

# LECTURE NOTES

## ELECTRICAL MACHINE - II

B.Tech, 4<sup>th</sup> Semester, EEE

Prepared by :

SNIGDHA PRIYAMBADA

Assistant Professor in EEE



**Vikash Institute of Technology, Bargarh**

*(Approved by AICTE, New Delhi & Affiliated to BPUT, Odisha)*

Barahaguda Canal Chowk, Bargarh, Odisha-768040

[www.vitbargarh.ac.in](http://www.vitbargarh.ac.in)

## **DISCLAIMER**

- This document does not claim any originality and cannot be used as a substitute for prescribed textbooks.
- The information presented here is merely a collection by SNIGDHA PRIYAMBADA with the inputs of students for their respective teaching assignments as an additional tool for the teaching- learning process.
- Various sources as mentioned at the reference of the document as well as freely available materials from internet were consulted for preparing this document.
- Further, this document is not intended to be used for commercial purpose and the authors are not accountable for any issues, legal or otherwise, arising out of use of this document.
- The author makes no representations or warranties with respect to the accuracy or completeness of the contents of this document and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose.

\*\*\*\*\*

## EEPC2003 ELECTRICAL MACHINES-II (3-0-0)

### Module I (08 Hours)

Three-phase synchronous generators:

Construction, Salient pole type and Cylindrical rotor structure, Armature windings, Winding factor, EMF equation, Armature reaction, Synchronous impedance, Alternator on load, Phasor diagrams, Open Circuit and Short Circuit tests, Short Circuit Ratio, Voltage regulation by EMF, MMF and ZPF methods, Two reactance concept of Salient pole Synchronous machines, Slip test, Power equations, Power angle characteristics.

### Module II (06 Hours)

Parallel operation of alternators:

Requirements for parallel operation, synchronizing of alternators, three dark lamp method, synchroscope, synchronizing current, synchronizing power, synchronizing torque, effect of increasing the excitation, effect of increasing the driving torque and effect of change in speed of one of the alternators, load sharing between two alternators.

### Module III (04 Hours)

Synchronous motors: Rotating magnetic field, operating principle of a synchronous motor, phasor diagrams, power equations, load angle, 'V' and inverted 'V' curves, synchronous condenser, starting methods, hunting.

### Module IV (06 Hours)

Three-phase induction motors: Construction, principle of operation, types, squirrel cage rotor, slip ring induction motor, slip, torque equations, starting torque, full load torque, maximum torque, torque-slip and torque-speed characteristics, effect of rotor resistance, effect of change in supply voltage, effect of change in frequency, power losses and efficiency, synchronous watt, equivalent circuit of induction motor, phasor diagrams, power output, testing of induction motors, No-load test, Blocked rotor test, load test, measurement of slip, circle diagram.

### Module V (06 Hours)

Starting and speed control of three-phase induction motors: DOL starting, stator resistance starting, auto transformer starting, star-delta starting, starting of slip ring induction motors, speed control by variation of supply voltage-supply frequency, rotor resistance control, crawling and cogging effects.

Single-phase induction motors: Construction, principle of operation, double field revolving theory, equivalent circuit, performance characteristics, starting methods, capacitor start-capacitor run single phase induction motors.

\*\*\*\*\*

**Course Outcomes (COs)**

- CO1: Explain the construction and working principles of synchronous generators, derive EMF equations, and analyze armature reaction and voltage regulation. (Knowledge, Understanding)
- CO2: Demonstrate the requirements and procedures for the parallel operation of alternators and analyze the impact of synchronizing current, power, and torque on system stability and load sharing. (Application, Analysis)
- CO3: Describe the construction, operating principles, and characteristics of synchronous motors, and analyze V and inverted V curves for performance assessment. (Knowledge, Understanding, Analysis)
- CO4: Explain the structure, operation, and torque characteristics of three-phase induction motors, evaluate effects of rotor resistance and supply variations, and analyze equivalent circuits. (Understanding, Application, Analysis)
- CO5: Analyze different starting and speed control methods for induction motors, assess performance of single-phase induction motors, and apply theories like double field revolving theory for performance analysis. (Analysis, Evaluation)

**Textbooks:**

1. “Theory & Performance of Electrical Machines” by J.B. Gupta, 15th edition, S. K. Kataria & Sons, reprint 2015.
2. Fitzgerald & Kingsley’s “Electric Machinery”, Stephen D. Umans, 7th edition, McGrawHill publishers, 2014.

**Reference books:**

1. “The Performance and Design of Alternating Current Machines”, by M. G. Say, CBS Publishers & Distributors, 2005.

\*\*\*\*\*

# **REFERENCES**

## **ELECTRICAL MACHINE -II**

### **B.Tech, 4<sup>th</sup> Semester, EEE**

#### **Books:**

1. Theory & Performance of Electrical Machines” by J.B. Gupta, 15th edition, S. K. Kataria & Sons, reprint 2015.
2. Fitzgerald& Kingsley’s “Electric Machinery”, Stephen D. Umans, 7th edition, McGrawHill publishers, 2014.
3. The Performance and Design of Alternating Current Machines”, by M. G. Say  
CBS Publishers & Distributors, 2005.

#### **Digital References:**

1. Google Web Pages

## Module I

### Three-phase synchronous generators

The synchronous generator, also known as an alternator, is an electrical device that transforms mechanical energy from a prime mover into AC [electrical power](#) at a specific voltage and frequency. The [synchronous motor](#) operates at a consistent speed known as the synchronous speed. A synchronous generator is a type of electrical generator that is used widely in power generation. The term "synchronous" refers to the synchronization between the rotational speed of the rotor and the frequency of the alternating current (AC) output.

#### What is Synchronous Generator?

The synchronous generator or alternator is an electrical [generator](#) that converts mechanical energy to electrical energy in the form of alternating current. For reasons of cost and simplicity, most alternators use a rotating [magnetic field](#) with a stationary armature. It is one of the most common types of generators used in power generation systems globally. In a synchronous generator, the rotary magnetic field is produced either by an electromagnet rotating with the armature conductors or by a rotating permanent magnet. The [frequency](#) of the generated AC voltage is proportional to the rotational speed of the magnetic field.

#### Synchronous Generator Construction

A synchronous generator consists of two main parts - the stator and the rotor. The stator is the stationary external part that contains windings, while the rotor is the rotating internal part that revolves within the stator.

- **Stator:** The stator is the non-moving component of the alternator. It houses the armature winding where the [voltage](#) is generated. The alternator's output is derived from the stator.
- **Rotor:** The rotor is the moving part of the alternator. It generates the primary field [flux](#).

There are two common types of rotor construction in synchronous generators - salient pole rotor and cylindrical rotor.

#### Stator

- The stator of the alternator comprises several components, including the frame, stator core, armature windings, and cooling mechanisms.
- The frame of the stator is typically made from cast iron in smaller machines and welded steel in larger machines.
- The stator core consists of laminations made from high-grade silicon steel. These laminations help minimize hysteresis and eddy-current losses within the stator core.
- Slots are cut along the inner perimeter of the stator core to accommodate a 3-phase armature winding.
- The armature winding in the alternator is configured in a star connection. The windings for each phase are spread across multiple slots, ensuring that when current passes through, it generates a fundamentally sinusoidal distribution of EMF in space.

#### Rotor

The rotor in the alternator contains the field winding, which receives direct current via two slip rings from an external DC source, often referred to as the exciter. Typically, this exciter is a small DC shunt generator that is mounted directly on the alternator's shaft.

For alternators, there are two primary types of rotor designs employed: the salient-pole type and the cylindrical rotor type.

#### Salient Pole Rotor

A salient pole rotor contains projections called poles that protrude from the surface. Individual field coils are wound over each pole. This type is mainly used in smaller synchronous machines. The term "salient" refers to something that projects outward. The field pole windings are connected in series, allowing opposite polarities between adjacent poles when energized by the DC exciter.

Salient pole type rotors are typically used in alternators that operate at low to medium speeds (from 120 to 400 RPM), such as those powered by diesel engines or water turbines.

This is due to several reasons:

- The structure of the salient pole type rotor is not robust enough to handle the mechanical stresses that occur at higher speeds.
- Operating a salient field pole type rotor at high speeds would lead to windage losses and could generate noise.
- The rotors in these low-speed alternators are built with a larger diameter to accommodate the poles, leading to rotors with a large diameter and short axial length, characteristic of the salient pole design.

#### Cylindrical Rotor

In a cylindrical rotor, the field winding is distributed uniformly over the entire cylindrical surface rather than individual poles. This type is used for larger machines to handle high power requirements. Cylindrical rotors are constructed from solid forgings of high-grade nickel-chrome-molybdenum steel.

Unlike salient pole rotors, cylindrical rotors do not have visible physical poles. Approximately two-thirds of the cylindrical rotor's outer periphery features slots cut at regular intervals and aligned parallel to the rotor shaft. These slots house the field windings, which are excited by a DC supply and are of the distributed type.

The portion of the rotor without slots constitutes the pole faces, and the resulting poles are non-salient, meaning they do not protrude from the rotor's surface.

Cylindrical rotor constructions are typically employed in high-speed alternators (1500 to 3000 RPM), such as those driven by steam turbines, for several reasons:

- They offer superior mechanical strength and allow for more precise dynamic balancing.
- Their design ensures noiseless operation at high speeds due to a uniform air gap.
- The flux distribution around the rotor's periphery closely resembles a sine wave, resulting in a superior EMF waveform.

Cylindrical rotor alternators, often referred to as turbo-alternators or turbo-generators, generally have a smaller diameter and longer axial length. These alternators are always installed in a horizontal configuration.

#### Working Principle of Synchronous Generator

The synchronous generator operates based on Faraday's laws of electromagnetic induction. These laws state that an electromotive force (EMF) is induced in an armature coil when it rotates within a uniform magnetic field. Similarly, EMF can also be generated if the magnetic field rotates while the conductor remains stationary. Essentially, it is the relative motion between the conductor and the magnetic field that induces the EMF in the conductor. The waveform of the voltage induced is consistently sinusoidal.

When the rotor field winding of the alternator is energized by the DC exciter, alternating north (N) and south (S) magnetic poles are established on the rotor. As the rotor spins anticlockwise, driven by a prime mover, the armature conductors on the stator are intersected by the magnetic field from these rotor poles. This interaction induces an electromotive force (EMF) in the armature conductors through electromagnetic induction.

The induced EMF is alternating because the N and S poles alternately pass by the armature conductors.

The direction of the generated EMF can be determined using Fleming's right-hand rule, and its frequency can be calculated by the formula:

$$f = \frac{N_s P}{120}$$

Where,

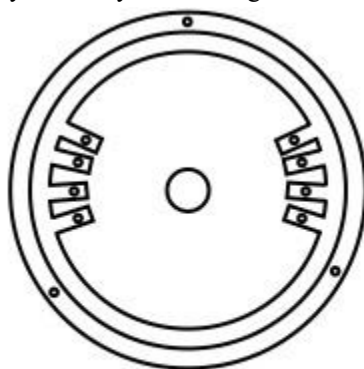
$N_s$  is the synchronous speed in RPM

$P$  is the number of rotor poles.

#### Types of Synchronous Generator

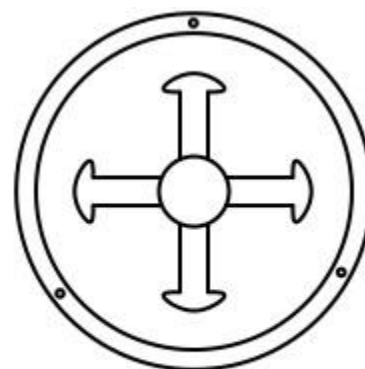
The main types are:

- Turbo generator: Driven by steam turbine in thermal power plants
- Hydro generator: Driven by water turbines in hydroelectric plants
- Diesel generator: Driven by a diesel engine
- Gas turbine generator: Driven by a gas turbine
- Wind turbine generator: Used in variable speed wind turbines
- Battery-driven synchronous generator: Used in EVs and UPS backup systems



**Cylindrical Rotor  
Synchronous Generator**

**Versus**



**Salient Pole  
Synchronous Generator**

#### Difference between Cylindrical Rotor and Salient-Pole Rotor Synchronous Generator

The following table highlights all the significant differences between a cylindrical rotor alternator and a salient-pole alternator –

| Basis of Difference | Cylindrical Rotor Synchronous Generator                                                                                       | Salient-Pole Synchronous Generator                                                                                               |
|---------------------|-------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|
| Description         | A synchronous generator that has its rotor which is cylindrical in shape is known as cylindrical rotor synchronous generator. | A synchronous generator whose rotor has poles projected on the rotor surface is called salient pole rotor synchronous generator. |

|                            |                                                                                                                                                   |                                                                                                                              |
|----------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------|
| Pole                       | In a cylindrical rotor alternator, the portion without the slots of the cylindrical rotor acts as poles. Thus, the poles are non-projecting type. | In salient pole alternator, the rotor poles are projecting out from the surface of the rotor.                                |
| Rotor diameter             | The diameter of the cylindrical rotor alternator is relatively smaller.                                                                           | The diameter of the salient-pole rotor alternator is larger than that of cylindrical rotor alternator.                       |
| Axial length               | Cylindrical rotor alternator has large axial length.                                                                                              | The axial length of the salient-pole rotor alternator is small.                                                              |
| Air gap                    | In cylindrical rotor alternator, the air-gap between stator and rotor is uniform because it has smooth cylindrical periphery.                     | Salient pole rotor alternator has non-uniform air gap because the poles are projecting out from the surface.                 |
| Also called                | Cylindrical rotor alternator is also called “non-salient rotor alternator” or “non-projected pole type alternator”.                               | Salient pole alternator is also known as “projected pole type rotor alternator”.                                             |
| Rating (For the same size) | The rating of the cylindrical rotor alternator is higher than salient pole alternator.                                                            | Salient pole alternator has smaller rating than the cylindrical type alternator.                                             |
| Mechanical strength        | The mechanical strength of the cylindrical rotor alternator is high.                                                                              | Salient pole alternator has relatively less mechanical strength.                                                             |
| Damper winding             | Cylindrical rotor alternator does not have separate damper winding.                                                                               | Salient pole alternator have separate damper winding.                                                                        |
| Suitability                | Cylindrical rotor alternator is suitable for high speed applications. Where, the speed ranges from 1500 to 3000 RPM.                              | Salient pole rotor alternator is most suitable for low speed applications, where the rotor speed ranges from 125 to 500 RPM. |
| Prime mover                | The prime movers used for running the cylindrical rotor alternator are steam turbines used in thermal, gas and nuclear power plants.              | The prime movers used for driving the salient pole alternators are hydro turbines and IC engines.                            |

#### Armature Windings:

The stator windings for alternating current motors and generators are alike. It should be noted that direct current and alternating current windings differ essentially by the former being of the closed-circuit type (through commutator), while alternating-current windings are of the open-circuit type. In most synchronous machines, stationary part is the armature. On the inner periphery of the stator core, number of slots (mostly open parallel sided slots) are provided. In these slots armature winding is placed.

. Types of Armature Winding: Various types of winding schemes can be adopted to wound the armature of an alternator, a few of them are given below:

. Single-phase and poly-phase windings: When only one winding is placed on the armature and only one emf is obtained at the output, winding is called single-phase winding. When more than one windings are placed on the armature and emfs induced are more than one, displaced from each other by some angle, the winding is called poly-phase winding. Mostly three-phase winding is provided on the armature.

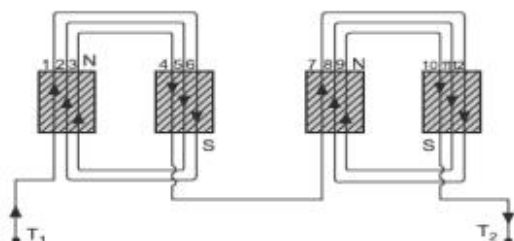
. Concentrated and distributed windings: When one slot per pole or slots equal to the number of poles are employed, the windings thus obtained are called concentrated windings. Such windings give maximum induced emfs for given number of conductors but the wave form of induced emf is not exactly sinusoidal. When number of slots per poles are more than one, the windings thus obtained are called distributed windings. Such windings give slightly less than maximum induced emf for a given number of conductors but the wave form of induced emf is more sinusoidal.

. Single layer and double layer windings: When only one coil side is placed in a slot, the winding is called single layer winding. However, when two coil sides are placed in one slot, one over the other, the winding is called double layer winding.

. Full pitched and short pitched windings: When the two coil sides of the same coil are 180 electrical degrees apart, the winding is called full pitch winding. When the two sides of the same coil are less than 180 electrical degrees apart, the winding is called short pitch winding. The emf induced in each coil is maximum with full pitch winding scheme is employed whereas emf induced in the short pitch winding is less than that.

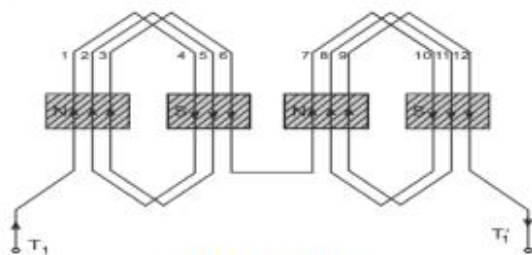
However, short pitch winding is preferred over full pitch winding because of the following reasons:

- It decreases the length at the end-connections and thus amount of copper required is saved.
  - It reduces the slot reactance and thus improves the wave shape of the generated emf, i.e., the generated emf can be made to approximately sinusoidal more easily by properly chording the winding
  - It reduces or eliminates distorting harmonics in the wave form of generated emf The only disadvantage of short pitch winding is that a few more turns are used to obtain the same voltage as it would be induced in full pitch winding.
- . Concentric (or spiral), Lap and Wave windings: When each group of coils under a pole is arranged into a sort of concentric shape i.e., when the current flow is traced through one such properly connected set of coils that the conductors seem to form a spiral around a portion of the core the winding is called concentric or chain or spiral winding. This type of winding scheme is preferred for large diameter, low speed synchronous machines



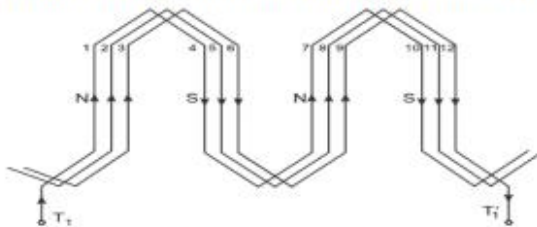
**Figure 3 Concentric Winding**

In the alternators, the lap and wave windings give the same emf as long as the other conditions are the same. In case of lap winding as shown in Fig. 4, coils or coil sides overlap the other consecutively and connections are made.



**Figure 4 Lap Winding**

Whereas in wave winding, as shown in Figure 5, the coils are always forward connected. The connections of a lap winding are simpler to that of the wave winding, therefore, lap winding is exclusively used.



**Figure 5 Wave Winding**

## 5. Important Terms About Armature Winding:

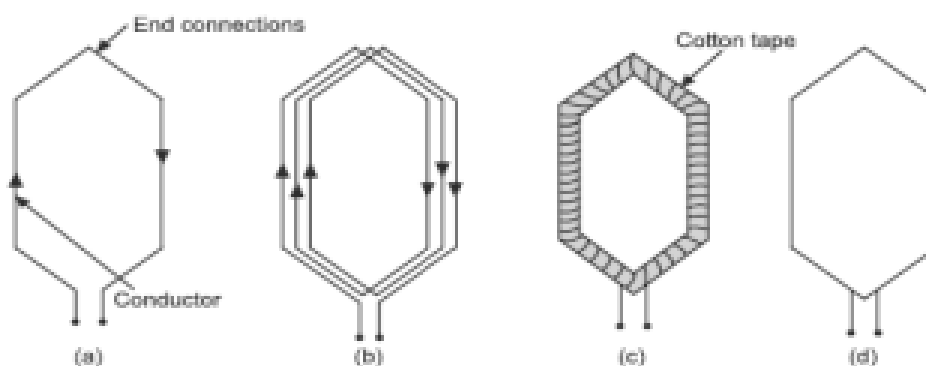
Some of the important terms used in the armature winding are given below:

- I. **Electrical angle:** When a conductor passes through a pair of poles, one cycle of emf is induced in it. Thus, a pair of poles represents an angle of 360 electrical degrees. There is a perfect relation between electrical and mechanical angle:

**Electrical angle = Mechanical angle  $\times$  Pair of poles.**

- II. **Pole pitch:** Distance between two neutral axes (or similar points) of adjacent poles is called pole pitch. The pole pitch can be expressed as number of slots per poles or electrical degrees (i.e.,  $180^\circ$  elect.), as in figure 7. If **S** is the number of slots on the whole periphery of armature and **P** is the number of poles, Then, **Pole pitch = No. of slots per pole =  $S/P$ .**

- III. **Coil:** Two conductors placed in the two slots displaced by pole pitch (in full pitch winding) or less than pole pitch (in short pitch winding), connected at one side by the end connections form a single turn coil as shown in Figure 6(a). When number of turns are connected in series and each side (coil side) is placed in the slot, it is called a multi-turn coil as shown in Figure 6(b) and (c). The multi-turn coil is shown in Figure 6(d) by a single line diagram.



**Figure 6 Single and Multi-Turn Coil**

- IV. **Coil pitch or coil span:** The distance between two active sides of a coil is called coil span. It is expressed in terms of number of slots or electrical degrees (figure 7).
- V. **Slot pitch:** The distance between centre points (or similar points) of two consecutive slots or teeth is called slot pitch. It is expressed in electrical degrees (figure 7).

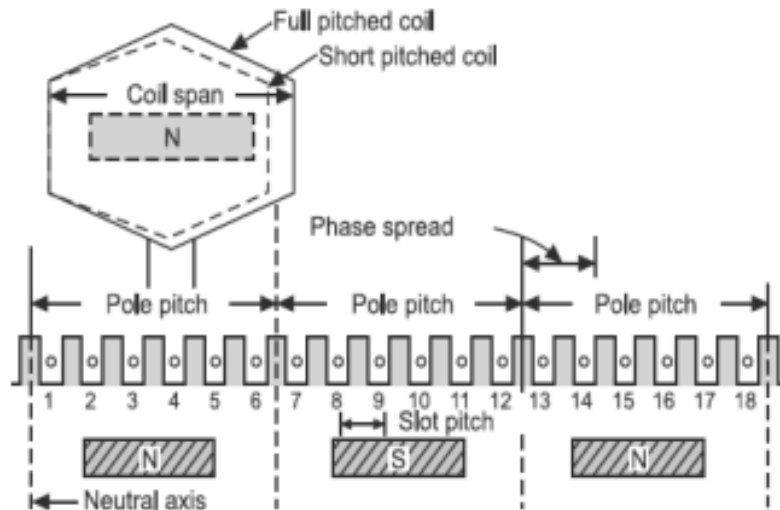
**Slot pitch,  $\beta = 180^\circ / (\text{No. of slots/pole})$**

**VI. Phase spread:** The angle or space of pole face over which coil sides of the same phase are spread is called phase spread, as shown in Figure 7. In a distributed winding, the conductors of one phase under one pole are spread in number of slots so that each phase has equal distribution.

