times 750} = \mathbf{0.989\ m.}$$

But $\dfrac{d}{D} = \dfrac{1}{6}$

$\therefore$ Dia. of jet, $d = \dfrac{1}{6} \times \text{D} = \dfrac{0.989}{6} = \mathbf{0.165\ m.\ Ans.}$

Discharge of one jet, $q = \text{Area of jet} \times \text{Velocity of jet}$

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(.165) \times 85.05 \text{ m}^3/\text{s} = 1.818 \text{ m}^3/\text{s}$$

Now
$$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{11772}{\frac{\rho g \times Q \times H}{1000}}$$

$$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}, \text{ where } Q = \text{Total discharge}$$

$\therefore$ Total discharge,
$$Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86} = 3.672 \text{ m}^3/\text{s}$$

$\therefore$ Number of jets
$$= \frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{Q}{q} = \frac{3.672}{1.818} = \textbf{2 jets. Ans.}$$

MODULE-IV

Flow in open channels is defined as the flow of a liquid with a free surface. A free surface is a surface having constant pressure such as atmospheric pressure. Thus, a liquid flowing at atmospheric pressure through a passage is known as flow in open channels. In most of cases, the liquid is taken as water. Hence flow of water through a passage under atmospheric pressure is called flow in open channels. The flow of water through pipes at atmospheric pressure or when the level of water in the pipe is below the top of the pipe, is also classified as open channel flow.

In case of open channel flow, as the pressure is atmospheric, the flow takes place under the force of gravity which means the flow takes place due to the slope of the bed of the channel only. The hydraulic gradient line coincides with the free surface of water.

Simply stated, Open channel flow is a flow of liquid in a conduit with free space. Open channel flow is particularly applied to understand the flow of a liquid in artificial (flumes, spillways, canals, weirs, drainage ditch, culverts) and natural (streams, rivers, flood plains). The two kinds of flow are similar in many ways but differ in one important respect. Open-channel flow must have a *free surface,* whereas pipe flow has none.

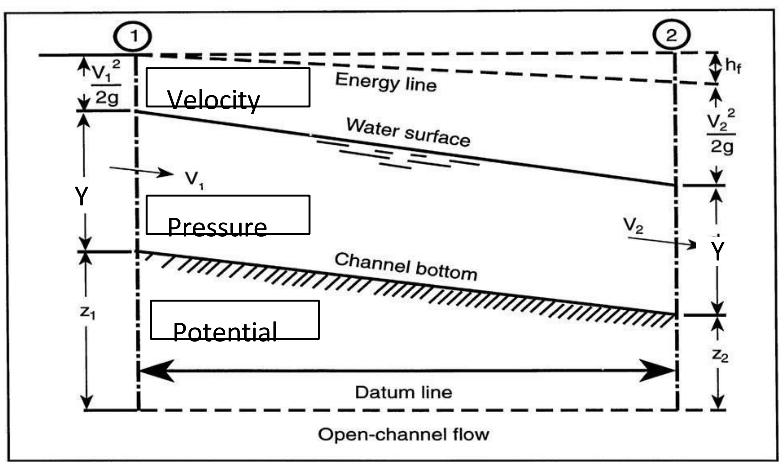


Figure 1: Schematic Presentation of Open Channel

1.1 Classification of Open Channel Flows:

A channel in which the cross-sectional shape and size and also the bottom slopes are constant is termed as a prismatic channel. Most of the man-made (artificial) channels are prismatic channels over long stretches. The rectangle, trapezoid, triangle and circle are some of the commonly used shapes in made channels. All natural channels generally have varying cross-sections and consequently are non-prismatic.

a) Steady and Unsteady Open Channel Flow: If the flow depth or discharge at a cross-section of an open channel flow is not changing with time, then the flow is steady flow, otherwise it is called as unsteady flow. Flood flows in rivers and rapidly varying surges in canals are some examples of unsteady flows. Unsteady flows are considerably more difficult to analyze than steady flows.

b) Uniform and Non-Uniform Open Channel Flow: If the flow depth along the channel is not changing at every cross-section for a taken time, then the flow is uniform flow. If the flow depth changes at every cross-section along the flow direction for a taken time, then it is non-uniform flow.

A prismatic channel carrying a certain discharge with a constant velocity is an example of uniform flow.

c) Uniform Steady Flow: The flow depth does not change with time at every cross section and at the same time is constant along the flow direction. The depth of flow will be constant along the channel length and hence the free surface will be parallel to the bed.

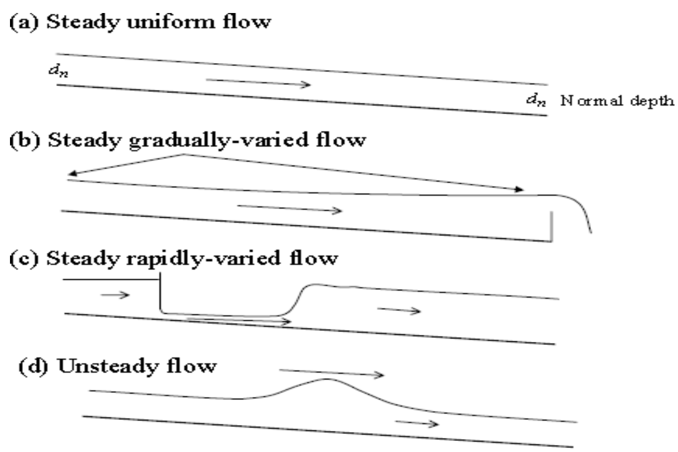


Figure 2: Schematic representation of different types of open channel flow

d) Non-Uniform Steady Flows: The water depth changes along the channel cross sections but does not change with time at every cross section with time. A typical example of this kind of flow is the backwater water surface profile at the upstream of a dam.

e) Rapidly Varied Flow (R.V.F.): Rapidly varied flow is defined as that flow in which depth of flow changes abruptly over a small length of the channel. When there is any obstruction in the path of flow of water, the level of water rises above the obstruction and then falls and again rises over a small length of channel. Thus, the depth of flow changes rapidly over a short length of the channel. For this short length of the channel the flow is called rapidly varied flow (R.V.F.).

f) Gradually Varied Flow (G.V.F.): If the depth of flow in a channel changes gradually over a long length of the channel, the flow is said to be gradually varied flow and is denoted by G.V.F.

g) Laminar Flow and Turbulent Flow: The flow in open channel is said to be laminar if the Reynold number (Re) is less than 500 or 600. Reynold number in case of open channels is defined as: R_e= pVR

where V = Mean velocity of flow of water, R = Hydraulic radius or Hydraulic mean depth

=Cross-section area of flow normal to the direction of flow/ Wetted

perimeter p and µ = Density and viscosity of water.

If the Reynold number is more than 2000, the flow is said to be turbulent in open channel flow. If R_e lies between 500 to 2000, the flow is considered to be in transition state.

1.2Difference between Open Channel & Pipe Flow

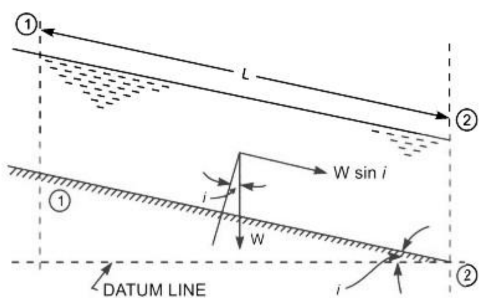
Despite the similarity between the two kinds of flow, it is much more difficult to solve problems of flow in open channels than in pipes. Flow conditions in open channels are complicated by the position of the free surface which will change with time and space. And also by the fact that depth of flow, the discharge, and the slopes of the channel bottom and of the free surface are all inter-dependent.

Physical conditions in open-channels vary much more than in pipes - the cross-section of pipes is usually round - but for open channel it can be any shape. Treatment of roughness also poses a greater problem in open channels than in pipes. Although there may be a great range of roughness in a pipe from polished metal to highly corroded iron, open channels may be of polished metal to natural channels with long grass and roughness that may also depend on depth of flow. And also, Open channel flows are found in large and small scale. Open channel flow is driven by gravity rather than by pressure work as in pipes.

	Pipe flow	Open Channel flow
Flow driven by	Pressure work	Gravity (potential energy)
Flow cross section	Known, fixed	Unknown in advance because the flow depth is unknown
Characteristics parameters	flow velocity deduced from continuity	Flow depth deduced simultaneously from solving both continuity and momentum equations
Specific boundary		Atmospheric pressure at conditions the free surface

➢ ***1.4 Discharge through Open Channel by Chezy's Constant***

Consider uniform flow of water in a channel as shown in Fig. 16.2. As the flow is uniform, it means the velocity, depth of flow and area of flow will be constant for a given length of the channel. Consider sections 1-1 and 2-2.



Let L = Length of channel, A= Area of flow of water, i = Slope of the bed, V = Mean velocity of flow of water, P = Wetted perimeter of the cross-section, f = Frictional resistance per unit velocity per unit area.

- The weight of water between sections 1-1 and 2-2.
  W = Specific weight of water x volume of water =wxAxL
- Component of W along direction of flow = W x sin i = wAL sin i
- Frictional resistance against motion of water= f x surface area x (velocity)^n
- The value of n is found experimentally equal to 2 and surface area = P x L
- Frictional resistance against motion = f x P x L x V^2

The forces acting on the water between sections 1-1 and 2-2 are:

1. Component of weight of water along the direction of flow,
2. Friction resistance against flow of water,
3. Pressure force at section 1-1,
4. Pressure force at section 2-2.

As the depths of water at the sections 1-1 and 2-2 are the same, the pressure forces on these two sections are same and acting in the opposite direction. Hence they cancel each other. In case of uniform flow, the velocity of flow is constant for the given length of the channel. Hence there is no acceleration acting on the water. Hence the resultant force acting in the direction of flow must be zero.