In a three-phase winding:

**Phase spread =  $180 / 3 = 60$  electrical degrees**

**or Phase spread = No. of slots/pole/phase**



**Figure 7 Distributed Windings**

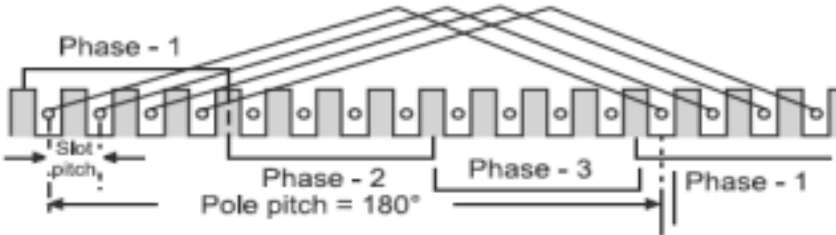
## 6. Windings Factors:

The stator winding of synchronous machine is distributed over the entire stator. The distributed winding produces nearly a sine waveform and the heating is more uniform. Likewise, the coils of armature winding are not full pitched i.e., the two sides of a coil are not at corresponding points under adjacent poles. The fractional pitched armature winding requires less copper per coil and at the same time waveform of output voltage is improved. The distribution and pitching of the coils affect the voltages induced in the coils. We shall discuss two winding factors:

### I. Distribution Factor ( $K_d$ ):

A winding with only one slot per pole per phase is called a concentrated winding. In this type of winding, the e.m.f. generated

per phase is equal to the arithmetic sum of the individual coil e.m.f.s in that phase. However, if the coils per phase are distributed over several slots in space (distributed winding), the e.m.f.s in the coils are not in phase (the phase difference between coils e.m.f.s is not zero) but are displaced from each by phase angle equal to the slot angle ( $\beta$ ) multiplied by the number of slots that the coils are distributed over as shown in figure 8.



**Figure 8 Distributed Windings**

The e.m.f. per phase will be the phasor sum of coil e.m.f.s. Thus, the distribution factor can be defined as:

$$K_d = \frac{\text{e.m.f. with distributed winding}}{\text{e.m.f. with concentrated winding}}$$

The final expression for the distribution factor is given below:

$$K_d = \frac{\sin m\beta/2}{m \sin \beta/2}$$

Where,  $\beta$  is the slot pitch,  $m$  is the number of slot per phase per pole.

## II. Pitch (Coil Span) Factor ( $K_p$ ):

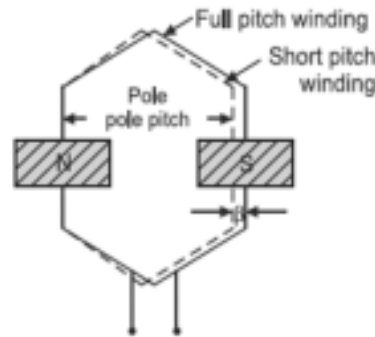
In a full pitch winding the coil span or coil pitch is always equal to the pole pitch which is equal to 180 electrical degrees. When the coil span is less than 180 electrical degrees, the winding is called short pitched or fractional pitch or chorded winding as shown in Figure 9. The pitch factor can be defined as:

$$K_p = \frac{\text{e.m.f. induced in short - pitch coil}}{\text{e.m.f. induced in full - pitch coil}}$$

The final expression for the pitch factor is given below:

$$K_p = \cos (\theta / 2),$$

Where,  $\theta$  is an integer multiple of the slot angle  $\beta$ .



**Figure 9 Short Pitch Winding**

## Winding Factor

The winding factor is the method of improving the rms generated voltage in a three-phase AC machine so that the torque and the output voltage do not consist of any harmonics which reduces the efficiency of the machine. Winding Factor is defined as the product of the Distribution factor ( $K_d$ ) and the coil span factor ( $K_c$ ). The distribution factor measured the resultant voltage of the distributed winding regards concentrate winding and the coil span is the measure of the number of armature slots between the two sides of a coil. It is

$$E_p = 4.44 K_w f \phi T_p \dots \dots (1)$$

denoted by  $K_w$ . The EMF equation is given below:

It is assumed that the induced voltage is sinusoidal. However, if the flux density distribution is non-sinusoidal, the induced voltage in the winding will be non-sinusoidal. The coil span factor, distribution factor, and winding factor will be different for each harmonic voltage. From equation (1), the fundamental EMF per phase is given by the equation shown below:

$$E_{p1} = 4.44 K_{w1} f \phi_1 T_p \dots \dots (2)$$

$$E_{p3} = 4.44 K_{w3} (3f) \phi_3 T_p \dots \dots (3)$$

The third harmonic, EMF per phase will be:

The  $n^{\text{th}}$  harmonic, EMF per phase will be here subscript 1,3 and n denote fundamental, third, and  $n^{\text{th}}$  harmonics respectively.

$$\frac{E_{pn}}{E_{p1}} = \frac{K_{wn}}{K_{w1}} \times \frac{n\phi_n}{\phi_1} \dots \dots \dots (5)$$

Therefore,

Where,

- $\phi_1$  is the total fundamental flux per pole.
- $\phi_1$  = average flux density x area under one pole

$$\phi_1 = \left( \frac{\text{peak flux density}}{\pi/2} \right) \times (\text{area under one pole})$$

$$\phi_1 = \left( \frac{B_{m1}}{\pi/2} \right) \left( \frac{\pi DL}{P} \right)$$

$$\phi_1 = \frac{2 DL}{P} B_{m1} \dots \dots \dots (6)$$

Where,

- $B_{m1}$  is the peak value of the fundamental component of the flux density wave
- D is the diameter of the armature or the mean air gap diameter
- L is the axial length of the armature or the active coil side length

$$\text{Pole pitch} = \frac{\pi D}{P_n} \dots \dots \dots (7)$$

$$\phi_n = \frac{2 DL}{nP} B_{mn} \dots \dots \dots (8)$$

Therefore,

$$\frac{E_{pn}}{E_{p1}} = \frac{K_{wn} B_{mn}}{K_{w1} B_{m1}} \dots \dots \dots (9)$$

Similarly for the  $n^{\text{th}}$  harmonic

$B_{mn}$  is the peak value of the  $n^{\text{th}}$  harmonic flux density

Winding Factor for  $n^{\text{th}}$  Harmonic

$$K_{wn} = K_{cn} K_{dn} \dots \dots (10)$$

The winding factor corresponding to the  $n^{\text{th}}$  harmonic voltage is given as:

Where  $K_{cn}$  and  $K_{dn}$  are the coil span factor and distribution factor for the  $n^{\text{th}}$  harmonic.

Therefore, the  $n^{\text{th}}$  order harmonic induced EMF per phase is given by the equation shown below:

$$E_p = 4.44 K_{cn} K_{dn} (nf) \phi_n T_p \dots \dots (11)$$

$$\phi_n = \frac{2 DL}{n P} B_{mn} \dots \dots (12)$$

Where,

In addition, to the fundamental flux, the induced voltage in a winding will contain harmonics because of the non-sinusoidal space flux density distribution. Since the positive and the negative halves of the flux density wave are identical, only odd harmonics can be present, and even harmonics are absent. Therefore, phase voltage may contain third, fifth, seventh, and higher-order harmonics.

Mainly the three-phase alternators are star-connected. The third-order harmonic voltages of all the phases are equal in-phase and magnitude. The phase of the star-connected machine is such that the voltage across any two lines is the phasor difference in the voltages of the corresponding phases. Hence, the third harmonic or multiple of the third harmonic are absent in the line voltage of the star-connected synchronous machine.

Since the strength of the harmonic components of voltage decreases with the increasing frequency, only fifth and seventh harmonics are important. These are known as Belt Harmonics.

Thus, the root mean square voltage of the induced voltage across lines of a 3 phase, star connected machine is given by the equation

$$E_{\text{line}} = \sqrt{3} \times \sqrt{E_1^2 + E_5^2 + E_7^2 + E_{11}^2 + \dots \dots}$$

shown below:

Where, subscripts 1, 5, 7, 11 ..... denotes fundamental fifth, seventh, eleventh harmonics respectively.

Let,

- P be the number of poles
- $\phi$  is Flux per pole in Webers
- N is the speed in revolution per minute (r.p.m)
- f be the frequency in Hertz
- $Z_{ph}$  is the number of conductors connected in series per phase
- $T_{ph}$  is the number of turns connected in series per phase
- $K_c$  is the coil span factor
- $K_d$  is the distribution factor

Flux cut by each conductor during one revolution is given as  $P\phi$  Weber. Time taken to complete one revolution is given by  $60/N$  sec

Average EMF induced per conductor will be given by the equation shown below:

$$\frac{P\phi}{60/N} = \frac{P\phi N}{60} \text{ volts}$$

Average EMF induced per phase will be given by the equation shown below:

$$\frac{P\phi N}{60} \times Z_{ph} = \frac{P\phi N}{60} \times 2T_{ph} \text{ and}$$

$$T_{ph} = \frac{Z_{ph}}{2}$$

$$\text{Average EMF} = 4 \times \phi \times T_{ph} \times \frac{PN}{120} = 4\phi f T_{ph}$$

The average EMF equation is derived with the following assumptions given below.

- Coils have got the full pitch.

- All the conductors are concentrated in one stator slot.

Root mean square (R.M.S) value of the EMF induced per phase is given by the equation shown below:

$E_{ph}$  = Average value x form factor

Therefore,

$$E_{ph} = 4\phi f T_{ph} \times 1.11 = 4.44 \phi f T_{ph} \text{ volts}$$

If the coil span factor  $K_c$  and the distribution factor  $K_d$ , are taken into consideration then the Actual EMF induced per phase is given as:

$$E_{ph} = 4.44 K_c K_d \phi f T_{ph} \text{ volts} \dots \dots (1)$$

Equation (1) shown above is the EMF equation of the Synchronous Generator.

#### Coil Span Factor

The Coil Span Factor is defined as the ratio of the induced emf in a coil when the winding is short-pitched to the induced emf in the same coil when the winding is full pitched.

#### Distribution Factor

The distribution factor is defined as the ratio of induced EMF in the coil group when the winding is distributed in a number of slots to the induced EMF in the coil group when the winding is concentrated in one slot.

#### Armature Reaction in Alternator

In an alternator like all other synchronous machines, the effect of armature reaction depends on the [power factor](#) i.e the phase relationship between the terminal [voltage](#) and armature current.

[Reactive power](#) (lagging) is the [magnetic field](#) energy, so if the generator supplies a lagging load, this implies that it is supplying magnetic energy to the load. Since this power comes from excitation of synchronous machine, the net reactive power gets reduced in the generator.

Hence, the armature reaction is demagnetizing. Similarly, the armature reaction has magnetizing effect when the generator supplies a leading load (as leading load takes the leading VAR) and in return gives lagging VAR (magnetic energy) to the generator. In case of purely resistive load, the armature reaction is cross magnetizing only.

The armature reaction of alternator or [synchronous generator](#), depends upon the phase angle between, stator armature current and induced voltage across the armature winding of alternator.

The phase difference between these two quantities, i.e. Armature current and voltage may vary from  $-90^\circ$  to  $+90^\circ$

If this angle is  $\theta$ , then,

$$-90^\circ \geq \theta \geq +90^\circ$$

To understand actual effect of this angle on armature reaction of alternator, we will consider three standard cases,

1. When  $\theta = 0$
2. When  $\theta = 90^\circ$
3. When  $\theta = -90^\circ$

#### Armature Reaction of Alternator at Unity Power Factor

At unity [power factor](#), the angle between armature current  $I$  and induced emf  $E$ , is zero. That means, armature current and induced emf are in same phase. But we know theoretically that emf induced in the armature is due to changing main field flux, linked with the armature conductor.

As the field is excited by DC, the main field flux is constant in respect to field magnets, but it would be alternating in respect of armature as there is a relative motion between field and armature in the alternator. If main field flux of the alternator in respect of armature can be represented as

$$\phi_f = \phi_{fm} \sin \omega t \dots \dots (1)$$

Then induced emf  $E$  across the armature is proportional to,  $d\phi/dt$ .

$$\text{Now, } \frac{d\phi_f}{dt} = -\omega \phi_{fm} \cos \omega t \dots \dots (2)$$

Hence, from these above equations (1) and (2) it is clear that the angle between,  $\phi_f$  and induced emf  $E$  will be  $90^\circ$ .

Now, armature flux  $\phi_a$  is proportional to armature current  $I$ . Hence, armature flux  $\phi_a$  is in phase with armature current  $I$ .

Again at unity [electrical power factor](#)  $I$  and  $E$  are in same phase. So, at unity power factor,  $\phi_a$  is phase with  $E$ . So at this condition, armature flux is in phase with induced emf  $E$  and field flux is in quadrature with  $E$ . Hence, armature flux  $\phi_a$  is in

quadrature with main field flux  $\phi_f$ .

As this two fluxes are perpendicular to each other, the armature reaction of the alternator at unity power factor is purely distorting or cross-magnetising type.

As the armature flux pushes the main field flux perpendicularly, distribution of main field flux under a pole face does not remain uniformly distributed. The flux density under the trailing pole tips increases somewhat while under the leading pole tips it decreases.

**Armature Reaction of Alternator at Lagging Zero Power Factor**

At lagging zero electrical power factor, the armature current lags by  $90^\circ$  to induced emf in the armature.

As the emf induced in the armature coil due to main field flux thus the emf leads the main field flux by  $90^\circ$ . From equation (1) we get, the field flux,

$$\phi_f = \phi_{fm} \sin \omega t$$

$$\text{Therefore, induced emf } E \propto - \frac{d\phi_f}{dt}$$

$$\Rightarrow E \propto - \omega \phi_{fm} \cos \omega t$$

Hence, at  $\omega t = 0$ , E is maximum and  $\phi_f$  is zero.

At  $\omega t = 90^\circ$ , E is zero and  $\phi_f$  has maximum value.

At  $\omega t = 180^\circ$ , E is maximum and  $\phi_f$  zero.

At  $\omega t = 270^\circ$ , E is zero and  $\phi_f$  has negative maximum value.

Here,  $\phi_f$  got maximum value  $90^\circ$  before E. Hence  $\phi_f$  leads E by  $90^\circ$ .

Now, armature current I is proportional to armature flux  $\phi_a$ , and I lags E by  $90^\circ$ . Hence,  $\phi_a$  lags E by  $90^\circ$ .

So, it can be concluded that, field flux  $\phi_f$  leads E by  $90^\circ$ .

Therefore, armature flux and field flux act directly opposite to each other. Thus, armature reaction of the alternator at lagging zero power factor is a purely demagnetising type. That means, armature flux directly weakens main field flux.

**Armature Reaction of Alternator at Leading Power Factor**

At leading power factor condition, armature current “I” leads induced emf E by an angle  $90^\circ$ . Again, we have shown just, field flux  $\phi_f$  leads, induced emf E by  $90^\circ$ .

Again, armature flux  $\phi_a$  is proportional to armature current I. Hence,  $\phi_a$  is in phase with I. Hence, armature flux  $\phi_a$  also leads E, by  $90^\circ$  as I leads E by  $90^\circ$ .

As in this case both armature flux and field flux lead, induced emf E by  $90^\circ$ , it can be said, field flux and armature flux are in the same direction. Hence, the resultant flux is simply arithmetic sum of field flux and armature flux. Hence, at last, it can be said that armature reaction of alternator due to a purely leading [electrical power factor](#) is the magnetizing type.

**Nature of Armature Reaction**

1. The armature reaction flux is constant in magnitude and rotates at synchronous speed.
2. The armature reaction is cross magnetising when the generator supplies a load at unity power factor.
3. When the generator supplies a load at leading power factor the armature reaction is partly demagnetising and partly cross-magnetising.
4. When the generator supplies a load at leading power factor the armature reaction is partly magnetising and partly cross-magnetising.
5. Armature flux acts independently of main field flux.

## Synchronous Impedance Method (EMF Method) for Finding Voltage Regulation of Alternator

The synchronous impedance method or EMF method is used to determine the voltage regulation of the larger alternators. The synchronous impedance method is based on the concept of replacing the effect of armature reaction by an imaginary reactance.

For an alternator,

At first, the synchronous impedance ( $Z_s$ ) is measured and then, the value of actual generated EMF ( $E_a$ ) is calculated. Thus, from the values of ( $E_a$ ) and V, the voltage regulation of the alternator can be calculated.

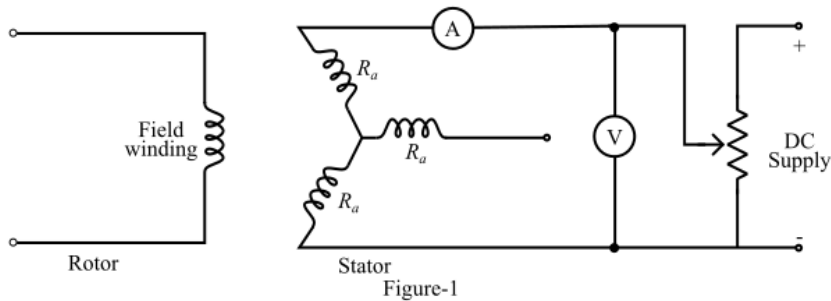
## Measurement of Synchronous Impedance

In order to determine the value of synchronous impedance, following tests are performed on an alternator –

- DC Resistance Test
- Open-Circuit Test
- Short-Circuit Test

### DC Resistance Test

The circuit diagram for the DC resistance test is shown in Figure-1.

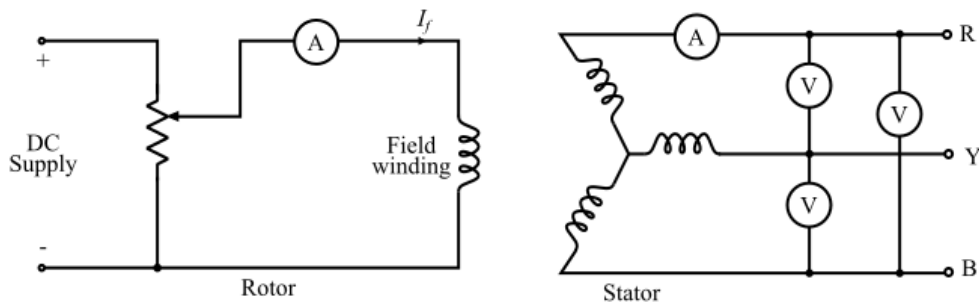


Consider the alternator is star-connected with the field winding open-circuited. Now, measure the DC resistance between each pair of terminals either by using Wheatstone's bridge or ammeter-voltmeter method. The average of three sets of resistance values  $R_t$  is taken. This value of  $R_t$  is divided by 2 to obtain the DC resistance per phase.

While performing the test, the alternator should be at rest, because the AC effective resistance is greater than DC resistance due to skin effect. The AC effective resistance per phase may be obtained by multiplying the DC resistance by a factor 1.20 to 1.75 depending upon the size of the alternator.

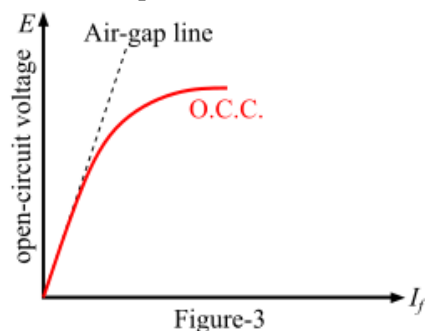
### Open-Circuit Test

To perform the open-circuit test, the load terminals are kept open and the alternator is run at rated synchronous speed. The circuit diagram of the open-circuit test is shown in Figure-2.



Initially, the field current is set to zero. Then, the field current is gradually increased in steps and the open-circuit terminal voltage  $E_t$  is measured in each step. The field current may be increased to obtain 25 % more than rated voltage of the alternator.

A graph is plotted between the open-circuit phase voltage ( $E_{ph} = E_t / \sqrt{3}$ ) and the field current ( $I_f$ ). The obtained characteristic curve is known as open-circuit characteristic (O.C.C.) of the alternator (see Figure-3).



The shape of O.C.C. is same as a normal magnetisation curve. When the linear portion of the O.C.C. is extended, it gives the air-gap line of the characteristic.

### Short-Circuit Test

For performing the short-circuit test, the armature terminals are short-circuited through three ammeters as shown in Figure-4.

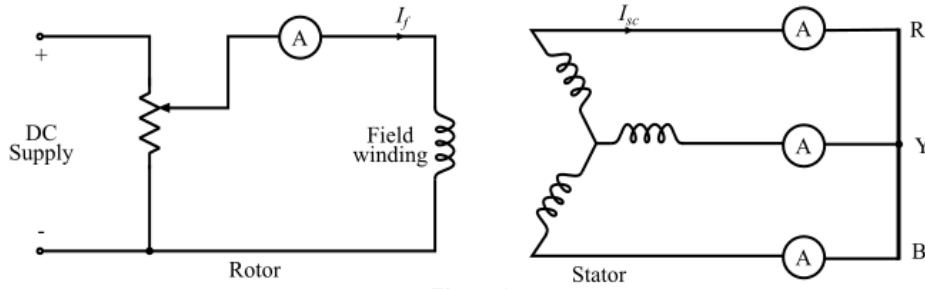


Figure-4

Before starting the alternator, the field current should be decreased to zero. Each ammeter should have a range more than the rated full-load value. Now, the alternator is run at synchronous speed. Then, the field current is gradually increased in steps and the armature current is measured at each step. The field current may be increased to obtain the armature currents up to 150 % of the rated value.

The field current ( $I_f$ ) and the average of the three ammeter readings is taken at each step. A graph is plotted between the armature current ( $I_a$ ) and the field current ( $I_f$ ). The obtained characteristic is known as short-circuit characteristic (S.C.C.) of the alternator and this characteristic is a straight line as shown in Figure-5.

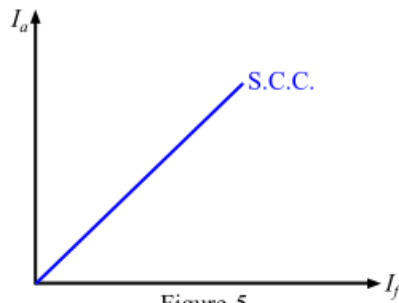


Figure-5

#### Calculation of Synchronous Impedance ( $Z_s$ )

In order to calculate the synchronous impedance of the alternator, the O.C.C. and the S.C.C. are drawn on the same curve sheet, as shown in Figure-6.

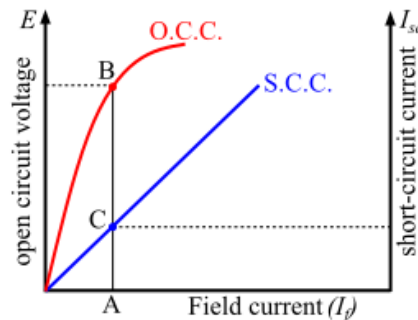


Figure-6

Then, determine the value of short-circuit current ( $I_{sc}$ ) at the field current that gives the rated voltage per phase of the alternator. The synchronous impedance ( $Z_s$ ) will then be equal to the ratio of the open-circuit voltage to the short-circuit current at the field current which gives the rated voltage per phase, i.e.,

$$Z_s = \frac{\text{open circuit voltage per phase}}{\text{short circuit armature current}} \dots (2)$$

From the figure-6, the synchronous impedance can be written as

$$\Rightarrow Z_s = \frac{AB \text{ (volts)}}{AC \text{ (amperes)}} \dots (3)$$

Also, the synchronous reactance of the alternator is

$$X_s = \sqrt{Z_s^2 - R_a^2} \dots (4)$$

Therefore, the percentage voltage regulation of the alternator will be,

$$\text{Percentage voltage regulation} = \frac{E_a - V}{V} \times 100 \dots (5)$$

**Alternator on load**

As the load on an alternator is varied, its terminal voltage is also found to vary as in d.c. generators. This variation in terminal voltage  $V$  is due to the following reasons:

1. voltage drop due to armature resistance  $R_a$
2. voltage drop due to armature leakage reactance  $X_L$
3. voltage drop due to armature reaction

#### (a) Armature Resistance

The armature resistance/phase  $R_a$  causes a voltage drop/phase of  $IR_a$  which is in phase with the armature current  $I$ . However, this voltage drop is practically negligible.

#### (b) Armature Leakage Reactance

When current flows through the armature conductors, fluxes are set up which do not cross the air-gap, but take different paths. Such fluxes are known as *leakage fluxes*. Various types of leakage fluxes are shown in Fig. 37.22.