$\therefore$ Resolving all forces in the direction of flow, we get

$$wAL \sin i - f \times P \times L \times V^2 = 0$$

or

$$wAL \sin i = f \times P \times L \times V^2$$

$$V^2 = \frac{wAL \sin i}{f \times P \times L} = \frac{w}{f} \times \frac{A}{P} \times \sin i$$

or

$$V = \sqrt{\frac{w}{f}} \times \sqrt{\frac{A}{P} \times \sin i}$$

But

$$\frac{A}{P} = m$$

= hydraulic mean depth or hydraulic radius,

$$\sqrt{\frac{w}{f}} = C = \text{Chezy's constant}$$

Substituting the values of $\frac{A}{P}$ and $\sqrt{\frac{w}{f}}$ in equation (*iii*), $V = C\sqrt{m \sin i}$

For small values of i, $\sin i \simeq \tan i \approx i \quad \therefore \quad V = C\sqrt{mi}$

$\therefore$ Discharge, $Q = \text{Area} \times \text{Velocity} = A \times V$

$$= A \times C\sqrt{mi}$$

Numerical related to Chezy's constant:

Q1. Find the velocity of flow and rate of flow of water through a rectangular channel of 6 m wide and 3 m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take Chery 's constant C = 55.

Width of rectangular channel, $b = 6$ m

Depth of channel, $d = 3$ m

$\therefore$ Area, $A = 6 \times 3 = 18 \text{ m}^2$

Bed slope, $i = 1 \text{ in } 2000 = \frac{1}{2000}$

Chezy's constant, $C = 55$

Perimeter, $P = b + 2d = 6 + 2 \times 3 = 12$ m

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{18}{12} = 1.5$ m

Velocity of flow is given by equation (16.4) as,

$$V = C\sqrt{mi} = 55\sqrt{1.5 \times \frac{1}{2000}} = \mathbf{1.506 \text{ m/s. Ans.}}$$

Rate of flow, $Q = V \times \text{Area} = V \times A = 1.506 \times 18 = \mathbf{27.108 \text{ m}^3\text{/s. Ans.}}$

Explanation: In the question width & depth of the channel was provided, using which Area is determined first. Later perimeter is calculated. After which we have area & perimeter, using which we can calculate the hydraulic mean depth of the channel. To calculate the velocity, we have slope and m, along with the C provided in question.

Rate of flow is nothing but Discharge in the channel, which is Velocity * Area of the channel.

Q2. Find the slope of the bed of a rectangular channel of width 5 m when depth of water is 2 m and rate of flow is given as 20 m^3/s. Take Chery's constant, C = 50.

Solution: Width = 5m Depth = 2m Q = 20 m^3/s

Chezy's constant $C = 50$

Let the bed slope $= i$

Using equation (16.5), we have $Q = AC\sqrt{mi}$

where $A = \text{Area} = b \times d = 5 \times 2 = 10 \text{ m}^2$

$$m = \frac{A}{P} = \frac{10}{b+2d} = \frac{10}{5+2\times 2} = \frac{10}{5+4} = \frac{10}{9} \text{ m}$$

$$\therefore \quad 20.0 = 10 \times 50 \times \sqrt{\frac{10}{9} \times i} \text{ or } \sqrt{\frac{10}{9}\, i} = \frac{20.0}{500} = \frac{2}{50}$$

Squaring both sides, we have $\frac{10}{9} i = \frac{4}{2500}$

$$\therefore \quad i = \frac{4}{2500} \times \frac{9}{10} = \frac{36}{25000} = \frac{1}{\frac{25000}{36}} = \frac{1}{\mathbf{694.44}}. \textbf{ Ans.}$$

$\therefore$ Bed slope is 1 in 694.44.

Geometric Properties Necessary for Analysis:

HYDRAULICS: ELEMENTS OF CHANNEL SECTIONS

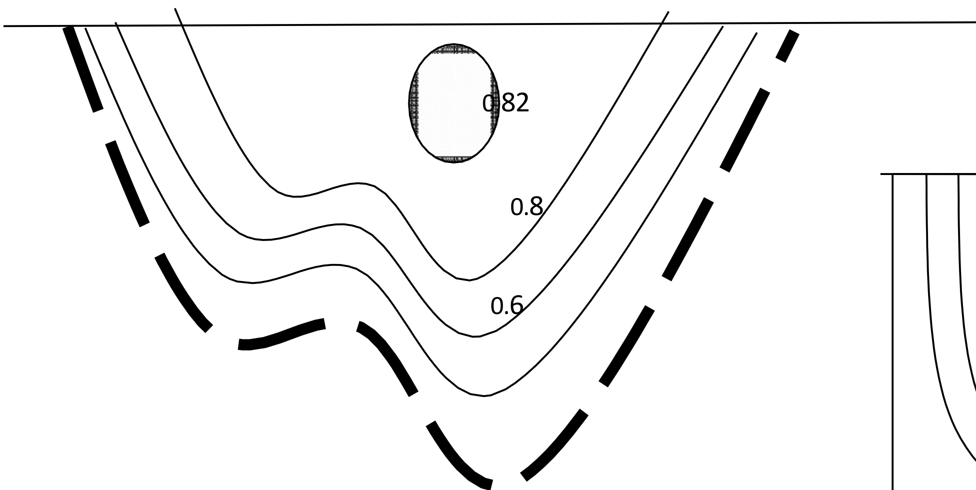
Section	Area a	Wetted Perimeter p	Hydraulic Radius r	Top Width T
Trapezoid	$bd+zd^2$	$b+2d\sqrt{z^2+1}$	$\frac{bd+zd^2}{b+2d\sqrt{z^2+1}}$	$b+2zd$
Rectangle	bd	$b+2d$	$\frac{bd}{b+2d}$	b
Triangle	zd^2	$2d\sqrt{z^2+1}$	$\frac{zd}{2\sqrt{z^2+1}}$	$2zd$
Parabola	$\frac{2}{3}dT$	$T+\frac{8d^2}{3T}$ [1]	$\frac{2dT^2}{3T^2+8d^2}$ [1]	$\frac{3a}{2d}$
Circle - < ½ full [2]	$\frac{D^2}{8}\left(\frac{\pi\theta}{180}-\sin\theta\right)$	$\frac{\pi D\theta}{360}$	$\frac{45D}{\pi\theta}\left(\frac{\pi\theta}{180}-\sin\theta\right)$	$D\sin\frac{\theta}{2}$ or $2\sqrt{d(D-d)}$
Circle - > ½ full [3]	$\frac{D^2}{8}\left(2\pi-\frac{\pi\theta}{180}+\sin\theta\right)$	$\frac{\pi D(360-\theta)}{360}$	$\frac{45D}{\pi(360-\theta)}\left(2\pi-\frac{\pi\theta}{180}+\sin\theta\right)$	$D\sin\frac{\theta}{2}$ or $2\sqrt{d(D-d)}$

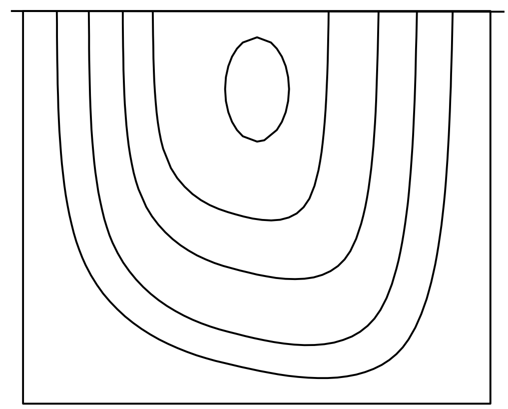
[1] Satisfactory approximation for the interval $0 < \frac{d}{T} \leq 0.25$. When $d/T > 0.25$, use $p = \frac{1}{2}\sqrt{16d^2+T^2} + \frac{T^2}{8d}\sinh^{-1}\frac{4d}{T}$

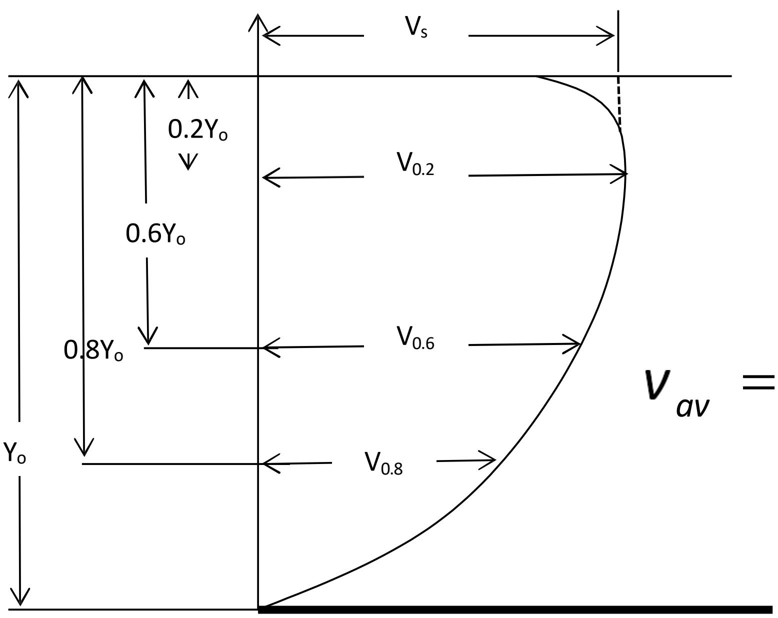
[2] $\theta = 4\sin^{-1}\sqrt{d/D}$ } Insert θ in degrees in above equations

[3] $\theta = 4\cos^{-1}\sqrt{d/D}$

1.5 Velocity Distribution in Open







$$v_{av} = v_{0.6} = \frac{v_{0.2} + v_{0.8}}{2}$$

Most Efficient Channel Sections

A section of a channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction of a channel depends upon the excavation and the lining. To keep the cost down or minimum, the wetted perimeter, for a given discharge, should be minimum. This condition is utilized for determining the dimensions of an economical sections of different form of channels.