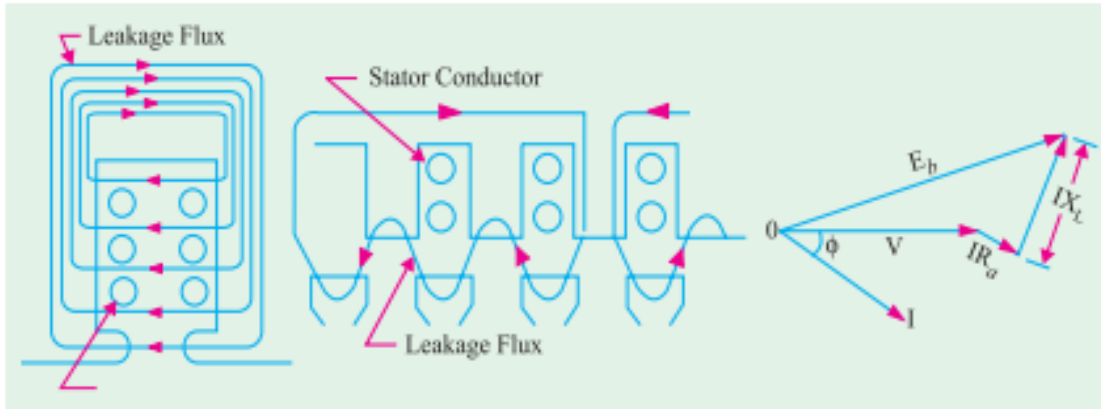


Fig. 37.22

Fig. 37.23

The leakage flux is practically independent of saturation, but is dependent on  $I$  and its phase angle with terminal voltage  $V$ . This leakage flux sets up an e.m.f. of self-inductance which is known as *reactance e.m.f.* and which is ahead of  $I$  by  $90^\circ$ . Hence, armature winding is assumed to possess leakage reactance  $X_L$  (also known as Potier reactance  $X_p$ ) such that voltage drop due to this equals  $IX_L$ . A part of the generated e.m.f. is used up in overcoming this reactance e.m.f.

$\therefore$

$$E = V + I(R + jX_L)$$

This fact is illustrated in the vector diagram of Fig. 37.23.

#### (c) Armature Reaction

As in d.c. generators, armature reaction is the effect of armature flux on the main field flux. **In the case of alternators, the power factor of the load has a considerable effect on the armature reaction.** We will consider three cases : (i) when load of p.f. is unity (ii) when p.f. is zero lagging and (iii) when p.f. is zero leading.

Before discussing this, it should be noted that in a 3-phase machine the combined ampere-turn wave (or m.m.f. wave) is sinusoidal which moves synchronously. This amp-turn or m.m.f. wave is fixed relative to the poles, its amplitude is proportional to the load current, but its position depends on the p.f. of the load.

Consider a 3-phase, 2-pole alternator having a single-layer winding, as shown in Fig. 37.24 (a). For the sake of simplicity, assume that winding of each phase is concentrated (instead of being distributed) and that the number of turns per phase is  $N$ . Further suppose that the alternator is loaded with a resistive load of unity power factor, so that phase currents  $I_a$ ,  $I_b$  and  $I_c$  are in phase with their respective phase voltages. Maximum current  $I_a$  will flow when the poles are in position shown in Fig. 37.24 (a) or at a time  $t_1$  in Fig. 37.24 (c). When  $I_a$  has a maximum value,  $I_b$  and  $I_c$  have one-half their maximum values (the arrows attached to  $I_a$ ,  $I_b$  and  $I_c$  are only polarity marks and are not meant to give the instantaneous directions of these currents at time  $t_1$ ). The instantaneous directions of currents are shown in Fig. 37.24 (a). At the instant  $t_1$ ,  $I_a$  flows in conductor  $\alpha$  whereas  $I_b$  and  $I_c$  flow out.

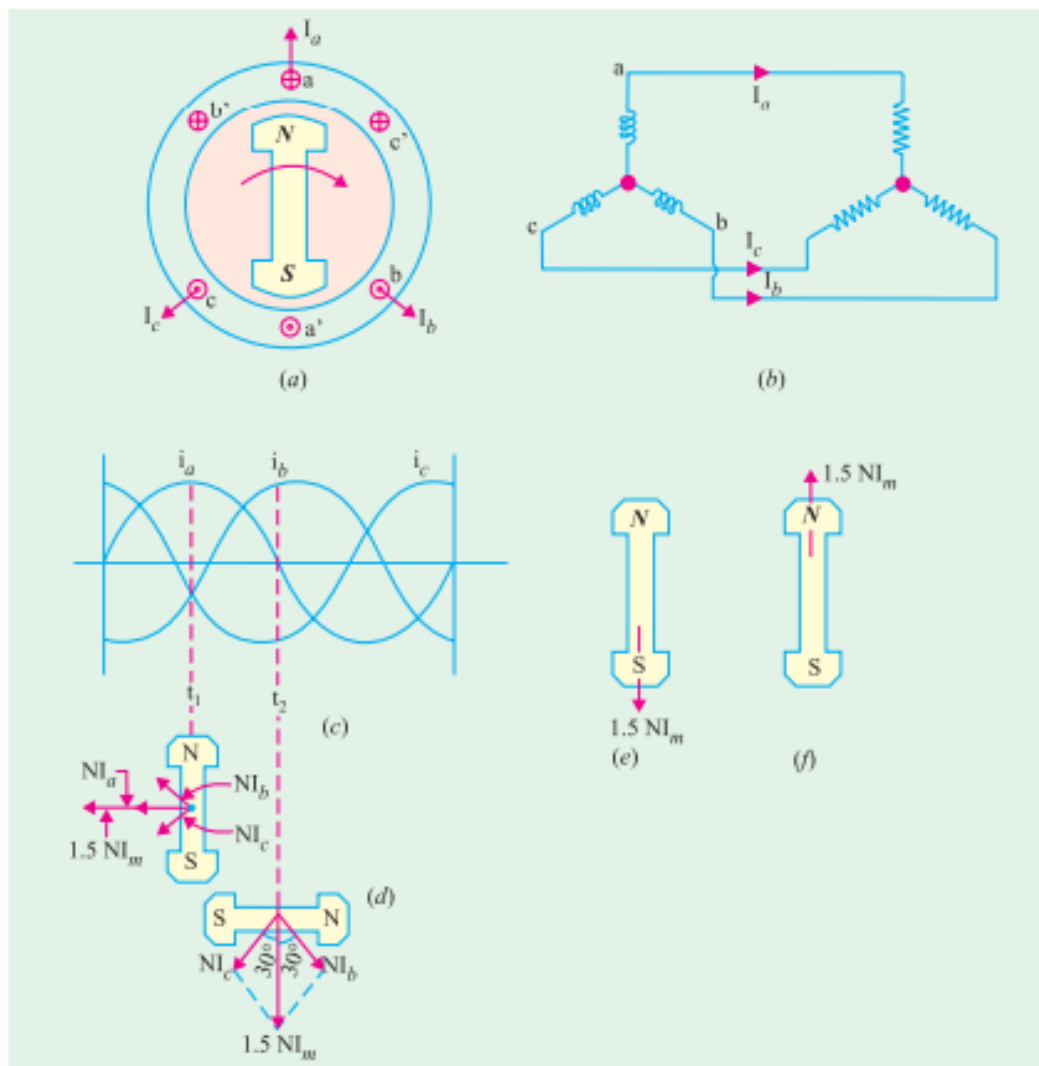


Fig. 37.24

As seen from Fig. 37.24 (d), the m.m.f. ( $= NI_m$ ) produced by phase  $a-a'$  is horizontal, whereas that produced by other two phases is  $(I_m/2) N$  each at  $60^\circ$  to the horizontal. The total armature m.m.f. is equal to the vector sum of these three m.m.fs.

$$\therefore \text{Armature m.m.f.} = NI_m + 2 \cdot (1/2 NI_m) \cos 60^\circ = 1.5 NI_m$$

As seen, at this instant  $t_1$ , the m.m.f. of the main field is upwards and the armature m.m.f. is behind it by  $90$  electrical degrees.

Next, let us investigate the armature m.m.f. at instant  $t_2$ . At this instant, the poles are in the horizontal position. Also  $I_a = 0$ , but  $I_b$  and  $I_c$  are each equal to  $0.866$  of their maximum values. Since  $I_c$  has not changed in direction during the interval  $t_1$  to  $t_2$ , the direction of its m.m.f. vector remains unchanged. But  $I_b$  has changed direction, hence, its m.m.f. vector will now be in the position shown in Fig. 37.24 (d). Total armature m.m.f. is again the vector sum of these two m.m.fs.

$$\therefore \text{Armature m.m.f.} = 2 \times (0.866 NI_m) \times \cos 30^\circ = 1.5 NI_m$$

If further investigations are made, it will be found that.

1. armature m.m.f. remains constant with time

2. it is  $90^\circ$  space degrees behind the main field m.m.f., so that it is only distortional in nature.
3. it rotates synchronously round the armature *i.e.* stator.

For a lagging load of zero power factor, all currents would be delayed in time  $90^\circ$  and armature m.m.f. would be shifted  $90^\circ$  with respect to the poles as shown in Fig. 37.24 (e). Obviously, armature m.m.f. would demagnetise the poles and cause a reduction in the induced e.m.f. and hence the terminal voltage.

For leading loads of zero power factor, the armature m.m.f. is advanced  $90^\circ$  with respect to the position shown in Fig. 37.24 (d). As shown in Fig. 37.24 (f), the armature m.m.f. strengthens the main m.m.f. In this case, armature reaction is wholly magnetising and causes an increase in the terminal voltage.

The above facts have been summarized briefly in the following paragraphs where the matter is discussed in terms of 'flux' rather than m.m.f. waves.

### 1. Unity Power Factor

In this case [Fig. 37.25 (a)] the armature flux is cross-magnetising. The result is that the flux at the leading tips of the poles is reduced while it is increased at the trailing tips. However, these two effects nearly offset each other leaving the average field strength constant. In other words, armature reaction for unity p.f. is distortional.

### 2. Zero P.F. lagging

As seen from Fig. 37.25 (b), here the armature flux (whose wave has moved backward by  $90^\circ$ ) is in direct opposition to the main flux.

Hence, the main flux is decreased. Therefore, it is found that armature reaction, in this case, is wholly **demagnetising**, with the result, that due to weakening of the main flux, less e.m.f. is generated. To keep the value of generated e.m.f. the same, field excitation will have to be increased to compensate for this weakening.

### 3. Zero P.F. leading

In this case, shown in Fig. 37.25 (c) armature flux wave has moved forward by  $90^\circ$  so that it is in phase with the main flux wave. This results in added main flux. Hence, in this case, armature reaction is wholly **magnetising**, which results in greater induced e.m.f. To keep the value of generated e.m.f. the same, field excitation will have to be reduced somewhat.

4. For intermediate power factor [Fig. 37.25 (d)], the effect is partly distortional and partly demagnetising (because p.f. is lagging).

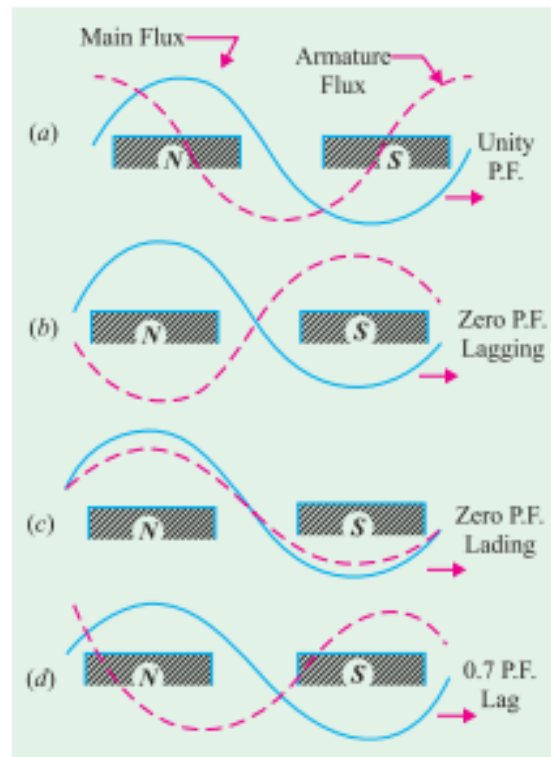


Fig. 37.25

## 37.17. Synchronous Reactance

From the above discussion, it is clear that for the same field excitation, terminal voltage is decreased from its no-load value  $E_0$  to  $V$  (for a lagging power factor). This is because of

1. drop due to armature resistance,  $IR_a$
2. drop due to leakage reactance,  $IX_L$
3. drop due to armature reaction.

The drop in voltage due to armature reaction may be accounted for by assuming the presence of a fictitious reactance  $X_a$  in the armature winding. The value of  $X_a$  is such that  $IX_a$  represents the voltage drop due to armature reaction.

The leakage reactance  $X_L$  (or  $X_p$ ) and the armature reactance  $X_a$  may be combined to give synchronous reactance  $X_s$ .

$$\text{Hence } X_s = X_L + X_a$$

Therefore, total voltage drop in an alternator under load is  $= IR_a + jIX_s = I(R_a + jX_s) = IZ_s$  where  $Z_s$  is known as synchronous impedance of the armature, the word 'synchronous' being used merely as an indication that it refers to the working conditions.

Hence, we learn that the vector difference between no-load voltage  $E_0$  and terminal voltage  $V$  is equal to  $IZ_s$ , as shown in Fig. 37.26.

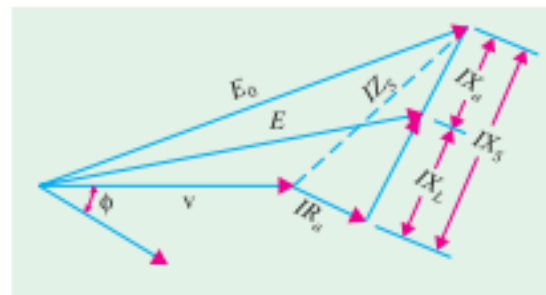


Fig. 37.26

### 37.18. Vector Diagrams of a Loaded Alternator

Before discussing the diagrams, following symbols should be clearly kept in mind.

$E_0$  = No-load e.m.f. This being the voltage induced in armature in the absence of three factors discussed in Art. 37.16. Hence, it represents the maximum value of the induced e.m.f.

$E$  = Load induced e.m.f. It is the induced e.m.f. after allowing for armature reaction.  $E$  is vectorially less than  $E_0$  by  $IX_a$ . Sometimes, it is written as  $E_a$  (Ex. 37.16).

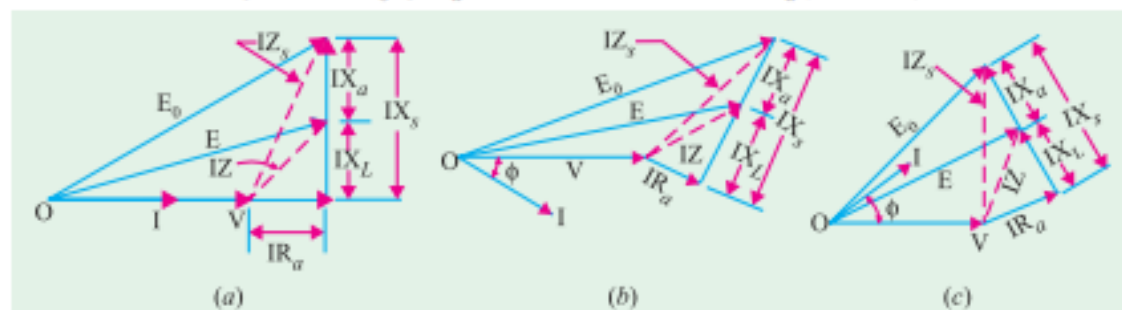


Fig. 37.27

$V$  = Terminal voltage, It is vectorially less than  $E_0$  by  $IZ_s$  or it is vectorially less than  $E$  by  $I_Z$  where

$$Z = \sqrt{(R_a^2 + X_L^2)}. \text{ It may also be written as } Z_a.$$

$I$  = armature current/phase and  $\phi$  = load p.f. angle.

In Fig. 37.27 (a) is shown the case for unity p.f., in Fig. 37.27 (b) for lagging p.f. and in Fig. 37.27 (c) for leading p.f. All these diagrams apply to one phase of a 3-phase machine. Diagrams for the other phases can also be drawn similarly.

**Example 37.16.** A 3-phase, star-connected alternator supplies a load of 10 MW at p.f. 0.85 lagging and at 11 kV (terminal voltage). Its resistance is 0.1 ohm per phase and synchronous reactance 0.66 ohm per phase. Calculate the line value of e.m.f. generated.

(Electrical Technology, Aligarh Muslim Univ. 1988)

**Solution.** F.L. output current  $= \frac{10 \times 10^6}{\sqrt{3} \times 11,000 \times 0.85} = 618 \text{ A}$

$IR_a \text{ drop} = 618 \times 0.1 = 61.8 \text{ V}$

$IX_s \text{ drop} = 618 \times 0.66 = 408 \text{ V}$

Terminal voltage/phase  $= 11,000 / \sqrt{3} = 6,350 \text{ V}$

$\phi = \cos^{-1}(0.85) = 31.8^\circ; \sin \phi = 0.527$

As seen from the vector diagram of Fig. 37.28 where  $I$  instead of  $V$  has been taken along reference vector,

$$\begin{aligned} E_0 &= \sqrt{(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_s)^2} \\ &= \sqrt{(6350 \times 0.85 + 61.8)^2 + (6350 \times 0.527 + 408)^2} \\ &= 6,625 \text{ V} \end{aligned}$$

Line e.m.f.  $= \sqrt{3} \times 6,625 = 11,486 \text{ volt}$

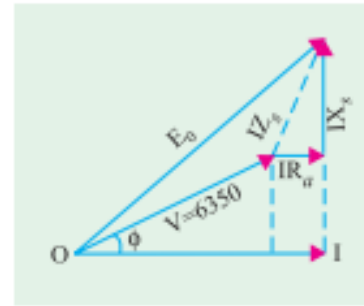


Fig. 37.28

### 37.19. Voltage Regulation

It is clear that with change in load, there is a change in terminal voltage of an alternator. The magnitude of this change depends *not only on the load but also on the load power factor.*

The voltage regulation of an alternator is defined as “the rise in voltage when full-load is removed (field excitation and speed remaining the same) divided by the rated terminal voltage.”

$\therefore \% \text{ regulation 'up'} = \frac{E_0 - V}{V} \times 100$

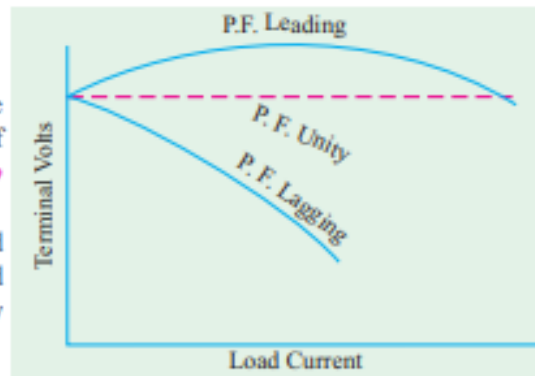


Fig. 37.29

**Note.** (i)  $E_0 - V$  is the arithmetical difference and not the vectorial one.

(ii) In the case of *leading* load p.f., terminal voltage will fall on removing the full-load. Hence, regulation is negative in that case.

(iii) The rise in voltage when full-load is thrown off is not the same as the fall in voltage when full-load is applied.

Voltage characteristics of an alternator are shown in Fig. 37.29.

### 37.20. Determination of Voltage Regulation

In the case of small machines, the regulation may be found by direct loading. The procedure is as follows :

The alternator is driven at synchronous speed and the terminal voltage is adjusted to its rated value  $V$ . The load is varied until the wattmeter and ammeter (connected for the purpose) indicate the rated values at desired p.f. Then the entire load is thrown off while the speed and field excitation are kept constant. The open-circuit or no-load voltage  $E_0$  is read. Hence, regulation can be found from

$$\% \text{ regn} = \frac{E_0 - V}{V} \times 100$$

In the case of large machines, the cost of finding the regulation by direct loading becomes prohibitive. Hence, other indirect methods are used as discussed below. It will be found that all these methods differ chiefly in the way the no-load voltage  $E_0$  is found in each case.



1. **Synchronous Impedance or E.M.F. Method.** It is due to Behn Eschenberg.
2. **The Ampere-turn or M.M.F. Method.** This method is due to Rothert.
3. **Zero Power Factor or Potier Method.** As the name indicates, it is due to Potier.

All these methods require—

1. Armature (or stator) resistance  $R_a$
2. Open-circuit/No-load characteristic.
3. Short-circuit characteristic (but zero power factor lagging characteristic for Potier method).

Now, let us take up each of these methods one by one.

#### (i) Value of $R_a$

Armature resistance  $R_a$  per phase can be measured directly by voltmeter and ammeter method or by using Wheatstone bridge. However, under working conditions, the effective value of  $R_a$  is increased due to 'skin effect'. The value of  $R_a$  so obtained is increased by 60% or so to allow for this effect. Generally, a value 1.6 times the d.c. value is taken.

#### (ii) O.C. Characteristic

As in d.c. machines, this is plotted by running the machine on no-load and by noting the values of induced voltage and field excitation current. It is just like the  $B$ - $H$  curve.

#### (iii) S.C. Characteristic

It is obtained by short-circuiting the armature (i.e. stator) windings through a low-resistance ammeter. The excitation is so adjusted as to give 1.5 to 2 times the value of full-load current. During this test, the speed which is not necessarily synchronous, is kept constant.

**Example 37.17 (a).** The effective resistance of a 2200V, 50Hz, 440 KVA, 1-phase, alternator is 0.5 ohm. On short circuit, a field current of 40 A gives the full load current of 200 A. The electromotive force on open-circuits with same field excitation is 1160 V. Calculate the synchronous impedance and reactance. (Madras University, 1997)

**Solution.** For the 1-ph alternator, since the field current is same for O.C. and S.C. conditions

$$Z_s = \frac{1160}{200} = 5.8 \text{ ohms}$$

$$X_s = \sqrt{5.8^2 - 0.5^2} = 5.7784 \text{ ohms}$$

**Example 37.17 (b).** A 60-KVA, 220 V, 50-Hz, 1- $\phi$  alternator has effective armature resistance of 0.016 ohm and an armature leakage reactance of 0.07 ohm. Compute the voltage induced in the armature when the alternator is delivering rated current at a load power factor of (a) unity (b) 0.7 lagging and (c) 0.7 leading. (Elect. Machines-I, Indore Univ. 1981)

**Solution.** Full load rated current  $I = 60,000/220 = 272.2 \text{ A}$

$$IR_a = 272.2 \times 0.016 = 4.3 \text{ V ;}$$

$$IX_L = 272.2 \times 0.07 = 19 \text{ V}$$

(a) **Unity p.f.** — Fig. 37.30 (a)

$$E = \sqrt{(V + IR_a)^2 + (IX_L)^2} = \sqrt{(220 + 4.3)^2 + 19^2} = 225 \text{ V}$$

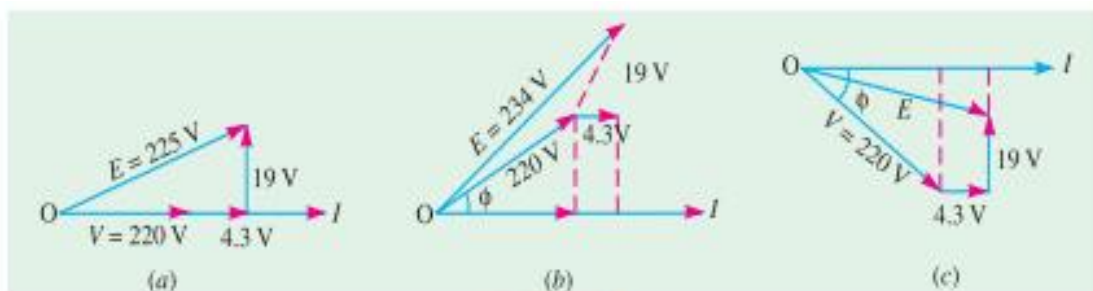


Fig. 37.30

(b) p.f. 0.7 (lag) —Fig. 37.30 (b)

$$E = [V \cos \phi + IR_a]^2 + (V \sin \phi + IX_L)^2]^{1/2}$$

$$= [(220 \times 0.7 + 4.3)^2 + (220 \times 0.7 + 19)^2]^{1/2} = 234 \text{ V}$$

(c) p.f. = 0.7 (lead) —Fig. 37.30 (c)

$$E = [(V \cos \phi + IR_a)^2 + (V \sin \phi - IX_L)^2]^{1/2}$$

$$= [(220 \times 0.7 + 4.3)^2 + (220 \times 0.7 - 19)^2]^{1/2} = 208 \text{ V}$$

**Example 37.18 (a).** In a 50-kVA, star-connected, 440-V, 3-phase, 50-Hz alternator, the effective armature resistance is 0.25 ohm per phase. The synchronous reactance is 3.2 ohm per phase and leakage reactance is 0.5 ohm per phase. Determine at rated load and unity power factor :

(a) Internal e.m.f.  $E_a$  (b) no-load e.m.f.  $E_0$  (c) percentage regulation on full-load (d) value of synchronous reactance which replaces armature reaction.