Most economical section is also called the best section or most efficient section as the discharge, assing through a most economical section of channel for a given cross-sectional area (A), slope of the bed (i) and a resistance co-efficient, is maximum. But the discharge, Q is given by equation as

$$Q = AC\sqrt{mi} = AC\sqrt{\frac{A \times i}{P}} \qquad \left(\because \ m = \frac{A}{P}\right)$$

For a given A, i and resistance co-efficient C, the above equation is written as

$$Q = K\frac{1}{\sqrt{P}}, \quad \text{where } K = AC\sqrt{Ai} = \text{constant}$$

Hence the discharge, Q will be maximum, when the wetted perimeter P is minimum. This condition will be used for determining the best section of a channel i.e., best dimensions of a channel for a given area.

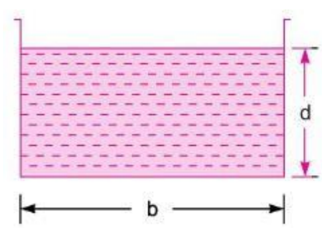
The conditions to be most economical for the following shapes of the channels will be considered :

1. Rectangular Channel,
2. Trapezoidal Channel, and
3. Circular Channel.

3.1.1 Most Efficient Rectangular Channel:

The condition for most economical section, is that for a given area, the perimeter should be minimum.

Consider a rectangular channel as shown in Fig.



Let, b = Width of Channel

d = Depth of the flow then Area, A = b * d

& Wetted Perimeter P = b + 2d

From above we get b = A / d,

Substituting this value in case of wetted perimeter, we get P = b + 2d = A/d + 2d

Now, for most efficient channel section, P should be minimum for a given area.

or $$\frac{dP}{d(d)} = 0$$

Differentiating the equation (*iii*) with respect to d and equating the same to zero, we get

$$\frac{d}{d(d)}\left[\frac{A}{d} + 2d\right] = 0 \quad \text{or} \quad -\frac{A}{d^2} + 2 = 0 \quad \text{or} \quad A = 2d^2$$

But, A = b * d which implies > b * d = $2d^2$ or **b = 2d** **(i)**

Now hydraulic mean depth, $m = \frac{A}{P} = \frac{b \times d}{b + 2d}$ $(\because \quad A = bd, P = b + 2d)$

= 2d × d / 2d + 2d = **$2d^2$/ 4d** **(ii)**

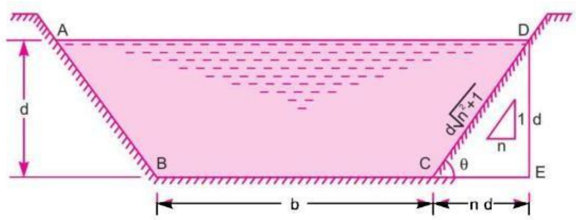
From eq^n (i) & (ii), it is clear that the rectangular channel will be most economical when:

- Either b = 2d which means width is two times the depth of flow.
- Or m = d/2 means hydraulic depth is half of depth of flow.

3.1.2 Most Efficient Trapezoidal Channel Section:

The trapezoidal section of a channel will be most economical when its wetted perimeter is minimum.

Consider a trapezoidal channel section as shown in fig. below:



Let, b = width of channel Bottom
d = Depth of the flow

ø = angle made by the sides with horizontal

(*i*) The side slope is given as 1 vertical to *n* horizontal.

$\therefore$ Area of flow, $A = \frac{(BC + AD)}{2} \times d = \frac{b + (b + 2nd)}{2} \times d$ $(\because \quad AD = b + 2nd)$

$$= \frac{2b + 2nd}{2} \times d = (b + nd) \times d \qquad ...(i)$$

$$\therefore \qquad \frac{A}{d} = b + nd$$

$$\therefore \qquad b = \frac{A}{d} - nd \qquad ...(ii)$$

Now wetted perimeter, $P = AB + BC + CD = BC + 2CD$ $(\because \quad AB = CD)$

$$= b + 2\sqrt{CE^2 + DE^2} = b + 2\sqrt{n^2d^2 + d^2} = b + 2d\sqrt{n^2 + 1} \qquad ...(iia)$$

Substituting the value of *b* from equation (*ii*), we get

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \qquad ...(iii)$$

For most economical section, *P* should be minimum or $\frac{dP}{d(d)} = 0$

$\therefore$ Differentiating equation (*iii*) with respect to *d* and equating it equal to zero, we get

$$\frac{d}{d(d)}\left[\frac{A}{d} - nd + 2d\sqrt{n^2 + 1}\right] = 0$$

or $$-\frac{A}{d^2} - n + 2\sqrt{n^2 + 1} = 0 \qquad (\because \quad n \text{ is constant})$$

or $$\frac{A}{d^2} + n = 2\sqrt{n^2 + 1}$$

Substituting the value of *A* from equation (*i*) in the above equation,

$$\frac{(b + nd)d}{d^2} + n = 2\sqrt{n^2 + 1} \quad \text{or} \quad \frac{b + nd}{d} + n = 2\sqrt{n^2 + 1}$$

$$\text{or} \qquad \frac{b+nd+nd}{d} = \frac{b+2nd}{d} = 2\sqrt{n^2+1} \quad \text{or} \quad \frac{b+2nd}{2} = d\sqrt{n^2+1}$$

But, $\frac{b+2nd}{2}$ = Half of top width

$d\sqrt{n^2+1} = CD$ = one of the sloping side

From above equation is the required condition for a trapezoidal section to be most economical, which can be expressed as half of the top width must be equal to one of the sloping sides of the channel.