(Electrical Engg. Bombay Univ. 1987)

**Solution.** (a) The e.m.f.  $E_a$  is the vector sum of (i) terminal voltage  $V$  (ii)  $IR_a$  and (iii)  $IX_L$  as detailed in Art. 37.17. Here,

$$V = 440 / \sqrt{3} = 254 \text{ V}$$

F.L. output current at u.p.f. is

$$= 50,000 / \sqrt{3} \times 440 = 65.6 \text{ A}$$

Resistive drop =  $65.6 \times 0.25 = 16.4 \text{ V}$

Leakage reactance drop  $IX_L = 65.6 \times 0.5 = 32.8 \text{ V}$

$$\therefore E_a = \sqrt{(V + IR_a)^2 + (IX_L)^2}$$

$$= \sqrt{(254 + 16.4)^2 + 32.8^2} = 272 \text{ volt}$$

$$\text{Line value} = \sqrt{3} \times 272 = 471 \text{ volt.}$$

(b) The no-load e.m.f.  $E_0$  is the vector sum of (i)  $V$  (ii)  $IR_a$  and (iii)  $IX_S$  or is the vector sum of  $V$  and  $IZ_S$  (Fig. 37.31).

$$\therefore E_0 = \sqrt{(V + IR_a)^2 + (IX_S)^2} = \sqrt{(254 + 16.4)^2 + (65.6 \times 3.2)^2} = 342 \text{ volt}$$

$$\text{Line value} = \sqrt{3} \times 342 = 592 \text{ volt}$$

$$(c) \% \text{ age regulation 'up'} = \frac{E_0 - V}{V} \times 100 = \frac{342 - 254}{254} \times 100 = 34.65 \text{ per cent}$$

$$(d) X_a = X_S - X_L = 3.2 - 0.5 = 2.7 \Omega \quad \dots \text{Art. 37.17}$$

**Example 37.18 (b).** A 1000 kVA, 3300-V, 3-phase, star-connected alternator delivers full-load current at rated voltage at 0.80 p.f. Lagging. The resistance and synchronous reactance of the

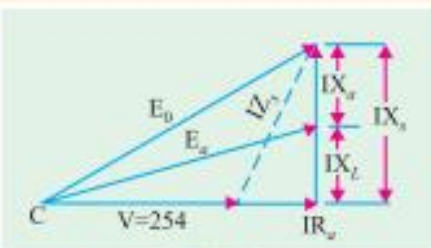


Fig. 37.31

machine per phase are 0.5 ohm and 5 ohms respectively. Estimate the terminal voltage for the same excitation and same load current at 0.80 p. f. leading. (Amravati University, 1999)

**Solution.**  $V_{ph} = \frac{3300}{\sqrt{3}} = 1905 \text{ volts}$

At rated load,  $I_{ph} = \frac{1000 \times 1000}{\sqrt{3} \times 3300} = 175 \text{ amp}$

From phasor diagram for this case [Fig. 37.32 (a)]

Component of E along Ref =  $OD = OA + AB \cos \phi + BC \sin \phi$   
 $= 1905 + (87.5 \times 0.80) + (875 \times 0.60) = 2500$

Component of E along perpendicular direction  
 $= CD = -AB \sin \phi + BC \cos \phi$   
 $= 87.5 \times 0.6 + 875 \times 0.80 = 647.5 \text{ volts}$

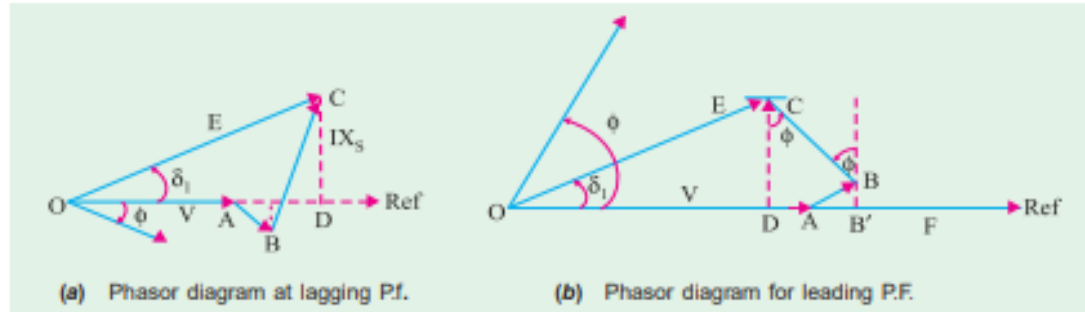


Fig. 37.32

$$OA = 1950, AB = I_r = 87.5, BC = I X_s = 875$$

$$OC = E = \sqrt{OD^2 + DC^2} = \sqrt{2500^2 + 647.5^2} = 2582.5 \text{ volts}$$

$$\delta_1 = \sin^{-1} \frac{CD}{OC} = \sin^{-1} (647.5 / 2582.5) = 14.52^\circ$$

Now, for  $E$  kept constant, and the alternator delivering rated current at 0.80 leading p.f., the phasor diagram is to be drawn to evaluate  $V$ .

Construction of the phasor diagram starts with marking the reference. Take a point  $A$  which is the terminating point of phasor  $V$  which starts from  $O$ .  $O$  is the point yet to be marked, for which the other phasors have to be drawn.

$$AB = 87.5, BC = 875$$

$$BAF = 36.8^\circ$$

$BC$  perpendicular to  $AB$ . From  $C$ , draw an arc of length  $E$ , i.e. 2582.5 volts to locate  $O$ .

**Note.** Construction of Phasor diagram starts from known  $AE$ ,  $V$  is to be found.

Along the direction of the current,  $AB = 87.5$ ,  $\angle BAF = 36.8^\circ$ , since the current is leading.  $BC = 875$  which must be perpendicular to  $AB$ . Having located  $C$ , draw a line  $CD$  which is perpendicular to the reference, with point  $D$ , on it, as shown.

Either proceed graphically drawing to scale or calculate geometrically :

Since  $CD = AB \sin \phi + BC \cos \phi = (87.5 \times 0.60) + (875 \times 0.80) = 752.5 \text{ volts}$

$CD = E \sin \delta_2$ ,  $\sin \delta_2 = 752.5 / 2582.5$  giving  $\delta_2 = 17^\circ$

$OD = E \cos \delta = 2470 \text{ volts}$

$DA = DB' - AB'$

$= BC \sin \phi - AB \cos \phi = 875 \times 0.6 - 87.5 \times 0.8 = 455 \text{ volts}$

## 1.14 Potier Reactance

For obtaining potier reactance Zero Power Factor test is conducted by connecting the alternator to ZPF load and exciting the alternator in such way that the alternator supplies the rated current at rated voltage running at rated speed. To plot ZPF characteristics only two points are required. One point is corresponding to the zero voltage and rated current that can be obtained from  $sc$  and the other at rated voltage and rated current under zpf load. This zero power factor curve appears like  $OCC$  but shifted by a factor  $I_a X_L$  vertically and horizontally by armature reaction mmf as shown below in Fig: 1.15. Following are the steps to draw ZPF characteristics.

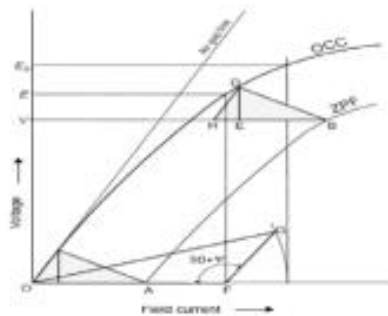


Fig: 1.15

By suitable tests plot OCC and SCC. Draw air gap line. Conduct ZPF test at full load for rated voltage and fix the point B. Draw the line BH with length equal to field current required to produce full load current on short circuit. Draw HD parallel to the air gap line so as to cut the OCC. Draw DE perpendicular to HB or parallel to voltage axis. Now, DE represents voltage drop  $I_a X_L$  and BE represents the field current required to overcome the effect of armature reaction.

Triangle BDE is called **Potier triangle** and  $X_L$  is the **Potier reactance**.

## 1.15 Voltage Regulation

When an alternator is subjected to a varying load, the voltage at the armature terminals varies to a certain extent, and the amount of this variation determines the regulation of the machine. When the alternator is loaded the terminal voltage decreases as the drops in the machine starts increasing and hence it will always

be different than the induced emf.

Voltage regulation of an alternator is defined as the change in terminal voltage from no load to full load expressed as a percentage of rated voltage when the load at a given power factor is removed without change in speed and excitation. Or the numerical value of the regulation is defined as the percentage rise in voltage when full load at the specified power-factor is switched off with speed and field current remaining unchanged expressed as a percentage of rated voltage.

Hence regulation can be expressed as

$$\% \text{ Regulation} = \left( \frac{E_0 - V_t}{V_t} \right) \times 100$$

where  $E_0$  = No-load induced emf /phase,  $V_t$  = Rated terminal voltage/phase at load

### **1.16 Methods of finding Voltage Regulation:**

The voltage regulation of an alternator can be determined by different methods. In case of small generators it can be determined by direct loading whereas in case of large generators it cannot be determined by direct loading but will be usually predetermined by different methods. Following are the different methods used for predetermination of regulation of alternators.

1. Direct loading method
2. EMF method or Synchronous impedance method
3. MMF method or Ampere turns method
4. ASA modified MMF method
5. ZPF method or Potier triangle method

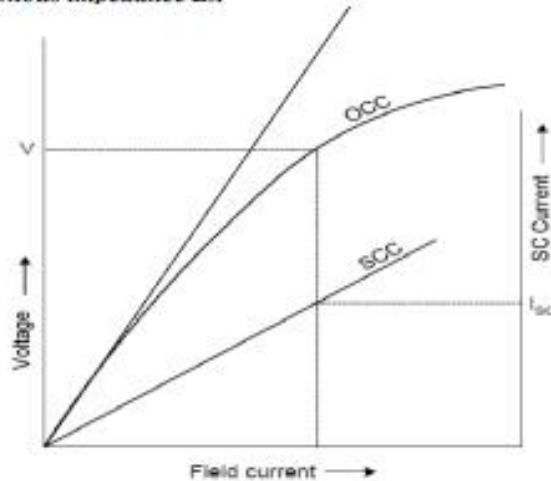
All the above methods other than direct loading are valid for non-salient pole machines only. As the alternators are manufactured in large capacity direct loading of alternators is not employed for determination of regulation. Other methods can be employed for predetermination of regulation. Hence the other methods of determination of regulations will be discussed in the following sections.

### 1.16.1 EMF method:

This method is also known as synchronous impedance method. Here the magnetic circuit is assumed to be unsaturated. In this method the MMFs (fluxes) produced by rotor and stator are replaced by their equivalent emf, and hence called emf method.

To predetermine the regulation by this method the following informations are to be determined. Armature resistance /phase of the alternator, open circuit and short circuit characteristics of the alternator.

#### *Determination of synchronous impedance $Z_s$ :*



**Fig: 1.16 OCC and SCC of alternator**

As the terminals of the stator are short circuited in SC test, the short circuit current is circulated against the impedance of the stator called the synchronous impedance. This impedance can be estimated from the oc and sc characteristics.

The ratio of open circuit voltage to the short circuit current at a particular field current, or at a field current responsible for circulating the rated current is called the synchronous impedance.

Synchronous impedance  $Z_s = (\text{open circuit voltage per phase}) / (\text{short circuit current per phase})$  for same  $I_f$

Hence  $Z_s = (V_{oc}) / (I_{sc})$  for same  $I_f$

From Fig: 1.16 synchronous impedance  $Z_s = V / I_{sc}$

Armature resistance  $R_a$  of the stator can be measured using Voltmeter – Ammeter method. Using synchronous impedance and armature resistance synchronous reactance and hence regulation can be calculated as follows using emf method.

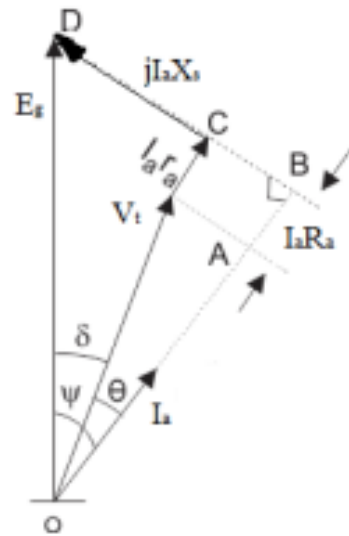


Fig: 1.17

$$Z_s = \sqrt{(R_a)^2 + (X_s)^2} \text{ and Synchronous reactance } X_s = \sqrt{(Z_s)^2 - (R_a)^2}$$

$$\text{Hence induced emf per phase can be found as } E_g = \sqrt{(V_t \cos \theta + I_a R_a)^2 + (V_t \sin \theta \pm I_a X_s)^2}$$

where  $V_t$  = phase voltage per phase =  $V_{ph}$ ,  $I_a$  = load current per phase

In the above expression in second term + sign is for lagging power factor and – sign is for leading power factor.

$$\% \text{ Regulation} = \left( \frac{E_g - V_t}{V_t} \right) \times 100$$

where  $E_g$  = no-load induced emf /phase,  $V_t$  = rated terminal voltage/phase

Synchronous impedance method is easy but it gives approximate results. This method gives the value of regulation which is greater (poor) than the actual value and hence this method is called pessimistic method.

The complete phasor diagram for the emf method is shown in Fig 1.18.

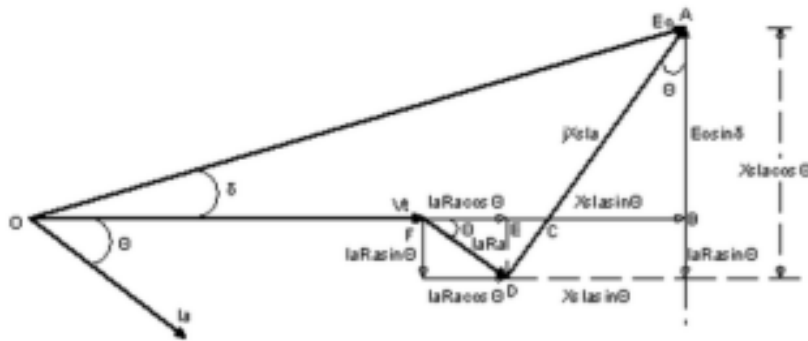


Fig: 1.18

### 1.13.2 MMF method

This method is also known as amp - turns method. In this method the all the emfs produced by rotor and stator are replaced by their equivalent MMFs (fluxes), and hence called mmf method. In this method also it is assumed that the magnetic circuit is unsaturated. In this method both the reactance drops are replaced by their equivalent mmfs. Fig: 1.19 shows the complete phasor diagram for the mmf method. Similar to emf method OC and SC characteristics are used for the determination of regulation by mmf method. The details are shown in Fig: 1.19. Using the details it is possible determine the regulation at different power factors.

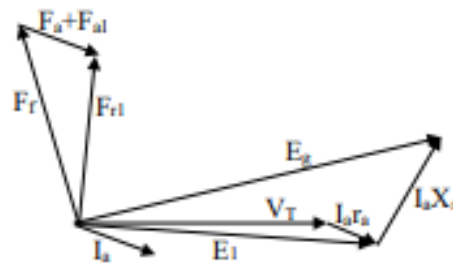


Fig: 1.19

From the phasor diagram it can be seen that the mmf required to produce the emf  $E_1 = (V + I_a R_a)$  is  $F_{r1}$ . In large machines resistance drop may be neglected. The mmf required to overcome the reactance drops is  $(F_r + F_{al})$  as shown in phasor diagram. The mmf  $(F_r + F_{al})$  can be found from SC characteristic as under SC condition both reactance drops will be present.

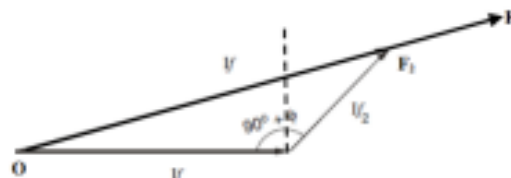
Following procedure can be used for determination of regulation by mmf method.

1. By conducting OC and SC test plot OCC and SCC.
2. From the OCC find the field current  $I_{f1}$  required to produce the voltage,  $E_1 = (V + IR_a)$ .
3. From SCC find the magnitude of field current  $I_{f2}$  ( $\approx F_a + F_{al}$ ) to produce the required armature current.  $F_a + F_{al}$  can also found from ZPF characteristics.
4. Draw  $I_{f2}$  at angle  $(90^\circ + \Phi)$  from  $I_{f1}$ , where  $\Phi$  is the phase angle of current w. r. t voltage. If current is leading, take the angle of  $I_{f2}$  as  $(90^\circ - \Phi)$ .
5. Determine the resultant field current,  $I_f$  and mark its magnitude on the field current axis.
6. From OCC. find the voltage corresponding to  $I_f$ , which will be  $E_0$  and hence find the regulation.

Because of the assumption of unsaturated magnetic circuit the regulation computed by this method will be less than the actual and hence this method of regulation is called optimistic method.

### 1.13.3 ASA Modified MMF Method:

ASA or modified mmf method consider saturation effect for calculation of regulation. In the mmf method the total mmf  $F$  computed is based on the assumption of unsaturated magnetic circuit which is unrealistic. In order to account for the partial saturation of the magnetic circuit it must be increased by a certain amount  $FF_2$  which can be computed from occ, sec and air gap lines as explained below referring to Fig: 1.20 (i) and (ii).





load. This zero power factor curve appears like OCC but shifted by a factor  $I_a X_L$  vertically and horizontally by armature reaction mmf as shown below in Fig: 1.21. Following are the steps to draw ZPF characteristics.

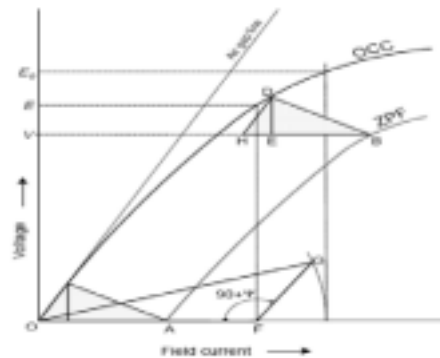


Fig: 1.21

By suitable tests plot OCC and SCC. Draw air gap line. Conduct ZPF test at full load for rated voltage and fix the point B. Draw the line BH with length equal to field current required to produce full load current on short circuit. Draw HD parallel to the air gap line so as to cut the OCC. Draw DE perpendicular to HB or parallel to voltage axis. Now, DE represents voltage drop  $IX_L$  and BE represents the field current required to overcome the effect of armature reaction.

Triangle BDE is called Potier triangle and  $X_L$  is the Potier reactance. Find E from  $V$ ,  $I_a R_a$ ,  $IX_L$  and  $\Phi$ . Use the expression  $E = \sqrt{(V_t \cos \Phi + I_a R_a)^2 + (V_t \sin \Phi + I_a X_L)^2}$  to compute E. Find field current corresponding to E. Draw FG with magnitude equal to BE at angle  $(90 + \Psi)$  from field current axis, where  $\Psi$  is the phase angle of current from voltage vector E (internal phase angle).

The resultant field current is given by OG. Mark this length on field current axis. From OCC find the corresponding  $E\theta$ . Find the regulation.

## 2.1 Salient pole alternators and Blondel's Two Reaction Theory

The details of synchronous generators developed so far is applicable to only round rotor or non-salient pole alternators. In such machines the air gap is uniform throughout and hence the effect of mmf will be same whether it acts along the pole axis or the inter polar axis. Hence reactance of the stator is same throughout and hence it is called synchronous reactance. But in case salient pole machines the air gap is non uniform and it is smaller along pole axis and is larger along the inter polar axis. These axes are called direct axis or d-axis and quadrature axis or q-axis. Hence the effect of mmf when acting along direct axis will be different than that when it is acting along quadrature axis. Hence the reactance of the stator cannot be same when the mmf is acting along d – axis and q- axis. As the length of the air gap is small along direct axis reluctance of the magnetic circuit is less and the air gap along the q – axis is larger and hence the along the quadrature axis will be comparatively higher. Hence along d-axis more flux is produced than q-axis. Therefore the reactance due to armature reaction will be different along d-axis and q-axis. These reactances are,

$$X_{ad} = \text{direct axis reactance; } X_{aq} = \text{quadrature axis reactance}$$

Hence the effect of armature reaction in the case of a salient pole synchronous machine can be taken as two components - one acting along the direct axis (coinciding with the main field pole axis) and the other acting along the quadrature axis (inter-polar region or magnetic neutral axis) and as such the mmf components of armature-reaction in a salient-pole machine cannot be considered as acting on the same magnetic circuit. Hence the effect of the armature reaction cannot be taken into account by considering only the synchronous reactance, in the case of a salient pole synchronous machine.

In fact, the direct-axis component  $F_{ad}$  acts over a magnetic circuit identical with that of the main field system and produces a comparable effect while the quadrature-axis component  $F_{aq}$  acts along the interpolar axis, resulting in an altogether smaller effect and, in addition, a flux distribution totally different from that of  $F_{ad}$  or the main field m.m.f. This explains why the application of cylindrical-rotor theory to salient-pole machines for predicting the performance gives results not conforming to the performance obtained from an actual test.

## 2.2 Direct-axis and Quadrature-axis Synchronous Reactances

Blondel's two-reaction theory considers the effects of the quadrature and direct-axis components of the armature reaction separately. Neglecting saturation, their different effects are considered by assigning to each an appropriate value of armature-reaction "reactance," respectively  $X_{ad}$  and  $X_{aq}$ . The effects of armature resistance and true leakage reactance ( $X_L$ ) may be treated separately, or may be added to the armature reaction coefficients on the assumption that they are the same, for either the direct-axis or quadrature-axis components of the armature current (which is almost true). Thus the combined reactance values can be expressed as,

$$X_{sd} = X_{ad} + X, \text{ and } X_{sq} = X_{aq} + X, \text{ for the direct- and cross-reaction axes respectively.}$$

In a salient-pole machine,  $X_{aq}$ , the quadrature-axis reactance is smaller than  $X_{ad}$ , the direct-axis reactance, since the flux produced by a given current component in that axis is smaller as the reluctance of the magnetic path consists mostly of the interpolar spaces. It is essential to clearly note the difference between the quadrature and direct-axis components  $I_{aq}$ , and  $I_{ad}$  of the armature current  $I_a$ , and the reactive and active components  $I_{sa}$  and  $I_{ra}$ . Although both pairs are represented by phasors in phase quadrature, the former are related to the induced emf  $E_i$  while the latter are referred to the terminal voltage  $V$ . These phasors are clearly indicated with reference to the phasor diagram of a (salient pole) synchronous generator supplying a lagging power factor (pf) load, shown in Fig.2.1

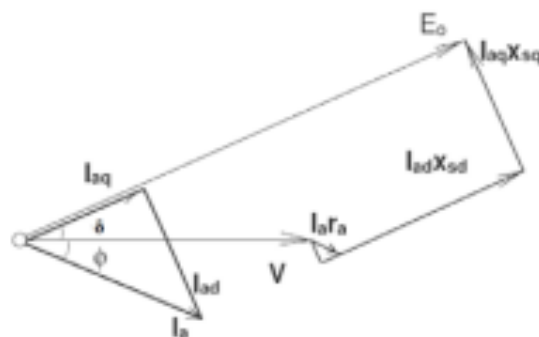


Fig: 2.1 Phasor diagram of salient-pole alternator

$$I_{aq} = I_a \cos(\delta + \phi); \quad I_{ad} = I_a \sin(\delta + \phi); \quad \text{and} \quad I_a = \sqrt{(I_{aq})^2 + (I_{ad})^2}$$

$$I_{aa} = I_a \cos \phi; \quad I_{ar} = I_a \sin \phi; \quad \text{and} \quad I_a = \sqrt{(I_{aa})^2 + (I_{ar})^2}$$

where  $\delta$  = torque or power angle and  $\phi$  = the p.f. angle of the load.

### 2.3 Power Angle Characteristic of Salient Pole Machine

Neglecting the armature winding resistance, the power output of the generator is given by:

$$P = V * I_a * \cos \phi$$

This can be expressed in terms of  $\sigma$ ,

$$I_a * \cos \phi = I_{aq} * \cos \sigma + I_{ad} * \sin \sigma$$

$$V * \cos \sigma = E_o - I_{ad} * x_{sd}$$

$$\text{and } V * \sin \sigma = I_{aq} * x_{sd}$$

Substituting these in the expression for power, we have.

$$\begin{aligned} P &= V[(V * \sin \sigma / x_{sd}) * \cos \sigma + (E_o - V * \cos \sigma) / x_{sd} * \sin \sigma] \\ &= (V * E_o / x_{sd}) * \sin \sigma + V^2 * (x_{sd} - x_{sq}) / (2 * x_{sq} * x_{sd}) * \sin 2\sigma \end{aligned}$$

It is clear from the above expression that the power is a little more than that for a cylindrical rotor synchronous machine, as the first term alone represents the power for a cylindrical rotor synchronous machine. A term in  $(\sin 2\sigma)$  is added into the power - angle characteristic of a non-salient pole synchronous machine. This also shows that it is possible to generate an emf even if the excitation  $E_o$  is zero. However this magnitude is quite less compared with that obtained with a finite  $E_o$ . Likewise we can show that the machine develops a torque - called the reluctance torque - as this torque is developed due to the variation of the reluctance in the magnetic circuit even if the excitation  $E_o$  is zero. Fig. 2.2 shows the typical power angle characteristic of a salient pole alternator.

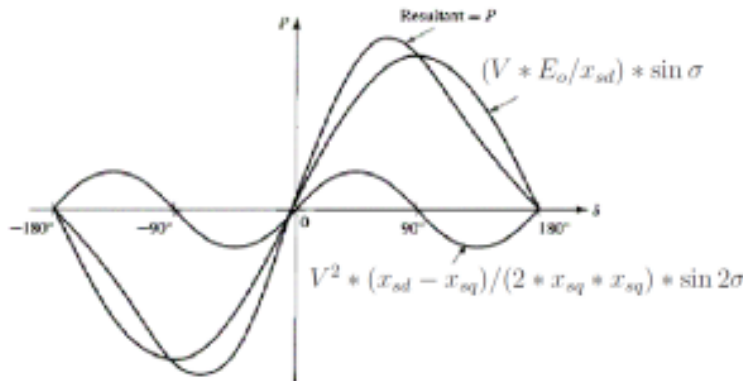


Fig. 2.2

## 2.4 Slip Test

From this test the values of  $X_d$  and  $X_q$  are determined by applying a balance reduced external voltage (say,  $V$  volts, around 25% of rated value) to the armature. The field winding remains unexcited. The machine is run at a speed a little less than the synchronous speed (the slip being less than 1%) using a prime mover (or motor). Connection diagram is shown in circuit diagram.

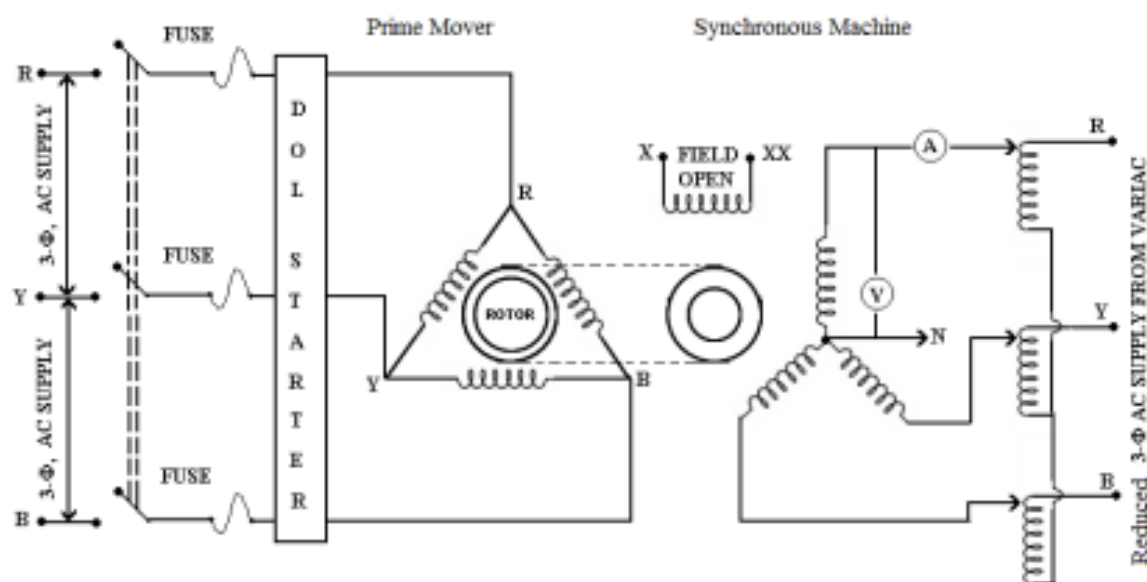


Fig: 2.3

Due to voltage  $V$  applied to the stator terminal a current  $I$  will flow causing a stator mmf. This stator mmf moves slowly relative to the poles and induced an emf in the field circuit in a similar fashion to that of rotor in an induction motor at slip frequency. The effect will be that the stator mmf will moves slowly relative to the poles.

The physical poles and the armature-reaction mmf are alternately in phase and out, the change occurring at slip frequency. When the axis of the pole and the axis of the armature reaction mmf wave coincide, the armature mmf acts through the field magnetic circuit. Since the applied voltage is constant, the air-gap flux would be constant. When crest of the rotating armature mmf is in line with the field-pole axis, minimum air-gap offers minimum reluctance thus the current required in armature for the establishment of constant air-gap flux must be minimum. Constant applied voltage minus the minimum impedance voltage drop in the armature terminal gives maximum armature terminal

DEPT

Vignansarathi Engineering College, Vellore

voltage. Thus the d-axis synchronous reactance is given by

$$X_d = \frac{\text{Maximum armature terminal voltage per phase}}{\text{Minimum armature current per phase}}$$

Similarly

$$X_q = \frac{\text{Minimum armature terminal voltage per phase}}{\text{Maximum armature current per phase}}$$

# Module II

## Parallel operation of alternators

The operation of connecting an alternator in parallel with another alternator or with common bus-bars is known as *synchronizing*. Generally, alternators are used in a power system where they are in parallel with many other alternators. It means that the alternator is connected to a live system of constant voltage and constant frequency. Often the electrical system, to which the alternator is connected, has already so many alternators and loads connected to it that no matter what power is delivered by the incoming alternator, the voltage and frequency of the system remain the same. In that case, the alternator is said to be connected to *infinite* bus-bars.

For proper synchronization of alternators, the following four conditions must be satisfied

1. The terminal voltage (effective) of the incoming alternator must be the same as bus-bar voltage.
2. The speed of the incoming machine must be such that its frequency ( $= PN/60$ ) equals bus-bar frequency.
3. The phase of the alternator voltage must be identical with the phase of the bus-bar voltage.
4. The phase angle between identical phases must be zero.

It means that the switch must be closed at (or very near) the instant the two voltages have correct phase relationship.

Condition (1) is indicated by a voltmeter, conditions (2), (3) and (4) are indicated by synchronizing lamps or a synchroscope.

The synchronizing lamp method is consists of 3 lamps connected between the phases of the running 3-ph generator and the incoming generator as shown in Fig: 2.4.

---

In three phase alternators, it is necessary to synchronize one phase only, the other two phases be will then synchronized automatically. However, first it is necessary that the incoming alternator is correctly 'phased out' *i.e.* the phases are connected in the proper order of  $R, Y$  &  $B$  not  $R, B, Y$  etc. Lamp  $L_1$  is connected between  $R$  and  $R'$ ,  $L_2$  between  $Y$  and  $B'$  (not  $Y$  and  $Y'$ ) and  $L_3$  between  $B$  and  $Y'$  (and not  $B$  and  $B'$ ) as shown in Fig: 2.5.

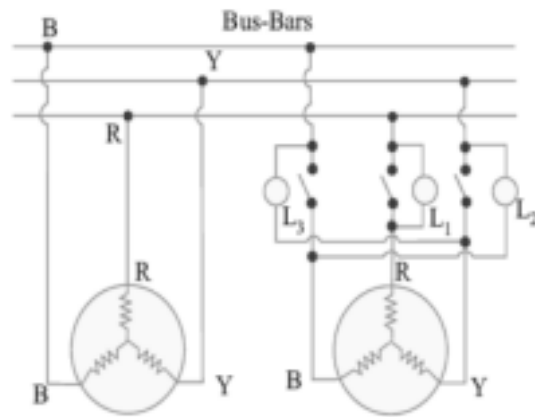


Fig: 2.4

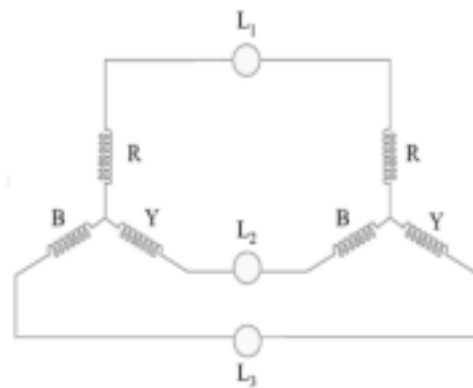


Fig: 2.5

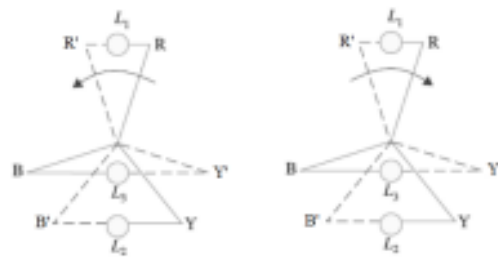


Fig: 2.6

Two set of star vectors will rotate at unequal speeds if the frequencies of the two are different. If the incoming alternator is running faster, then voltage star  $R'Y'B'$  appear to rotate anticlockwise with respect to the bus-bar voltage star  $RYB$  at a speed corresponding to the difference between their frequencies. With reference to Fig: 2.6, it is seen that voltage across  $L1$  is  $RR'$  to be increasing from zero, and that across  $L2$  is  $YB'$  which is decreasing, having just passed through its maximum, and that across  $L3$   $BY'$  which is increasing and approaching its maximum. Hence the lamps will light up one after the other in the order 2, 3, 1, 2, 3, 1 or 1, 2, 3. If the incoming alternator is running slower, then the sequence of light up will be 1, 3, 2. Synchronization is done at the moment the uncrossed lamp  $L1$  is in the middle of the dark period and other two lamps are equally bright. Hence this method of synchronization is known as two bright one dark lamp method.

It should be noted that synchronization by lamps is not quite accurate, because to a large extent, it depends on the sense of correct judgment of the operator. Hence, to eliminate the element of personal judgment in routine operation of alternators, the machines are synchronized by a more accurate device called a synchronoscope as shown in Fig: 2.7. It consists of 3 stationary coils and a rotating iron vane which is attached to a pointer. Out of three coils, a pair is connected to one phase of the line and the other to the corresponding machine terminals, potential transformer being usually used. The pointer moves to one side or the other from its vertical position depending on whether the incoming machine is too fast or too slow. For correct speed, the pointer points vertically up.

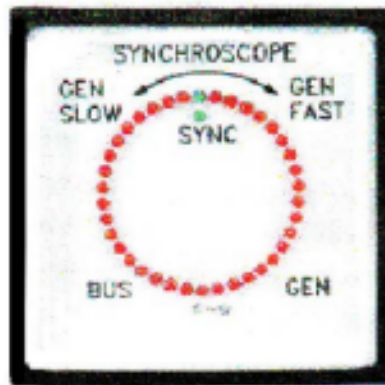


Fig: 2.7

### 2.5.1 Synchronizing Current:

If two alternators generating exactly the same emf are perfectly synchronized, there is no resultant emf acting on the local circuit consisting of their two armatures connected in parallel. No current circulates between the two and no power is transferred from one to the other. Under this condition emf of alternator 1, i.e.  $E_1$  is equal to and in phase opposition to emf of alternator 2, i.e.  $E_2$  as shown in the Figure. There is, apparently, no force tending to keep them in synchronism, but as soon as the conditions are disturbed a synchronizing force is developed, tending to keep the whole system stable. Suppose one alternator falls behind a little in phase by an angle  $\theta$ . The two alternator emfs now produce a resultant voltage and this acts on the local circuit consisting of the two armature windings and the joining connections. In alternators, the synchronous reactance is large compared with the resistance, so that the resultant circulating current  $I_s$  is very nearly in quadrature with the resultant emf  $E_r$  acting on the circuit. Figure represents a single phase case, where  $E_1$  and  $E_2$  represent the two induced emfs, the latter having fallen back slightly in phase. The resultant emf,  $E_r$ , is almost in quadrature with both the emfs, and gives rise to a current,  $I_s$ , lagging behind  $E_r$  by an angle approximating to a right angle. It is, thus, seen that  $E_1$  and  $I_s$  are almost in phase. The first alternator is generating a power  $E_1 I_s \cos \Phi_1$ , which is positive, while the second one is generating a power  $E_2 I_s \cos \Phi_2$ , which is negative, since  $\cos \Phi_2$  is negative. In other words, the first alternator is supplying

the second with power, the difference between the two amounts of power represents the copper losses occasioned by the current  $I_s$  flowing through the circuit which possesses resistance. This power output of the first alternator tends to retard it, while the power input to the second one tends to accelerate it till such a time that  $E_1$  and  $E_2$  are again in phase opposition and the machines once again work in perfect synchronism. So, the action helps to keep both machines in stable synchronism. The current,  $I_s$ , is called the synchronizing current.

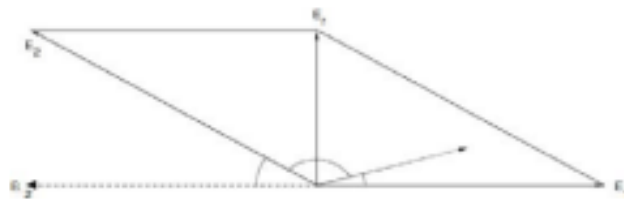


Fig: 2.7

### 2.5.2 Effect of Change of Excitation:

A change in the excitation of an alternator running in parallel with other affects only its KVA output; it does not affect the KW output. A change in the excitation, thus, affects only the power factor of its output. Let two similar alternators of the same rating be operating in parallel, receiving equal power inputs from their prime movers. Neglecting losses, their kW outputs are therefore equal. If their excitations are the same, they induce the same emf, and since they are in parallel their terminal voltages are also the same. When delivering a total load of  $I$  amperes at a power-factor of  $\cos \phi$ , each alternator delivers half the total current and  $I_1 = I_2 = I/2$ .

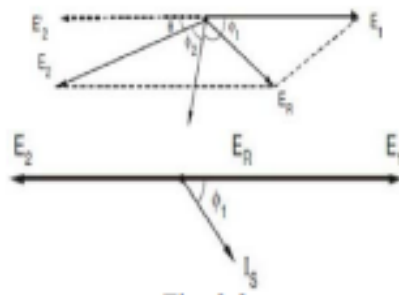


Fig: 2.8

Since their induced emfs are the same, there is no resultant emf acting around the local circuit formed by their two armature windings, so that the synchronizing current,  $I_s$ , is zero. Since the armature

resistance is neglected, the vector difference between  $E_1 = E_2$  and  $V$  is equal to,  $I_1 X_{s1} = I_2 X_{s2}$ , this vector leading the current  $I$  by  $90^\circ$ , where  $X_{s1}$  and  $X_{s2}$  are the synchronous reactances of the two alternators respectively.

Now consider the effect of reducing the excitation of the second alternator.  $E_2$  is therefore reduced as shown in Figure. This reduces the terminal voltage slightly, so let the excitation of the first alternator be increased so as to bring the terminal voltage back to its original value. Since the two alternator inputs are unchanged and losses are neglected, the two kW outputs are the same as before. The current  $I_2$  is changed due to the change in  $E_2$ , but the active components of both  $I_1$  and  $I_2$  remain unaltered. It can be observed that there is a small change in the load angles of the two alternators, this angle being slightly increased in the case of the weakly excited alternator and slightly decreased in the case of the strongly excited alternator. It can also be observed that  $I_1 + I_2 = I$ , the total load current.

### **2.5.3 Effect of Change of Input Torque**

The amount of power output delivered by an alternator running in parallel with others is governed solely by the power input received from its prime mover. If two alternators only are operating in parallel the increase in power input may be accompanied by a minute increase in their speeds, causing a proportional rise in frequency. This can be corrected by reducing the power input to the other alternator, until the frequency is brought back to its original value. In practice, when load is transferred from one alternator to another, the power input to the alternator required to take additional load is increased, the power input to the other alternator being simultaneously decreased. In this way, the change in power output can be effected without measurable change in the frequency. The effect of increasing the input to one prime mover is, thus, seen to make its alternator take an increased share of the load, the other being relieved to a corresponding extent. The final power-factors are also altered, since the ratio of the reactive components of the load has also been changed. The power-factors of the two alternators can be brought back to their original values, if desired, by adjusting the excitations of alternators.

### 2.5.4 Load Sharing

When several alternators are required to run in parallel, it probably happens that their rated outputs differ. In such cases it is usual to divide the total load between them in such a way that each alternator takes the load in the same proportion of its rated load in total rated outputs. The total load is not divided equally. Alternatively, it may be desired to run one large alternator permanently on full load, the fluctuations in load being borne by one or more of the others. If the alternators are sharing the load equally the power triangles are as shown in Fig. 2.9.

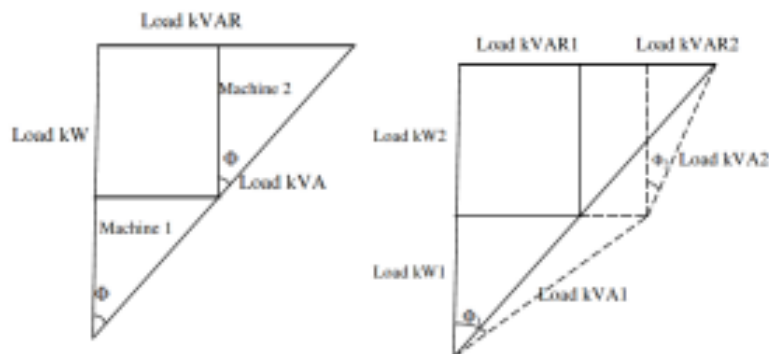
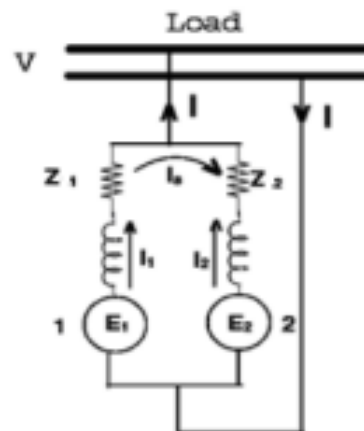


Fig: 2.9

### 2.5.5 Sharing of load when two alternators are in parallel

Consider two alternators with identical speed load characteristics connected in parallel as shown in Fig. 2.10.



Let  $E_1, E_2$  be the induced emf per phase,  
 $Z_1, Z_2$  be the impedances per phase,  
 $I_1, I_2$  be the current supplied by each machine per phase  
 $Z$  be the load impedance per phase,  
 $V$  be the terminal voltage per phase

From the circuit we have  $V = E_1 - I_1 Z_1 = E_2 - I_2 Z_2$   
and hence,  $I_1 = E_1 - V/Z_1$  and  $I_2 = E_2 - V/Z_2$

and also  $V = (I_1 + I_2) Z = IZ$   
solving above equations

$$I_1 = [(E_1 - E_2) Z + E_1 Z_2] / [Z(Z_1 + Z_2) + Z_1 Z_2]$$

$$I_2 = [(E_2 - E_1) Z + E_2 Z_1] / [Z(Z_1 + Z_2) + Z_1 Z_2]$$

The total current  $I = I_1 + I_2 = [E_1 Z_2 + E_2 Z_1] / [Z(Z_1 + Z_2) + Z_1 Z_2]$

And the circulating current or synchronizing current  $I_s = (E_1 - E_2) / (Z_1 + Z_2)$

Synchronization by a Synchroscope

A stationary alternator must not be connected to live busbars. It is because the EMF induced by the alternator is zero at standstill and a short circuit will result. In order to check the synchronization of alternators, an equipment named Synchroscope is used



Here, the phase sequence of the alternator is checked carefully at the time of its installation. The conditions 1 and 2 required for the synchronization are assured by means of the Synchroscope (shown in the figure). The Synchroscope compares the voltage from one phase of the incoming alternator with that of the corresponding phase of the 3-phase system.

The position of the pointer of the Synchroscope indicates the phase difference between the voltages of the incoming alternator and the infinite busbar.

- When the frequencies of the two voltages are equal, the pointer remains stationary.
- When the frequencies differ, the pointer rotates in one direction or the other.

The direction of the rotation of the pointer shows whether the incoming alternator is running too fast or too slow, i.e., whether the frequency of the incoming alternator is higher or lower than that of the infinite busbar. The speed of the rotation of the pointer is equal to the difference between the frequency of the incoming alternator and the frequency of the infinite busbar.

The frequency and phase positions are controlled by controlling the input to the prime mover of the incoming alternator. When the pointer of the Synchroscope moves very slowly, that is the two frequencies are almost same and passes through the zero-phase point, the circuit breaker is closed and the incoming alternator is connected to the busbar.

# Module III

## Synchronous motors

### Working Principle of Synchronous Motor

A synchronous motor is a type of AC motor that operates on the principle of synchronism. It consists of stator and rotor. When the stator is energised it creates a magnetic field and interacts with the rotor, causing it to rotate at a synchronous speed. Once synchronous motors are synchronized, they operate efficiently.

### What is a Synchronous Motor?

A synchronous motor is a type of AC motor that operates at a speed directly proportional to the frequency of current supplied. It helps the synchronous motor to maintain synchronous speed regardless of changes in load.

Unlike induction motors, which rely on the principle of electromagnetism and experience a change in speed due to a change in load. However, synchronous motors maintain their synchronous speed with the frequency supply irrespective of changes in load making them ideal for applications that require constant speed control such as clocks, timers, and various industrial processes.