(ii) Hydraulic Mean Depth

Hydraulic mean depth, $m = \frac{A}{P}$

Value of A from (i), $A = (b + nd) \times d$

Value of P from (iia), $P = b + 2d\sqrt{n^2+1} = b + (b + 2nd)$ Also,

$b + 2nd = 2d\sqrt{n^2+1}$)

$= 2b + 2nd = 2(b + nd)$

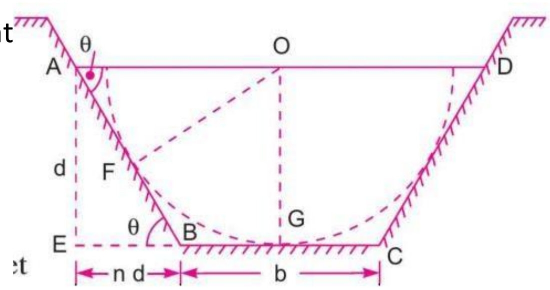
$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{(b+nd)d}{2(b+nd)} = \frac{d}{2}$

Hence for a trapezoidal section to be most economical hydraulic mean depth must be equal to half the depth of flow,

(iii) The three sides of the trapezoidal section of most economical section are tangential to the semi-circle described on the water line. This is proved as:

Let Fig. below shows the trapezoidal channel of most economical

section. Let ø = angle made by the sloping side with horizont



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and

O = the centre of the top width, AD.

Draw OF perpendicular to the sloping side AB.

Triangle OAF is a right-angled triangle and angle OAF = ø

$\therefore \quad \sin\theta = \frac{OF}{OA} \qquad \therefore \quad OF = AO \sin\theta$

In ΔAEB, $\sin\theta = \frac{AE}{AB} = \frac{d}{\sqrt{d^2+n^2d^2}}$

$= \frac{d}{} = \frac{1}{\sqrt{1+n^2}}$

Substituting $OF = AO \times \frac{1}{\sqrt{1+n^2}}$ ı equation

But $\quad AO$ = half of top width

$$= \frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

Substituting this value of AO in above equation, we get

$$OF = \frac{d\sqrt{n^2+1}}{\sqrt{n^2+1}} = d \text{ depth of flow}$$

Thus, if a semi-circle is drawn with O as centre and radius equal to the depth of flow d, the three sides of most economical trapezoidal section will be tangential to the semi-circle.

Hence the conditions for the most economical trapezoidal section are:

- $\frac{b + 2nd}{2} = d\sqrt{n^2+1}$
- m = d/ 2
- A semi-circle drawn from O with radius equal to depth of flow will touch the three sides of the channel.

3.1.3 Best Side Slope for Most Economical Trapezoidal Section

Area of trapezoidal section, $A = (b + nd)d$ ...(*i*)

where b = width of trapezoidal channel, d = depth of flow, and

n = slope of the side of the channel

From equation (*i*), $b = \frac{A}{d} - nd$ ...(*ii*)

Perimeter (wetted) of channel, $P = b + 2d\sqrt{n^2+1}$

Substituting the value of b from equation (*ii*), perimeter becomes

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2+1} \qquad ...(iii)$$

For the most economical trapezoidal section depth of flow d, and area A, is the only variable. Best side slope will be when section is most economical or in other words, P is minimum. For P minimum we have, dP/dn = 0

Hence differentiating equation (*iii*) with respect to n,

$$\frac{d}{dn}\left[\frac{A}{d} - nd + 2d\sqrt{n^2+1}\right] = 0$$

or $\quad -d + 2d \times \frac{1}{2} \times (n^2+1)^{1/2-1} \times 2n = 0$ or $-d + 2nd \times \frac{1}{\sqrt{n^2+1}} = 0$

Cancelling d and re-arranging, we get $2n = \sqrt{n^2+1}$

Squaring to both sides,

$$4n^2 = n^2 + 1 \text{ or } 3n^2 = 1 \text{ or } n = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

If the sloping side makes and Angle ø , with the horizontal then we have,

Tan ø = 1/n = 1/ 1√3 = tan 60°

Ø = 60°

Hence best side slope is at 60 to the horizontal or the value of n for the best side slope is given by eq^n

$$4n^2 = n^2 + 1 \text{ or } 3n^2 = 1 \text{ or } n = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