The construction of a [synchronous motor](#) includes two main components: stator and rotor. The stator is the stationary part of the motor and consists of three-phase windings similar to the induction motor. While the rotor is the rotating part of the motor, it can be either a salient pole or cylindrical.

Synchronous motor works on the principle of interaction between the stator and rotor. When a three-phase AC supply is provided to windings of stator, they generate a rotating magnetic field (RMF) which interacts with the magnetic field of the rotor, locks the rotor and causes it to rotate at synchronous speed.

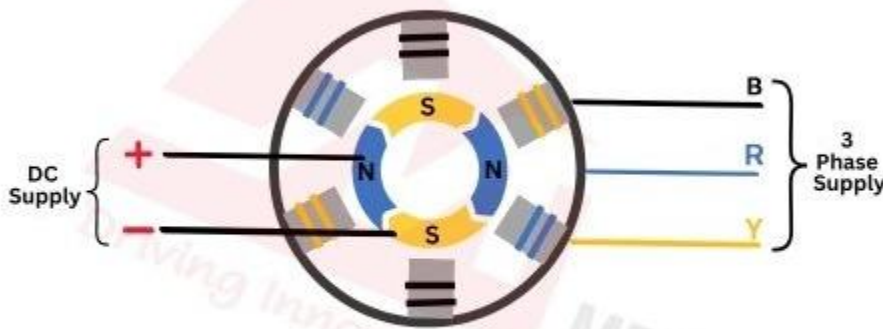
However, unlike induction motors, synchronous motors are not self-starting. At idle conditions, the rotor's magnetic field experiences alternating torque due to the rotating magnetic field (RMF) of the stator which is insufficient to initiate the rotation of the rotor. To start the rotation, auxiliary methods such as damper windings or an external motor are used to bring the rotor up to synchronous speed before DC excitation is applied.

### Working of Synchronous Motor

The working of a synchronous motor is based on the principle of synchronism. In this principle, the rotation of the rotor is synchronised with the frequency of the alternating current (AC) supply. Similar to other AC motors, synchronous motors also consist of a stationary part called the stator.

The stator contains wire coils, supplied with alternating current (AC) supply. When these coils are energised with an AC supply, it produces a rotating magnetic field. This rotating magnetic field helps the rotor to rotate.

The rotor is the rotating part of the motor. It contains either electromagnets or permanent magnets arranged in specific patterns. These magnets are then arranged with a rotating magnetic field produced by the stator. As a result of this interaction, the rotor starts to rotate. The most important feature of synchronous motors is their ability to maintain constant speed. They have the ability to synchronize the rotation of the rotor with the frequency of AC supply.



Working of Synchronous Motor

When the rotor rotates with the magnetic field produced by the stator, an electromotive force (EMF) is induced in rotor windings. This EMF creates a magnetic field in the rotor, which aligns itself with the magnetic field of the stator. As a result, the rotor rotates at the same speed as the rotating magnetic field.

The synchronous speed ( $N_s$ ) of the synchronous motor is determined by the formula  $N_s = 120f/P$  where  $f$  is the frequency of the AC supply and  $P$  is the number of poles in the motor. This formula shows that the speed of a synchronous motor is inherently related to the frequency of the AC power supply, typically 50Hz or 60Hz.

Mechtex has a [110V Synchronous motor](#) and a [230V Synchronous Motor](#) that operates at 50Hz or 60Hz frequency. These synchronous motors are used in various industrial machinery, pumps, compressors, wind turbines, and daily live applications.

However, precise speed control is achieved by adjusting the frequency of AC supply. By increasing or decreasing, the frequency of supply the motor speed can be adjusted to meet the requirements of specific applications.

In some synchronous motors, an additional DC supply is required for rotor winding to create the magnetic field produced for synchronisation. This process is known as excitation. It helps synchronous motors to maintain their synchronization with the AC power supply. However synchronous motors are not self-starting.

Unlike other AC motors, which can start and operate without any external source, [synchronous motors](#) require an external source for initial rotation to synchronise with the magnetic field produced by the stator. Once synchronous motors are synchronised, they operate efficiently.

However synchronous motors are not self-starting. Unlike other AC motors, which can start and operate without any external source, synchronous motors require an external source for initial rotation to synchronise with the magnetic field produced by the stator. Once synchronous motors are synchronised, they operate efficiently.

Let's discuss the simplest way to draw the phasor diagram for a synchronous motor and its advantages. First, we need to understand the notations for each quantity:

$E_f$  to represent the excitation voltage

$V_t$  to represent the terminal voltage

$I_a$  to represent the armature current

$\theta$  to represent the angle between terminal voltage and armature current

$\psi$  to represent the angle between the excitation voltage and armature current

$\delta$  to represent the angle between the excitation voltage and terminal voltage

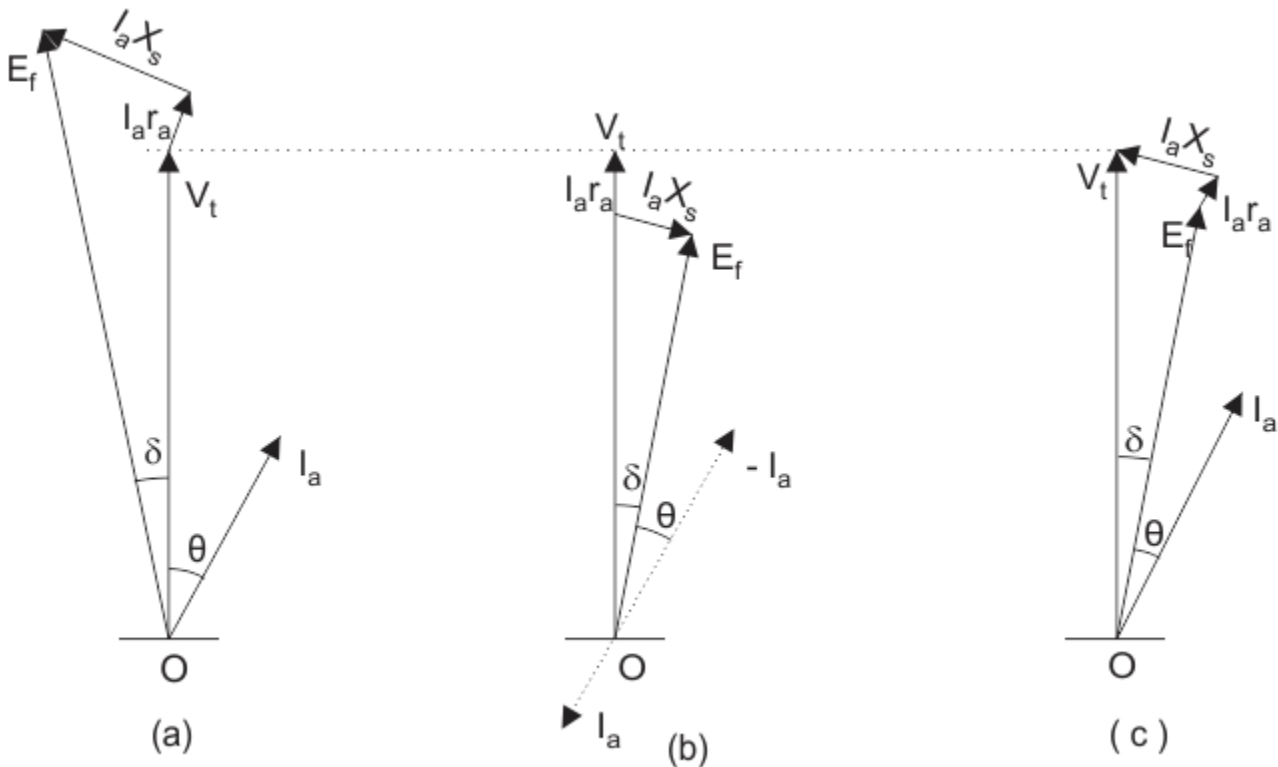
$r_a$  to represent the armature per phase resistance.

We will take  $V_t$  as the reference phasor in order to draw the phasor diagram for synchronous motor. In order to draw the phasor diagram one should know these two important points which are written below:

(1) We know that if a machine is made to work as a [asynchronous motor](#) then direction of armature current will be in phase opposition to that of the excitation emf.

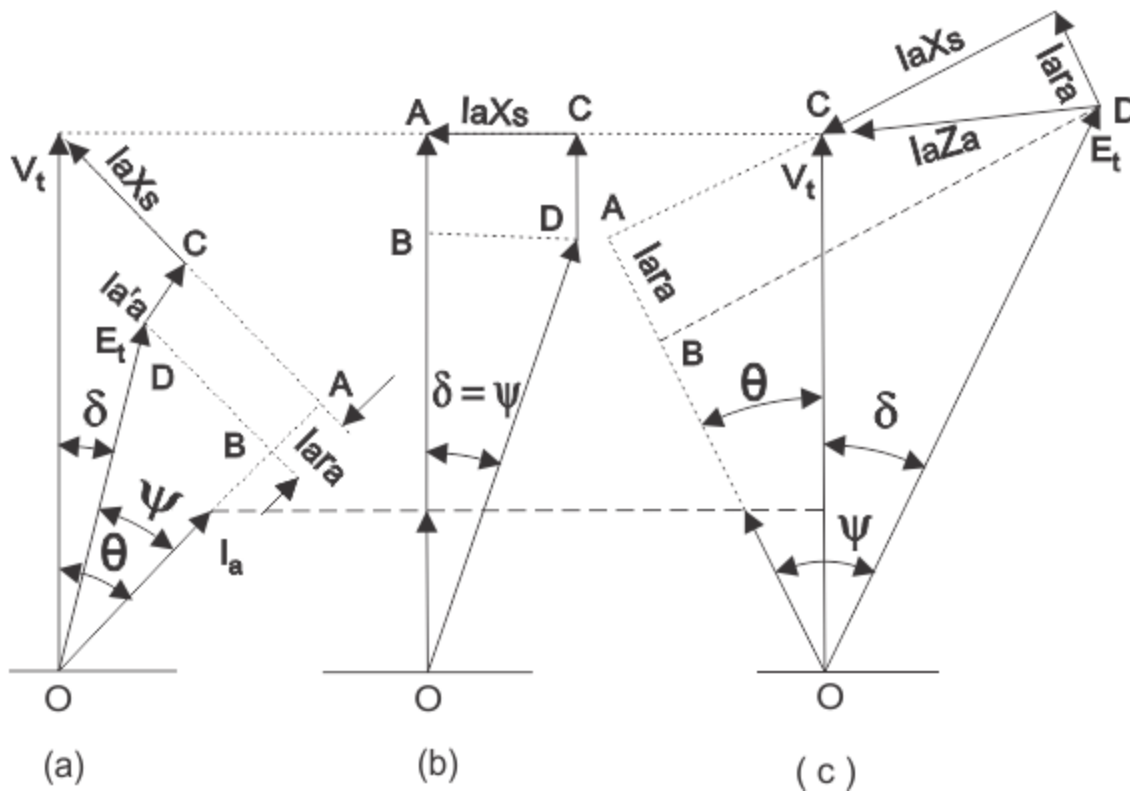
(2) Phasor excitation emf is always behind the phasor terminal voltage.

These two points are enough to draw the phasor diagram for a synchronous motor. The diagram is shown below:



In the first phasor diagram, the armature current direction is opposite to the excitation emf phase.

Usually, we omit the negative sign of the armature current in the phasor diagram. In the second phasor, we have omitted this sign. Now, let's draw the complete phasor diagram and derive the excitation emf expression for each of the three cases:



(a) Motoring operation at lagging power factor.

(b) Motoring operation at unity power factor.

(c) Motoring operation at leading power factor.

Given below are the phasor diagrams for all the operations.

a) Motoring operation at lagging power factor: In order to derive the expression for the excitation emf for the lagging operation we first take the component of the terminal voltage in the direction of armature current  $I_a$ . Component in the direction of armature current is  $V_t \cos \theta$ .

As the direction of armature is opposite to that of the terminal voltage therefore [voltage drop](#) will be  $-I_a r_a$  hence the total voltage drop is  $(V_t \cos \theta - I_a r_a)$  along the armature current. Similarly we can calculate the [voltage](#) drop along the direction perpendicular to armature current. The total voltage drop comes out to be  $(V_t \sin \theta - I_a X_s)$ . From the triangle BOD in the first phasor diagram we can write the expression for excitation emf as

$$E_f^2 = (V_t \cos \theta - I_a \times r_a)^2 + (V_t \sin \theta - I_a \times X_s)^2$$

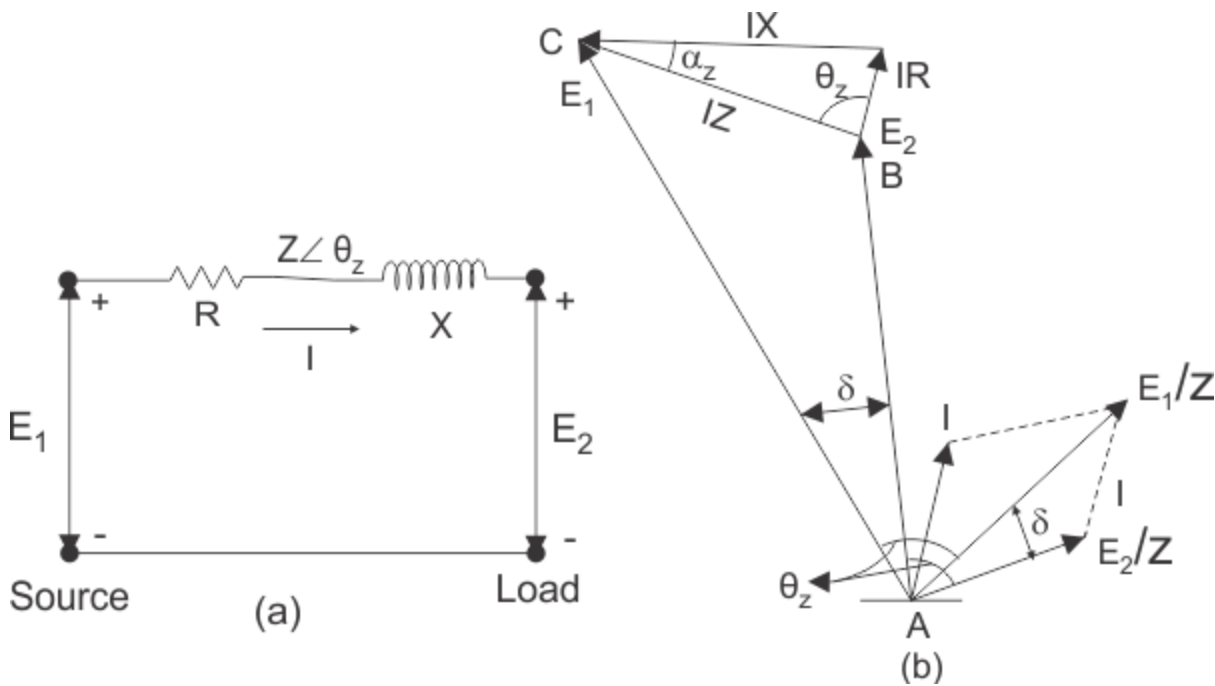
(b) Motoring operation at unity power factor: In order to derive the expression for the excitation emf for the unity power factor operation we again first take the component of the terminal voltage in the direction of armature current  $I_a$ . But here the value of theta is zero and hence we have  $\psi = \delta$ . From the triangle BOD in the second phasor diagram we can directly write the expression for excitation emf as

$$E_f^2 = (V_t - I_a \times r_a)^2 + (I_a \times X_s)^2$$

(c) Motoring operation at leading power factor: In order to derive the expression for the excitation emf for the leading power factor operation we again first take the component of the terminal voltage in the direction of armature current  $I_a$ . Component in the direction of armature current is  $V_t \cos \theta$ . As the direction of armature is opposite to that of the terminal voltage therefore voltage drop will be  $(-I_a r_a)$  hence the total voltage drop is  $(V_t \cos \theta - I_a r_a)$  along the armature [current](#). Similarly we can calculate the voltage drop along the direction perpendicular to armature current. The total voltage drop comes out to be  $(V_t \sin \theta + I_a X_s)$ . From the triangle BOD in the first phasor diagram we can write the expression for excitation emf as

$$E_f^2 = (V_t \cos \theta - I_a \times r_a)^2 + (V_t \sin \theta + I_a \times X_s)^2$$

In order to derive various conditions for power in both [alternators](#) and [synchronous motors](#), let us consider the general problem of power flow through inductive impedance. The circuit diagram shown below consists of [voltage source](#)  $E_1$ , voltage source  $E_1$  and load which consists of one [resistor](#) in series with an [inductor](#). Now if we assume that the voltage source  $E_1$  is greater than the voltage source  $E_2$  then the [voltage](#) equation for this circuit is given by the equation,



$$E_1 = E_2 + IZ$$

Where,  $Z$  is  $R + jX$  as shown in the above circuit diagram.

From the above expression we write the expression of [current](#) as

$$I = \frac{E_1}{Z} - \frac{E_2}{Z}$$

Phasor Diagram for above Circuit

Let's explore how to draw a simple phasor diagram for the circuit. First, we'll list the notations for each quantity used:

$\theta_z$  to represent angle between the voltage  $E_1$  and current  $E_1/Z$  or voltage  $E_2$  and current  $E_2/Z$

$I$  to represent current in the above circuit and  $\delta$  to represent angle between  $E_1$  and  $E_2$ .

Given below is the phasor diagram for the above circuit: In order to draw the phasor diagram first draw the  $E_1$  voltage and the current  $E_1/Z$ , mark angle  $E_1$  and  $E_1/Z$  as  $\theta_z$ . Similarly draw phasor  $E_2$  and  $E_2/Z$ , such that the angle between  $E_1$  and  $E_2$  should be  $\delta$ . Complete the phasor diagram by drawing the [voltage drops](#)  $IX$  and  $IR$  as shown above. Now Let us derive the expression for power supplied by the source  $E_1$ .

Let the power supplied by the source  $E_1$  be  $P_1$ . We define power as the product of the [voltage](#) and current, using this we can write from the phasor diagram as  $P_1 = E_1 \times (\text{component of current } I \text{ in phase with the } \text{voltage source } E_1)$ . Component of current in phase with the voltage source  $E_1$  is

$$\frac{E_1}{Z} \cos \theta_z - \frac{E_2}{Z} \sin(\theta_z + \delta)$$

On substituting this expression in the above equation we have

$$P_1 = E_1 \times \left[ \frac{E_1}{Z} \cos \theta_z - \frac{E_2}{Z} \sin(\theta_z + \delta) \right]$$

$$\Rightarrow P_1 = E_1 \times \frac{E_1}{Z} \cos \theta_z - E_1 \times \frac{E_2}{Z} \sin(\theta_z + \delta)$$

From the phasor diagram, we have  $\theta_z = 90^\circ - \alpha_z$ . On substituting the value of the angle  $\theta_z$  in the above expression we have

$$P_1 = E_1 \times \frac{E_1}{Z} - E_1 \times \frac{E_2}{Z} \sin(\delta - \alpha_z) \dots \dots \dots (1)$$

This is the required expression for the power supplied by the source  $E_1$ .

Let the power supplied by the source  $E_2$  be  $P_2$ . We define power as the product of the [voltage](#) and current, using this we can write from the phasor diagram as

$$P_2 = E_2 \times (\text{component of current } I \text{ in phase with the voltage source } E_2)$$

Component of current in phase with the [voltage source](#)  $E_2$  is

$$\frac{E_1}{Z} \cos(\theta_z - \delta) - \frac{E_2}{Z} \sin \theta_z$$

On substituting this expression in the above equation we have

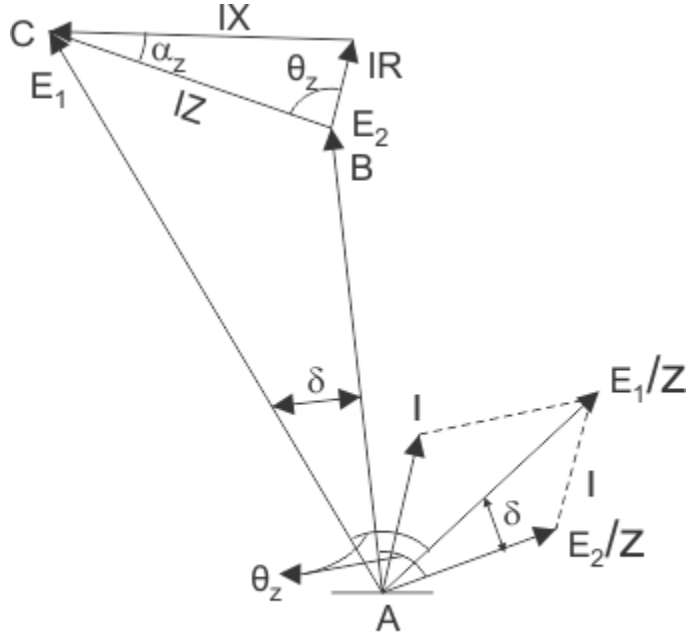
$$P_2 = E_2 \times \left[ \frac{E_1}{Z} \cos(\theta_z - \delta) - \frac{E_2}{Z} \sin \theta_z \right]$$

$$\Rightarrow P_2 = E_2 \times \frac{E_1}{Z} \cos(\theta_z - \delta) - E_2 \times \frac{E_2}{Z} \sin \theta_z$$

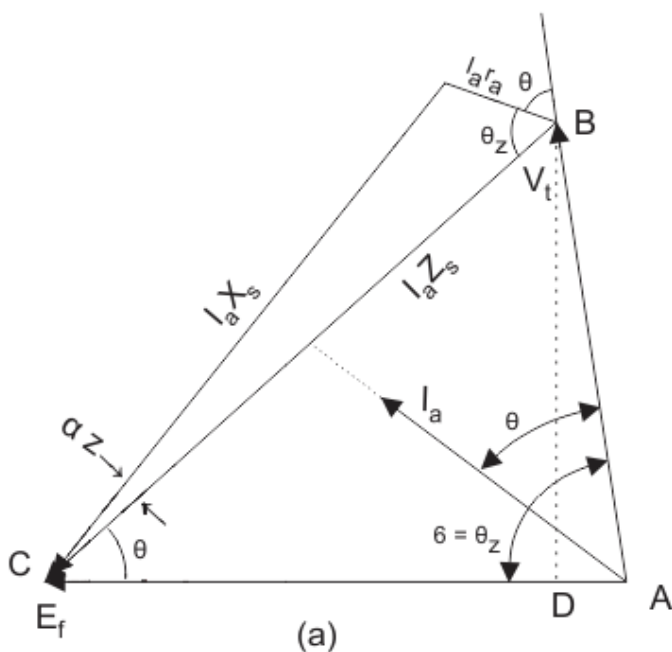
From the phasor diagram, we have  $\theta_z = 90^\circ - \alpha_z$ . On substituting the value of the angle  $\theta_z$  in the above expression we have

$$P_2 = -E_2 \times \frac{E_2}{Z} + E_1 \frac{E_2}{Z} \sin(\delta + \alpha_z) \dots \dots \dots (2)$$

This is the required expression for the power supplied by the source  $E_2$ .

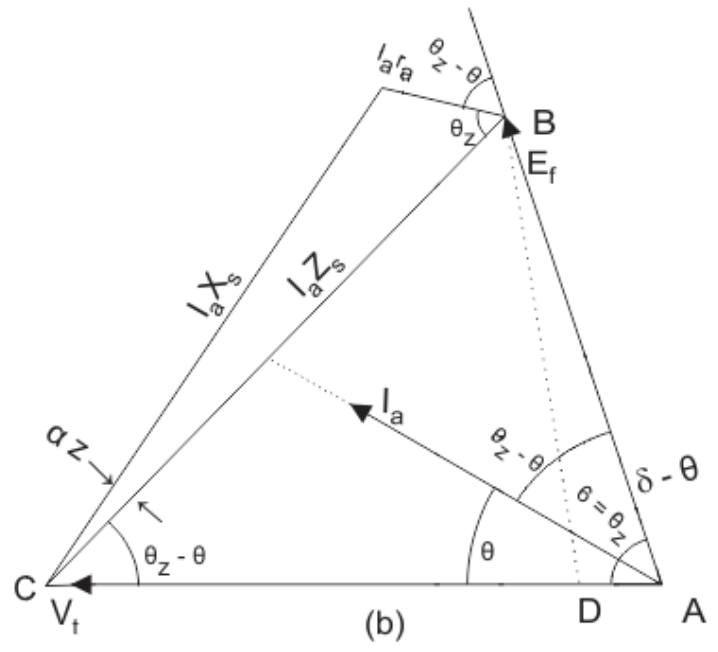


Phasor Diagram

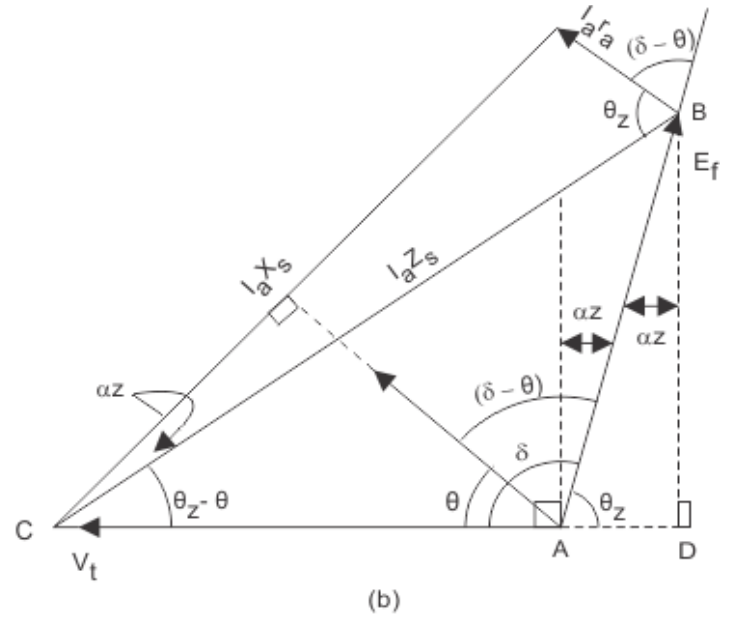
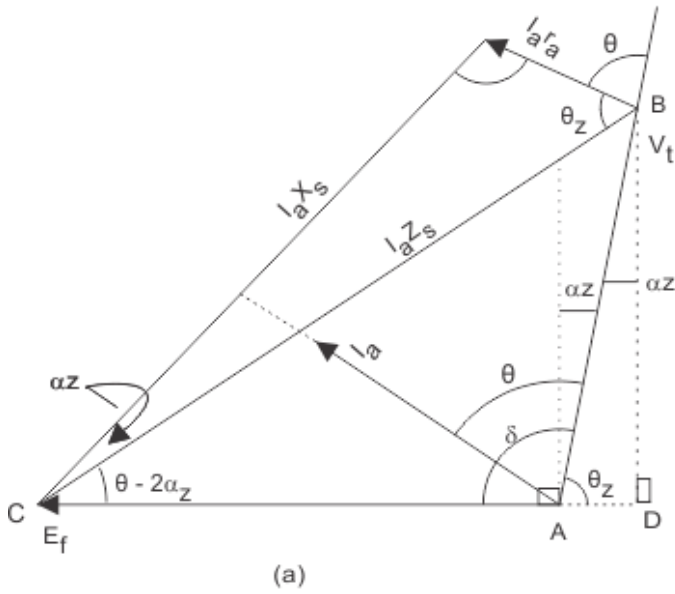


(a)

(a) Generator and (b) motor



(b)



(a) generator and (b) motor

Now let us derive various equations for the power flow the cylindrical rotor alternator. In order to derive various power equation for an alternator let us substitute voltage source  $E_1$  equal to the excitation voltage ( $E_f$ ), voltage source  $E_2$  equals to the terminal voltage ( $V_t$ ), inductive impedance of the above circuit equals to synchronous impedance ( $Z_s$ ) and  $Z_s = r_a + jX_s$ . After replacing all these, we will have power input by the source  $E_1$  is equal to power input to the generator ( $P_{ig}$ ). So,

$$P_{ig} = P_1 = E_f \times r_a \times \frac{E_f}{Z_s} - E_f \times \frac{V_t}{Z_s} \sin(\delta - \alpha_z)$$

Similarly we have output of the generator

$$P_{og} = P_2 = -V_t \times r_a \times \frac{V_t}{Z_s} - E_f \times \frac{V_t}{Z_s} \sin(\delta + \alpha_z)$$

Similarly we have output of the generator

An important result from these equations is that the difference between the power input and output of the generator represents ohmic losses. To prove this, we subtract the output power from the input power:

$$P_{ig} - P_{og} = \left[ E_f \times r_a \times \frac{E_f}{Z_s} - E_f \times \frac{V_t}{Z_s} \sin(\delta - \alpha_z) \right] - \left[ -V_t \times r_a \times \frac{V_t}{Z_s} - E_f \times \frac{V_t}{Z_s} \sin(\delta + \alpha_z) \right]$$

On expanding the expression, we have

$$P_{ig} - P_{og} = \frac{r_a}{Z_s^2} \times (E_f^2 + V_t^2 - 2 \times E_f V_t \cos \delta)$$

From the phasor diagram we have

$$E_f^2 + V_t^2 - 2E_f V_t \cos \delta = I_a^2 \times r_a$$

So substituting the value, from this equation we have

$$P_{ig} - P_{og} = I_a^2 \times r_a$$

Usually we neglect the value of armature resistance, due to this  $\alpha_z$  becomes zero and  $Z_s$  becomes equal to  $X_s$ . Hence we have,

$$P_{ig} = P_{og} = \frac{E_f \times V_t}{X_s} \sin \delta$$

Now we are in state to derive the expression for maximum power output conditions for the generator. In order to derive the maximum power output conditions we will first differentiate the expression of the power output equation of the generator that we have already derived above, after this we equate the equation with zero. On equating with zero we will get the angle relationship between alpha and delta at maximum power out conditions. Mathematically we have L

$$\frac{dP_{og}}{d\delta} = \theta = \frac{E_f \times V_t}{Z_s} \cos(\delta + \alpha_z)$$

$$\delta + \alpha_z = 90^\circ \Rightarrow \delta = 90^\circ - \alpha_z$$

This is the required condition for the maximum power output, at maximum power output we have load angle is equal to the impedance angle.

On substituting the above relation in output power relation we have maximum power output equals to

$$\frac{E_f \times V_t}{Z_s} - \left( \frac{V_t}{Z_s} \right)^2 \times r_a$$

Which is the required expression for the maximum power output of the generator.

Here we are interested in drawing the phasor diagrams for the maximum power output in case of generator. Given below is the phasor diagram for the generator in case of maximum power output. All the symbols have their usual meanings in the phasor diagram.

Similarly, we can derive the expression for maximum input to the generator. In order to derive the maximum power input conditions we will first differentiate the expression of the power input equation of the generator that we have already derived above, after this we equate the equation with zero. On equating with zero we will get the angle relationship between alpha and delta at maximum power out conditions. Mathematically, we have

$$\frac{dP_{ig}}{d\delta} = \theta = \frac{E_f \times V_t}{Z_s} \cos(\delta - \alpha_z)$$

$$\Delta - \alpha_z = 90^\circ \Rightarrow \delta = 180^\circ - \theta_z$$

This is the required condition for the maximum power input, at maximum power input we have load angle is equal to the 180 degree minus impedance angle. On substituting the above relation in input power relation we have maximum power input equals to

$$\frac{E_f \times V_t}{Z_s} + \left( \frac{E_f}{Z_s} \right)^2 \times r_a$$

Which is the required expression for the maximum power input for the generator?

Here, we are interested in drawing the phasor diagrams for the maximum power input in case of generator. Given below is the phasor diagram for the generator in case of maximum power input. All the symbols have their usual meanings in the phasor diagram. From the phasor diagram of maximum power input we can derive various conditions for the [power factor](#) and these conditions are written below:

- (a) When  $(E_f \cos \delta - I_a r_a \cos \theta)$  is less than terminal voltage then the power factor will be leading.
- (b) When  $(E_f \cos \delta - I_a r_a \cos \theta)$  is equal to terminal voltage then the power factor will be unity.
- (c) When  $(E_f \cos \delta - I_a r_a \cos \theta)$  is greater than terminal voltage then the power factor will be lagging.

Let us now derive the expression for Reactive power flow in case of [synchronous generator](#). We can derive expression for the reactive power at the output terminal of the generator as

$$Q_{og} = E_2 \times (\text{component of current in phase quadrature lagging with } E_2)$$

$$\Rightarrow Q_{og} = V_t \times \left[ \frac{E_f}{Z_s} \sin(\theta_z - \delta) - \frac{V_t}{Z_s} \sin \theta_z \right]$$

Further we can write this equation as

$$Q_{og} = \frac{V_t \times E_f}{Z_s} \cos(\delta + \alpha_z) - \left( \frac{V_t}{Z_s} \right)^2 \times X_s$$

From the above equation in generating mode if we have armature resistance equals to zero then the above equation will reduce to

$$Q_{og} = \frac{V_t}{X_s} [E_f \cos \delta - V_t]$$

From the above equation we can derive various conditions for the [power factor](#) and reactive power, these conditions are written below:

- (a) When  $E_f \cos \delta$  is less than terminal voltage then the power factor will be leading and reactive power is negative at the output terminal.
- (b) When  $E_f \cos \delta$  is equal to terminal voltage then the power factor will be unity and the reactive power is zero at the output terminal of the generator.
- (c) When  $E_f \cos \delta$  is greater than terminal voltage then the power factor will be lagging and reactive power is positive.

Now let us derive various equations for the power flow the cylindrical rotor synchronous motor. In order to derive various power equation for a [synchronous motor](#) let us substitute voltage source  $E_1$  equal to the excitation voltage ( $V_t$ ), [voltage source](#)  $E_2$  equals to the

terminal voltage ( $E_t$ ), inductive impedance of the above circuit equals to synchronous impedance ( $Z_s$ ) and  $Z_s = r_a + jX_s$ . After replacing all these, we will have power input by the source  $E_1$  is equal to power input to the generator ( $P_{ig}$ ). So,

$$P_{im} = P_1 = V_t \times r_a \times \frac{V_t}{Z_s} + E_f \times \frac{V_t}{Z_s} \sin(\delta - \alpha_z)$$

Similarly we have output of the synchronous motor

$$P_{om} = P_2 = -E_f \times r_a \times \frac{E_f}{Z_s} + E_f \times \frac{V_t}{Z_s} \sin(\delta + \alpha_z)$$

One important result can be derived from these equations. The difference of the power input to the synchronous motor and the power output to the synchronous motor gives ohmic losses in the generator. So in order to prove above statement let us subtract output power from the input power to the synchronous motor:

$$P_{im} - P_{om} = \left[ E_f \times r_a \frac{E_f}{Z_s} - E_f \times \frac{V_t}{Z_s} \sin(\delta - \alpha_z) \right] - \left[ -V_t \times r_a \frac{V_t}{Z_s} + E_f \times \frac{V_t}{Z_s} \sin(\delta + \alpha_z) \right]$$

On expanding the expression, we have

$$P_{im} - P_{om} = \frac{r_a}{Z_s^2} \times [E_f^2 + V_t^2 - 2E_f V_t \cos \delta]$$

From the phasor diagram we have

$$E_f^2 + V_t^2 - 2E_f V_t \cos \delta = I_a^2 \times r_a$$

So substituting the value, from this equation we have,

$$P_{im} - P_{om} = I_a^2 \times r_a$$

Usually we neglect the value of armature resistance, due to this  $\alpha_z$  becomes zero and  $Z_s$  becomes equal to  $X_s$ . Hence we have,

$$P_{im} = P_{om} = \frac{E_f \times V_t}{X_s} \sin \delta$$

Now we are in state to derive the expression for maximum power output conditions for the [synchronous motor](#). In order to derive the maximum power output conditions we will first differentiate the expression of the power output equation of the synchronous motor that we have already derived above, after this we equate the equation with zero. On equating with zero we will get the angle relationship between alpha and delta at maximum power out conditions. Mathematically we have

$$\frac{dP_{om}}{d\delta} = 0 = \frac{E_f \times V_t}{Z_s} \cos(\delta + \alpha_z)$$

$$\delta + \alpha_z = 90^\circ \Rightarrow \delta = 90^\circ - \alpha_z$$

This is the required condition for the maximum power output, at maximum power output we have load angle is equal to the impedance angle. On substituting the above relation in output power relation we have maximum power output equals to

$$\frac{E_f \times V_t}{Z_s} - \frac{E_f^2}{Z_s^2} \times r_a$$

Which is the required expression for the maximum power output for the synchronous motor.

Here we are interested in drawing the phasor diagrams for the maximum power output in case of synchronous motor. Given below is the phasor diagram for the synchronous motor in case of maximum power output. All the symbols have their usual meanings in the phasor diagram.

Similarly, we can derive the expression for maximum input to the motor. In order to derive the maximum power input conditions we will first differentiate the expression of the power input equation of the generator that we have already derived above, after this we equate the equation with zero. On equating with zero we will get the angle relationship between alpha and delta at maximum power out conditions. Mathematically, we have

$$\frac{dP_{im}}{d\delta} = 0 = \frac{E_f \times V_t}{Z_s} \cos(\delta - \alpha_z)$$

$$\delta - \alpha_z = 90^\circ \Rightarrow \delta = 90^\circ + \alpha_z$$

This is the required condition for the maximum power input, at maximum power input we have load angle is equal to the 180 degree

minus impedance angle.

On substituting the above relation in input power relation we have maximum power input equals to

$$\frac{E_f \times V_t}{Z_s} + \left( \frac{V_t}{Z_s} \right)^2 \times r_a$$

Which is the required expression for the maximum power input for the [synchronous motor](#).

Here we are interested in drawing the phasor diagrams for the maximum power input in case of synchronous motor. Given below is the phasor diagram for the synchronous motor in case of maximum power input. All the symbols have their usual meanings in the phasor diagram.

From the phasor diagram of maximum power input we can derive various conditions for the [power factor](#) and these conditions are written below:

- (a) When  $(E_f \cos \delta + I_a r_a \cos \theta)$  is less than terminal [voltage](#) then the power factor will be lagging.
- (b) When  $(E_f \cos \delta + I_a r_a \cos \theta)$  is equal to terminal voltage then the power factor will be unity.
- (c) When  $(E_f \cos \delta + I_a r_a \cos \theta)$  is greater than terminal voltage then the power factor will be leading.

Let us now derive the expression for Reactive power flow in case of synchronous motor. We can derive expression for the reactive power at the input terminal of the [synchronous motor](#)

$$Q_{im} = E_2 \times (\text{component of current in phase quadrature lagging with } E_2)$$

$$Q_{im} = -V_t \times \left[ \frac{E_f}{Z_s} \cos(\alpha_z - \delta) + \frac{V_t}{Z_s^2} X_s \right]$$

Further we can write this equation as

$$Q_{im} = -\frac{V_t \times E_f}{Z_s} \cos(\alpha_z - \delta) - \left( \frac{V_t}{Z_s} \right)^2 X_s$$

From the above equation in motoring mode if we have armature resistance equals to zero then the above equation will reduce to

$$Q_{im} = -\frac{V_t}{X_s} E_f \cos \delta - \frac{V_t^2}{X_s}$$

From the above equation we can derive various conditions for the power factor and reactive power, these conditions are written below:

- (a) When  $E_f \cos \delta$  is less than terminal [voltage](#) then the [power factor](#) will be lagging and reactive power is positive at the input terminal.
- (b) When  $E_f \cos \delta$  is equal to terminal voltage then the power factor will be unity and the reactive power is zero at the input terminal of the [synchronous motor](#).
- (c) When  $E_f \cos \delta$  is greater than terminal voltage then the power factor will be leading and reactive power is negative.

### Synchronous Motor V Curves

The graphs plotted between armature current ( $I_a$ ) and field current ( $I_f$ ) for different constant loads are known as the V curves of the synchronous motor.

The power factor of a synchronous motor can be controlled by changing the field excitation, i.e., by variation of field current ( $I_f$ ). Also, the armature current ( $I_a$ ) changes with the change in the excitation or field current ( $I_f$ ).

let us assume that the synchronous motor is operating at no-load. If the field current ( $I_f$ ) is increased from a small value, the armature current ( $I_a$ ) decreases until  $I_a$  becomes minimum. The power factor of the motor corresponding to this minimum armature current is unity. Up to the point of minimum armature current, the motor was operating at lagging power factor. If a graph is plotted between armature current ( $I_a$ ) and field current ( $I_f$ ) at no-load, the lowest curve in Figure-1 is obtained. In order to obtain a family of curves as shown in Figure-1, the above procedure is to be repeated for various increased loads.

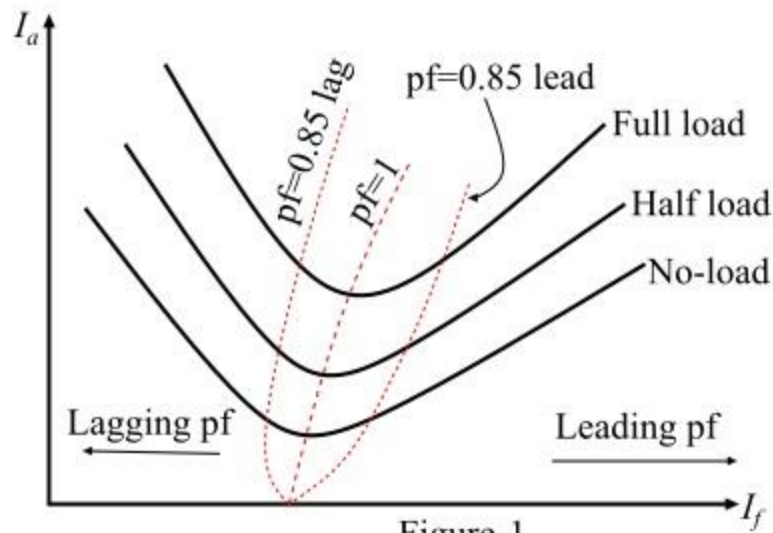


Figure-1

Because the shape of the curves plotted between armature current ( $I_a$ ) and field current ( $I_f$ ) resembles the letter "V", thus these curves are known as V curves of a synchronous motor.

The point corresponding to unity power factor is the point at which the armature current is minimum. The curve connecting the unity power factor points (or lowest points) of all V curves for various loads is called the unity power factor compounding curve. Similarly, the compounding curves for 0.85 power factor lag and 0.85 power factor lead are shown by dotted curves in Figure-1.

Therefore, The compounding curves may be defined as the loci of constant power factor points on the V curves of a synchronous motor. The compounding curves give the information about the manner in which the field current ( $I_f$ ) of the motor should be varied to maintain constant power factor under changing loads.

Hence, the V curves are useful in adjusting the field current of the synchronous motor. From the V curves shown in Figure-1, it is clear that decreasing the field current below that for minimum armature current results in lagging power factor. Similarly, increasing the field current beyond the level of minimum armature current results in leading power factor. Therefore, by changing the field excitation of a synchronous motor, the reactive power supplied to or consumed from the power system can be controlled.

#### Inverted V Curves of Synchronous Motor

Inverted V curves of a synchronous motor are defined as the graphs plotted between power factor and field current ( $I_f$ ) of the motor.

A family of inverted V curves of a synchronous motor obtained by plotting the power factor versus field current is shown in Figure-2.

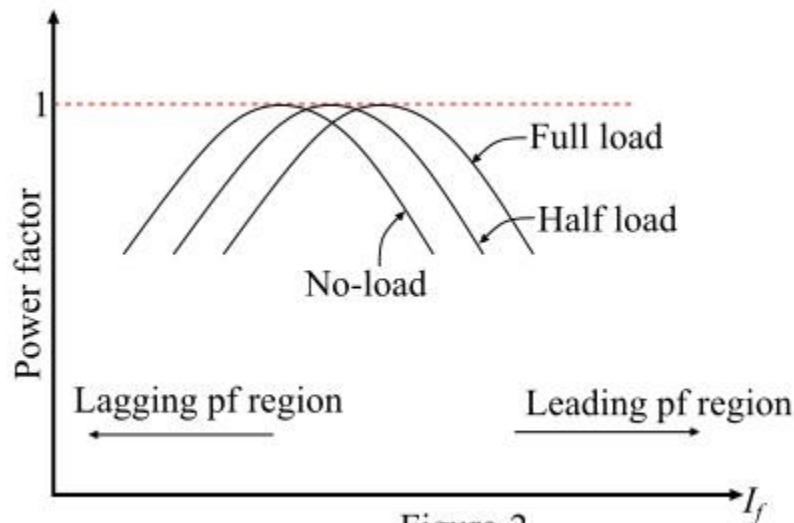


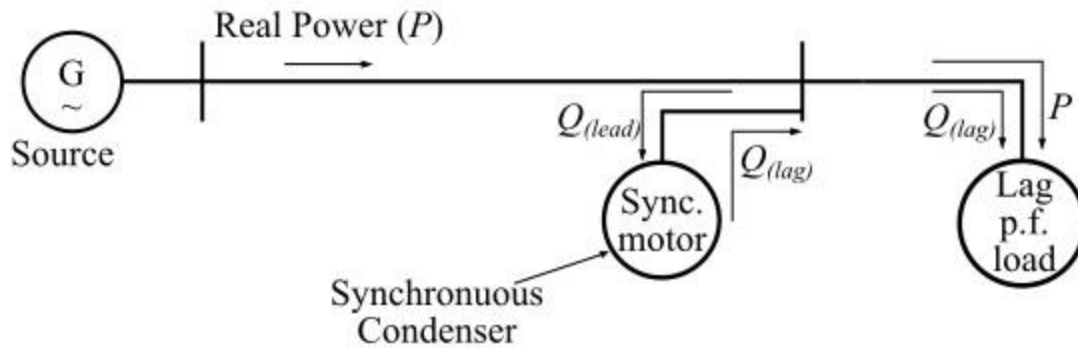
Figure-2

The peak point on each of these curves indicates unity power factor. From the curves, it can be seen that the field current ( $I_f$ ) for unity power factor at full-load is greater than the field current ( $I_f$ ) for unity power factor at no-load.

The inverted V curves also show that if the synchronous motor at full-load is operating at unity power factor then removal of the mechanical load from the shaft of the motor causes the motor to operate at a leading power factor.

#### Synchronous Condenser

An over-excited synchronous motor running on no-load is called the synchronous condenser. It is also known as *synchronous capacitor* or *synchronous compensator* or *synchronous phase modifier*.



A synchronous motor can deliver or absorb reactive power by changing the DC excitation of its field winding. It can be made to draw a leading current from the supply with over-excitation of its field winding and therefore, it supplies lagging reactive power (or absorbs leading reactive power).

#### Under-excited Synchronous Motor

When the synchronous motor is under-excited, then it draws a lagging current from the source and hence supplies leading reactive power (or absorbs lagging reactive power). Therefore, the current drawn by a synchronous condenser can be changed from lagging to leading smoothly by varying its field excitation.

#### Over-excited Synchronous Motor at No Load

When an over-excited synchronous motor is operated at no-load, it takes a leading current and hence, behaves as a capacitor. As the over-excited motor draws a leading current from the supply, it absorbs leading reactive power and delivers the lagging reactive power. When such a machine connected in parallel with induction motors or other inductive load, i.e., devices that operate at lagging power factor and absorbs the lagging reactive power, this lagging reactive power demand is met by the synchronous condenser. Thus, the inductive load does not take the lagging reactive power from the supply and the power factor of the plant has been improved.