Half of top width = length of one sloping side

or $$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

Substitute the value of n from above equation:

$$\frac{b + 2 \times \frac{1}{\sqrt{3}} \times d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} = \frac{2d}{\sqrt{3}} \quad \text{or} \quad \frac{\sqrt{3}b + 2d}{2 \times \sqrt{3}} = \frac{2d}{\sqrt{3}}$$

or $$\sqrt{3}b + 2d = 2 \times \sqrt{3} \times \frac{2d}{\sqrt{3}} = 4d$$

∴ $$b = \frac{4d - 2d}{\sqrt{3}} = \frac{2d}{\sqrt{3}} \qquad \text{...}(iv)$$

Now, wetted perimeter, $$P = b + 2d\sqrt{n^2 + 1}$$

$$= \frac{2d}{\sqrt{3}} + 2d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} \qquad \left(\because\ b = \frac{2d}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}\right)$$

$$= \frac{2d}{\sqrt{3}} + 2d \times \frac{2}{\sqrt{3}} = \frac{2d}{\sqrt{3}} + \frac{4d}{\sqrt{3}}$$

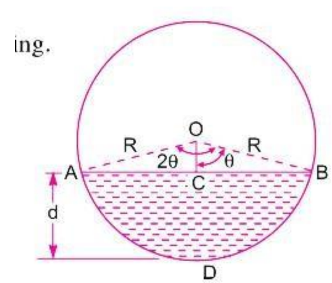
or $$P = \frac{6d}{\sqrt{3}} = 3 \times \frac{2d}{\sqrt{3}} = 3 \times b \qquad \left(\because\ \text{From } (iv), \frac{2d}{\sqrt{3}} = b\right)$$

For a slope of 60°, the length of sloping side is equal to the width of the trapezoidal section.

- **Flow through Circular Channel**

The flow of a liquid through a circular pipe, when the level of liquid in the pipe is below the top of the pipe is classified as an Open Channel flow. The rate of flow through circular channel is determined from the depth of flow and angle subtended by the liquid surface at the centre of the circular channel.

Fig. below shows a circular channel through which water is flowing.



Let d = depth of water,

2ø = angle subtended by water surface AB at the centre in radians

R = radius of the channel

Then the wetted perimeter and wetted area is determined as:

Wetted perimeter,

$$P = \frac{2\pi R}{2\pi} \times 2\theta = 2R\theta$$

Wetted area, A = Area $ADBA$

$= $ Area of sector $OADBO$ − Area of ΔABO

$$= \frac{\pi R^2}{2\pi} \times 2\theta - \frac{AB \times CO}{2} = R^2\theta - \frac{2BC \times CO}{2} \qquad (\because \quad AB = 2BC)$$

$$= R^2\theta - \frac{2 \times R \sin\theta \times R\cos\theta}{2} \qquad (\because \; BC = R\sin\theta,\; CO = R\cos\theta)$$

$$= R^2\theta - \frac{R^2 \times 2\sin\theta\cos\theta}{2} = R^2\theta - \frac{R^2 \sin 2\theta}{2} \qquad (\because \; 2\sin\theta\cos\theta = \sin 2\theta)$$

$$= R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

Then hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{R^2\left(\theta - \dfrac{\sin 2\theta}{2}\right)}{2R\theta} = \dfrac{R}{2\theta}\left(\theta - \dfrac{\sin 2\theta}{2}\right)$

And discharge, Q is given by, $Q = AC\sqrt{mi}$.

- **Most Economical Circular Section**

As discussed above that for a most economical section the discharge for a constant cross-sectional area, slope of bed and resistance co-efficient, is maximum. But in case of circular channels, the area of flow cannot be maintained constant. With the change of depth of flow in a circular channel of any radius, the wetted area and wetted perimeter changes. Thus, in case of circular channels, for most economical section, two separate conditions are obtained. They are:

i. Conditions for maximum velocity, and
ii. Condition for maximum discharge.

1. Condition for Maximum Velocity for Circular Section:

Fig. shows a circular channel through which water is flowing.

Let, d = depth of water,

2ø = Angle subtended at the corner by water surface,

R = radius of channel, and
i = slope of the bed

The velocity of flow according to Chezy's formula is given as

$$V = C\sqrt{mi} = C\sqrt{\frac{A}{P}\,i} \qquad \left(\because \quad m = \frac{A}{P}\right)$$

The Velocity of flow through a circular channel is maximum when the hydraulic mean depth m or A/P is maximum for a given value of C and i. In case of circular pipe, the variable is ø only.

Hence for maximum value A/P we have the condition,

$$\frac{d\left(\frac{A}{P}\right)}{d\theta} = 0$$ Where A & P both are function of ø.

The value of wetted area, A is given by equation as $A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$

The value of wetted perimeter, P is given by eq^n as:

P = 2Rø

Differentiating eq^n with respect to ø, we have:

$$\frac{P\frac{dA}{d\theta} - A\frac{dP}{d\theta}}{P^2} = 0 \quad \text{or} \quad P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$$

From equation (*ii*), $\frac{dA}{d\theta} = R^2\left(1 - \frac{\cos 2\theta}{2} \times 2\right) = R^2(1 - \cos 2\theta)$

From equation (*iii*), $\frac{dP}{d\theta} = 2R$

Substituting the values of A, $P\frac{dA}{d\theta}$ and $\frac{dP}{d\theta}$ in equation (*iv*),

$$2R\theta\left[R^2(1 - \cos 2\theta)\right] - R^2\left(\theta - \frac{\sin 2\theta}{2}\right)(2R) = 0$$

or $$2R^3\theta(1 - \cos 2\theta) - 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

or $$\theta(1 - \cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

$$\theta - \theta\cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

or $$\theta\cos 2\theta = \frac{\sin 2\theta}{2} \quad \text{or} \quad \frac{\sin 2\theta}{\cos 2\theta} = 2\theta$$

$\therefore$ $$\tan 2\theta = 2\theta$$

The solution of this equation by hit and trial, gives

$$2\theta = 257° \; 30' \qquad \text{(approximately)}$$

or $$\theta = 128° \; 45'$$

The depth of flow for maximum velocity from Fig. 16.24, is

$$d = OD - OC = R - R\cos\theta$$

$$= R[1 - \cos\theta] = R[1 - \cos 128° \; 45'] = R[1 - \cos(180° - 51° \; 15')]$$

$$= R[1 - (-\cos 51° \; 15')] = R[1 + \cos 51° \; 15']$$

$$= R[1 + 0.62] = 1.62\,R = 1.62 \times \frac{D}{2} = 0.81\,D$$

Where D = diameter of the circular channel.

Thus, for maximum velocity of flow, the depth of water in the circular channel should be equal to 0.81 times the diameter of the channel.

Hydraulic mean depth for maximum velocity is:

$$m = \frac{A}{P} = \frac{R^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2R\theta} = \frac{R}{2\theta}\left[\theta - \frac{\sin 2\theta}{2}\right]$$

where $\theta = 128° \; 45' = 128.75°$

$$= 128.75 \times \frac{\pi}{180} = 2.247 \text{ radians}$$

$$\therefore \quad m = \frac{R}{2 \times 2.247}\left[2.247 - \frac{\sin 257° \; 30'}{2}\right] = \frac{R}{4.494}\left[2.247 - \frac{\sin(180° + 87.5°)}{2}\right]$$

$$= \frac{R}{4.494}\left[2.247 + \frac{\sin 87.5°}{2}\right] = 0.611 \, R$$

$$= 0.611 \times \frac{D}{2} = 0.3055 \, D = 0.3 \, D$$

Thus, for maximum velocity, the hydraulic mean depth is equal to 0.3 times the diameter of circular channel.

- Condition for Maximum Discharge for Circular Section

Discharge through a channel is given by:

$$Q = AC\sqrt{mi} = AC\sqrt{\frac{A}{P}i}$$

$$= C\sqrt{\frac{A^3}{P}i}$$

The discharge will be maximum for constant values of C and i, when $\frac{A^3}{P}$ is maximum. $\frac{A^3}{P}$ will be maximum when $\frac{d}{d\theta}\left(\frac{A^3}{P}\right) = 0$.

Differentiating this equation with respect to θ and equation the same to zero, we get

$$\frac{P \times 3A^2 \frac{dA}{d\theta} - A^3 \frac{dP}{d\theta}}{P^2} = 0 \quad \text{or} \quad 3PA^2 \frac{dA}{d\theta} - A^3 \frac{dP}{d\theta} = 0$$

Dividing by A^2, $3P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$ ...(*i*)

But from equation (16.16), $P = 2R\theta$

$$\therefore \quad \frac{dP}{d\theta} = 2R$$

But, P = 2Rø,

So dP/dR = 2ø

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$\frac{dA}{d\theta} = R^2 (1 - \cos 2\theta)$$

Substituting the values of P, A, $\frac{dP}{d\theta}$ and $\frac{dA}{d\theta}$ in equation (i)

$$3 \times 2R\theta \times R^2 (1 - \cos 2\theta) - R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \times 2R = 0$$

$$6R^3\theta (1 - \cos 2\theta) - 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

Dividing by $2R^3$, we get

$$3\theta (1 - \cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2}\right) = 0 \quad \text{or} \quad 3\theta - 3\theta \cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

or

$$2\theta - 3\theta \cos 2\theta + \frac{\sin 2\theta}{2} = 0 \quad \text{or} \quad 4\theta - 6\theta \cos 2\theta + \sin 2\theta = 0$$

The solution of this equation by hit and trial, gives

$$2\theta = 308°$$

$$\therefore \quad \theta = \frac{308°}{2} = 154°$$

Depth of Flow for maximum discharge:

$$d = OD - OC = R - R\cos\theta$$
$$= R[1 - \cos\theta] = R[1 - \cos 154°]$$
$$= R[1 - \cos(180° - 26°)] = R[1 + \cos 26°] = 1.898\ R$$
$$= 1.898 \times \frac{D}{2} = 0.948\ D \simeq 0.95\ D$$

Where D = Diameter of the circular channel

Thus for maximum discharge through a circular channel the depth of flow is equal to 0.95 times its diameter.