The synchronous capacitors, i.e., over-excited synchronous motors on no-load are installed in electric power systems only for power factor improvement of the system. The synchronous condensers are economical in large sizes than the static capacitors

- A [synchronous motor](#) is not inherently self-starting. Therefore, it requires some auxiliary means of starting. In order to start a synchronous motor, there are following two methods – Starting with an external prime mover
- Starting with damper windings

#### Synchronous Motor Starting with an External Prime Mover

In this method of starting a synchronous motor, an external motor is used to drive the synchronous motor as shown in Figure-1.



Figure-1

The external motor brings the synchronous motor to synchronous speed and then the synchronous motor is synchronised with the AC supply as a synchronous generator. Then the prime mover (i.e., the external motor) is disconnected. Once synchronised, the synchronous machine will operate as a motor.

Now, the mechanical load can be connected to the shaft of the synchronous motor. Since the load is not connected to the synchronous motor before synchronising, thus the prime mover motor has to overcome only the inertia of the synchronous motor at no-load. Consequently, the rating of the prime mover or starting motor is much smaller than the rating of the synchronous motor.

#### Synchronous Motor Starting with Damper Windings

At present, damper windings are most widely used method of starting a synchronous motor. A damper winding is made up of heavy copper bars inserted in slots cut into the pole faces of the rotor as shown in Figure-2. These copper bars are then short-circuited by end rings at both ends of the rotor.

Hence, this arrangement of copper bars and end rings form a squirrel cage winding.

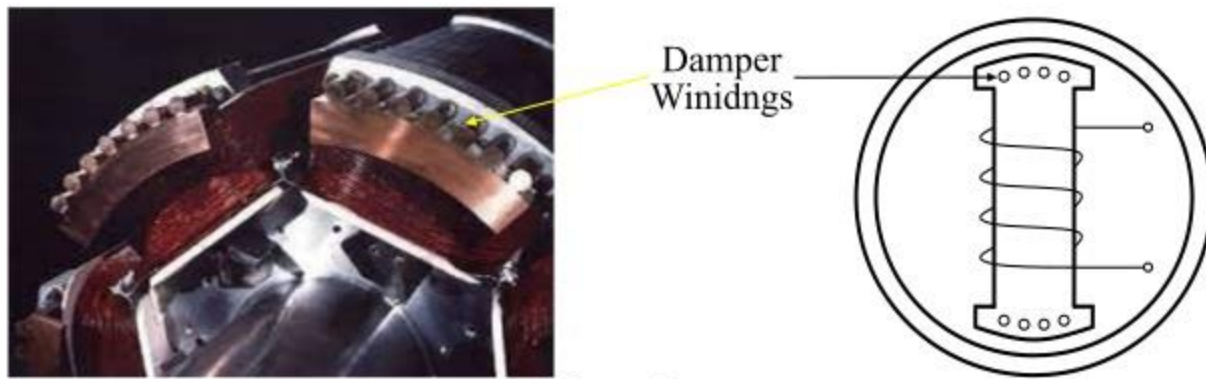


Figure-2

When the armature of the synchronous motor is connected to a three-phase supply, the synchronous motor with damper winding will start as a 3-phase squirrel cage induction motor.

When the motor attains a speed nearer to the synchronous speed, the DC field excitation is applied to the rotor field windings. Then, the rotor will pull into step with the [rotation magnetic field](#) of the armature and hence the motor runs at synchronous speed.

What is Hunting?

An unloaded synchronous machine starts with a zero-degree load angle. As the shaft load increases, so does the load angle. If a load,  $P_1$ , is suddenly applied to an unloaded machine, the machine will momentarily slow down.

Additionally, the load angle ( $\delta$ ) increases from zero to  $\delta_1$ . Initially, the electrical power developed matches the mechanical load,  $P_1$ .

Since equilibrium isn't reached, the rotor swings past  $\delta_1$  to  $\delta_2$ , generating more [electrical power](#) than before.

The rotor reaches synchronous speed but doesn't maintain it, accelerating beyond this speed. This acceleration causes the load angle to decrease, preventing the attainment of equilibrium once more.

Consequently, the rotor swings or oscillates around its new equilibrium position, a process known as hunting or phase swinging. Hunting occurs in both [synchronous motors](#) and generators when there's an abrupt load change.

Causes of Hunting in Synchronous Motor

1. Sudden change in load.
2. Sudden change in field current.
3. A load containing [harmonic](#) torque.
4. Fault in supply system.

Effects of Hunting in Synchronous Motor

1. It may lead to loss of synchronism.
2. Produces mechanical stresses in the rotor shaft.
3. Increases machine losses and cause temperature rise.
4. Cause greater surges in [current](#) and [power flow](#).
5. It increases possibility of resonance.

Reduction of Hunting in Synchronous Motor

Two techniques should be used to reduce hunting. These are –

- Use of Damper Winding: It consists of low [electrical resistance](#) copper / aluminum brush embedded in slots of pole faces in salient pole machine. Damper winding damps out hunting by producing torque opposite to slip of rotor. The magnitude of damping torque is proportional to the slip speed.
- Use of Flywheels: The prime mover is provided with a large and heavy flywheel. This increases the inertia of prime mover and helps in maintaining the rotor speed constant.
- Designing synchronous machine with suitable synchronizing power coefficients.

## Module IV

# Three-phase induction motors

What is a 3 Phase Induction Motor?

A three phase induction motor is an AC-type induction motor that operates on three-phase supply like a single-phase induction motor where a single-phase supply is needed to operate it. An electromagnetic field is produced by three three-phase supply currents in the stator winding which will produce the torque in the rotor winding of the three phase induction motor in the presence of the magnetic field.

Construction of 3 Phase Induction Motor

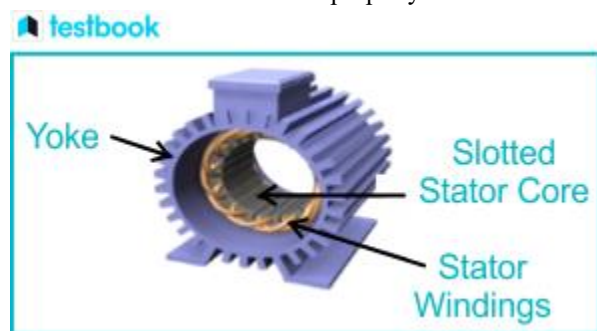
A 3 Phase induction motor has two main parts:

- Stator
- Rotor

By a small air gap ranging from 0.5 mm to 4 mm, the rotor and stator are separated depending on the power rating of the motor. here we will discuss the construction of three phase induction of motor in detail.

Stator of Three Phase Induction Motor

The stationary part of the motor is the stator. It is made of a steel frame which encloses a hollow cylindrical core. The core of the three phase induction motor is made of silicon steel lamination of thin layers to minimize the hysteresis losses and eddy current. Evenly spaced slots are given on the inner periphery of the laminated core as shown in the figure. The insulated conductors are kept in these stator slots and are connected properly to form a balanced 3-phase delta or star-connected stator winding.



The 3-phase stator windings are configured with a specific number of poles, depending on the requirement of speed, i.e., the greater the speed of the motor, the lesser the number of poles and vice-versa. When a balanced 3-phase supply is fed to the stator winding, a rotating magnetic flux (RMF) of constant magnitude is produced and this RMF induces currents in the rotor circuit by electromagnetic induction.

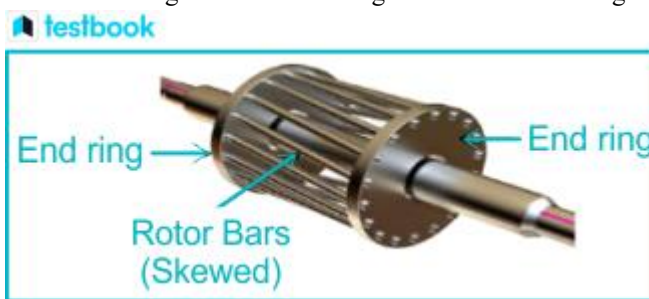
Rotor of 3 Phase Induction Motor

The rotor of an Induction motor is a laminated core hollow cylindrical, slots are constructed on its outer periphery. On these rotor slots, the rotor windings are placed. Depending upon the winding placement, the rotor of a 3-phase induction motor is of two types:

1. Squirrel Cage Type Rotor
2. Wound Type or Slip-Ring Type Rotor

Squirrel Cage Type Rotor

The squirrel cage rotor is made of a cylindrical laminated core, Skewed slots are kept on its outer periphery, which are nearly parallel to the shaft axis. An uninsulated aluminum or copper bar (rotor conductor) is placed in each slot. At both ends of the rotor, the rotor bar conductors are connected by heavy end rings made from the same material (see the figure) creating a short-circuit. This is indestructible winding because it is formed by permanently short-circuited winding. This whole arrangement resembles a cage which was once



normally used for keeping squirrels hence the name.

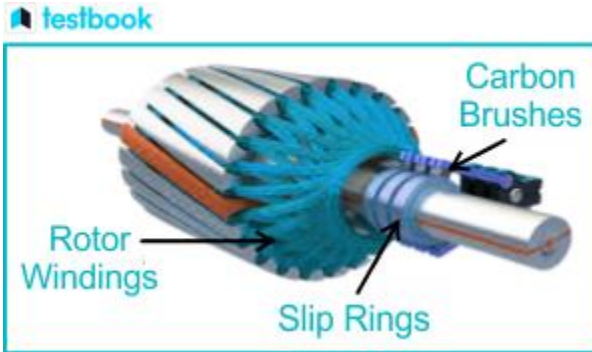
Currents are induced in the rotor by the electromagnetic induction from the stator, and hence rotor is not connected electrically to the supply. Those 3 phase induction motors which are known as squirrel cage induction motors are those in which squirrel cage rotors are employed. In the industries Squirrel cage rotor is mostly used in 3-phase induction motors because it has simple and robust construction enabling it to work in the most versatile environment. However, it has a disadvantage of low starting torque.

Squirrel cage rotor conductors are skewed because this offers the following advantages –

- The noise is reduced during operation.
- More uniform torque is produced.
- The magnetic locking tendency or cogging of the rotor is reduced. Due to magnetic action cogging occurs, in which the rotor and stator teeth lock with each other.

#### Wound Rotor or Slip Ring Rotor

The slip ring rotor is made of a laminated cylindrical armature core. The slots are constructed on the outer periphery and insulated conductors are placed in the slots. To form a 3-phase double layer distributed winding similar to the stator winding, the rotor conductors are connected. The rotor windings are connected in star form (shown in the figure).



The star connection's open ends are taken outside the rotor and connected to three insulated slip rings. The slip rings are placed on the rotor shaft with brushes resting on them. The brushes are linked to three variable resistors arranged in a star configuration. Here, Brushes and slip rings are used to give a means for connecting external resistors in the rotor circuit. The equivalent circuit of the wound (Slip ring) rotor is shown in the figure below.

The external resistors provide the variation of each rotor phase resistance to serve the following two purposes –

- To Reduce the starting current from the supply and increase the starting torque.
- To control the speed of the motor.

#### Working Principle of 3 Phase Induction Motor

This motor works on the principle of electromagnetic induction, that's why it is known as the induction motor. Electromagnetic induction refers to the phenomenon in which the electromotive force is induced across the electrical conductor when it is placed in a rotating magnetic field. The stator and rotor are two main parts of the motor. The stationary part is a stator, and the overlapping windings are carried by it. While the rotor carries the main or field winding. The windings of the stator are evenly displaced from each other by an angle of  $120^\circ$ .

In an induction motor the supply is applied only to one part, the stator so it is known as the single excited motor. The process of inducing the magnetic field on the parts of the motor is known as excitation. When the three-phase supply is fed to the stator, the rotating magnetic field is induced on it. The below-given fig shows the rotating magnetic field set up in the stator:

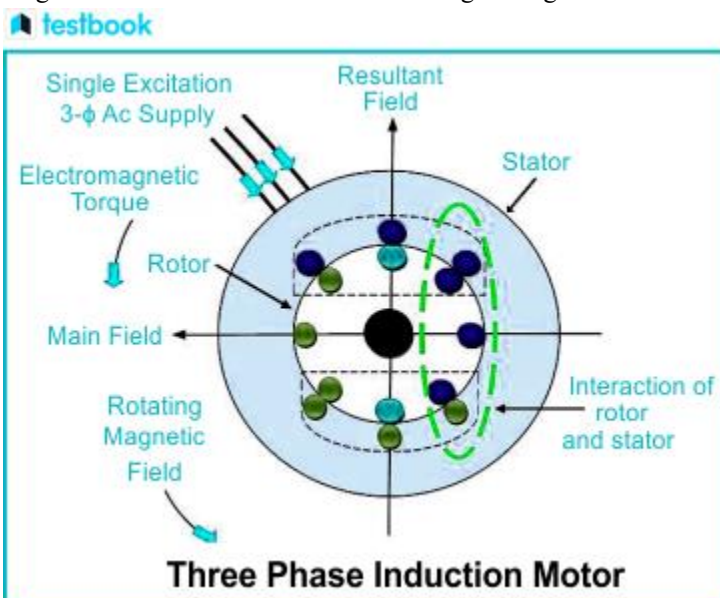


Fig -3 Phase Induction Motor Diagram

Consider that the direction of the induced rotating magnetic field is anticlockwise. The rotating magnetic field has moving polarities. The magnetic field polarities vary by taking into account both the positive and negative half cycles of the supply. The magnetic field rotation happens when we change polarities. The conductors of the rotor are stationary. The rotating magnetic field of the stator is cut by

this stationary conductor, and because of this electromagnetic induction, In the rotor, EMF will be induced. This EMF is called the rotor-induced EMF, and it happens due to the electromagnetic induction phenomenon.

The rotor conductors are short-circuited with the help of either the external resistance or by the end rings. In the rotor conductor, current will be induced by the relative motion of the rotating magnetic field and the rotor conductor. As the current flows through the conductor, the flux will be induced on it. The direction of rotor flux is the same as the direction of the rotor current. Now we have two fluxes, one because of the rotor and another because of the stator. These fluxes interact with each other. The Fluxes cancel each other on one end of the conductor, and on the other end, the flux density is very high. So, the high-density flux tries to push the rotor conductor towards the low-density flux region.

This phenomenon generates torque on the conductor, and this torque is referred to as electromagnetic torque. The electromagnetic torque and the rotating magnetic field share the same direction. Thus, the rotor initiates rotating in the same direction as that of the rotating magnetic field. The speed of the rotor is always lesser than the synchronous speed or rotating magnetic field. The rotor tries to run at synchronous speed, but it always slips away. So, the motor never runs at the speed of the rotating magnetic field, and because of this reason, the induction motor is also called the Asynchronous motor.

Types of 3 Phase Induction Motor

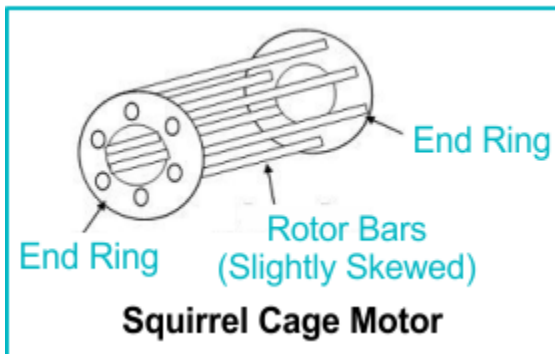
Three phase induction motors are mainly classified in two types based on the rotor winding slip ring (wound rotor motor) and (Armature coil winding) i.e. squirrel cage.

1. Squirrel Cage Induction Motor
2. Slip-ring or Wound Rotor Induction Motor

3 Phase Squirrel Cage Induction Motor

This motor is called a squirrel cage induction motor because the shape of this rotor is similar to the shape of the cage of a squirrel.





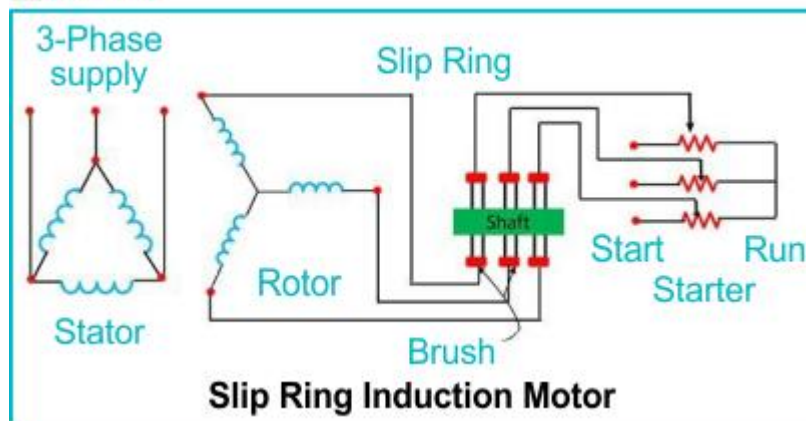
The construction of a squirrel cage type of rotor is very simple and rough. So, mostly induction motor is a squirrel cage induction motor. The rotor is made of a cylindrical laminated core and slots are constructed on the outer periphery. The slots are skewed at some angle. Skewed slots help to prevent cogging. It provides smooth operation and reduces the humming noise. Because of this, the length of the rotor conductor is increased due to this the rotor resistance is increased.

The squirrel cage rotor consists of rotor bars instead of the rotor winding. The rotor bars are made up of brass, copper, or aluminum. End rings short the rotor bars permanently, Creating a fully closed path in the rotor circuit. To provide mechanical support the rotor bars are welded with the end rings. Because of short-circuited rotor bars, there is no possibility to add external resistance to the rotor circuit. The construction of this type of motor is simpler and more robust because in this type of rotor, the slip rings and brushes are not used.

Slip-ring or Wound Rotor Induction Motor

The motor that uses the wound rotor is called a slip ring induction motor or phase wound motor. It is made of a laminated cylindrical core which features a semi-closed slot at the outer periphery and carries three-phase insulated winding. The rotor is wound with an





identical number of poles as the stator.

Forming a star point, three complete terminals are connected. and the three-star terminals are connected to three copper slip rings fixed on the shaft. Through the center of the rotor, the mild steel shaft is passed and fixed to the key. The aim of the shaft is to deliver the mechanical power.

#### Difference between Slip Ring & 3 Phase Squirrel Cage Induction Motor

The most common or popular type of [AC motor](#) is the squirrel cage induction motor. It is the most used motor in industries because it is very cheap, robust, efficient, simple, and reliable. The slip ring motor has very few applications in industries. Rarely 5% – 10% slip ring motors are used in industries because it has several disadvantages like it requires frequent maintenance, having a high copper loss, etc. One of the main differences between the squirrel cage and the slip ring motor is that for controlling the speed of the motor, the slip ring motor has an external resistance circuit. Whereas in a squirrel cage motor, we can't add any external circuit because of the permanently slotted bar of the motor at the end of the ring.

| Feature                  | Squirrel Cage Induction Motor                                            | Slip Ring Induction Motor                                                                |
|--------------------------|--------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| Rotor Construction       | Squirrel cage rotor with shorted conductors                              | Wound rotor with external connected slip rings and brushes                               |
| Control of Speed         | Limited speed control without external devices                           | Speed can be controlled by varying external resistance connected to slip rings           |
| Applications             | Widely used in various industries due to simplicity and reliability      | Limited applications (5% - 10% usage) due to maintenance issues and higher copper losses |
| Maintenance Requirements | Minimal maintenance, as there are no external components like slip rings | Requires frequent maintenance due to slip rings, brushes, and external resistance        |
| Efficiency               | Generally higher efficiency                                              | Lower efficiency due to additional losses in slip rings and brushes                      |
| Cost                     | Cost-effective and inexpensive                                           | Generally more expensive due to the complexity of slip rings and additional components   |
| Starting Torque          | Good starting torque                                                     | Can provide high starting torque, especially with external resistance                    |
| Common Usage             | Most commonly used in industrial applications                            | Rarely used, typically in situations where variable speed control is essential           |
| Complexity               | Simple construction with fewer components                                | More complex construction with additional components like slip rings and brushes         |

#### Advantages and Disadvantages of 3 Phase Induction Motor

The merits and demerits of a 3 phase Induction Motor are mentioned below.

##### Advantages of 3 Phase Induction Motor

Following are the advantages of a 3 phase induction motor –

- It has a simple and rugged construction.
- It requires less maintenance.
- It has good efficiency and better power factor.
- It is less expensive.
- It has self-starting torque.

##### Disadvantages of 3 Phase Induction Motor

The disadvantages of a 3-phase induction motor are given below –

- Speed control of 3-phase induction motor is very difficult because these motors are constant speed motors
- 3 phase induction motors have low starting torque and high inrush currents (about 4 to 8 times of the rated current).
- They always operate under a lagging power factor and during light loads, they operate at the very worst power factor (about 0.3 to 0.5 lagging).

##### Application of 3 Phase Induction Motor

In industrial applications, an induction motor is used. In residential as well as industrial applications, the squirrel cage induction motors are used especially when the speed control of motors is not needed, such as:

##### Uses of Squirrel Cage Induction Motor

- Pumps and submersible
- Pressing machine
- Lathe machine
- Grinding machine
- Conveyor
- Flour mills

- Compressor
- Other low mechanical power applications

Uses of Slip Ring Induction Motor

The slip ring motors are used where heavy load applications and high initial torque are required, such as:

- Steel mills
- Lift
- Crane Machine
- Hoist
- Line shafts
- and other heavy mechanical workshops, etc.

## ○ Torque equation of three phase induction motor

- Firstly the magnitude of rotor current, secondly the **flux** which interact with the rotor of three phase induction motor and is responsible for producing emf in the rotor part of **induction motor**, lastly the **power factor** of rotor of the three phase induction motor.

By integrating these factors, we derive the torque equation as follows:

$$T \propto \phi I_2 \cos \theta_2$$

Where, T is the torque produced by the induction motor,

$\phi$  is flux responsible for producing induced emf,

$I_2$  is rotor current,

$\cos \theta_2$  is the power factor of rotor circuit.

- The flux  $\phi$  produced by the stator is proportional to stator emf  $E_1$ .

i.e  $\phi \propto E_1$

We know that transformation ratio K is defined as the ratio of secondary **voltage** (rotor voltage) to that of primary voltage (stator voltage).

$$K = \frac{E_2}{E_1}$$

$$\text{or, } K = \frac{E_2}{\phi}$$

$$\text{or, } E_2 = \phi$$

Rotor **current**  $I_2$  is defined as the ratio of rotor induced emf under running condition,  $sE_2$  to total impedance,  $Z_2$  of rotor side,

$$\text{i.e } I_2 = \frac{sE_2}{Z_2}$$

- and total impedance  $Z_2$  on rotor side is given by ,

$$Z_2 = \sqrt{R_2^2 + (sX_2)^2}$$

Putting this value in above equation we get,

$$I_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$s$  = slip of **induction motor**

- We know that **power factor** is defined as ratio of **resistance** to that of impedance. The power factor of the rotor circuit is

$$\cos \theta_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Putting the value of **flux**  $\phi$ , rotor current  $I_2$ , power factor  $\cos \theta_2$  in the equation of torque we get,

$$T \propto E_2 \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

- Combining similar term we get,

$$T \propto sE_2^2 \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Removing proportionality constant we get,

$$T = KsE_2^2 \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{This constant } K = \frac{3}{2\pi n_s}$$

Where,  $n_s$  is synchronous speed in r. p. s,  $n_s = N_s / 60$ . So, finally the equation of torque becomes,

$$T = sE_2^2 \times \frac{R_2}{R_2^2 + (sX_2)^2} \times \frac{3}{2\pi n_s} N - m$$

How the constant K is derived in the torque equation.

In a [three phase induction motor](#), copper losses typically occur in the rotor. These rotor copper losses are expressed as

$$P_c = 3I_2^2 R_2$$

We know that rotor current,

$$I_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Substitute this value of  $I_2$  in the equation of rotor copper losses,  $P_c$ . So, we get

$$P_c = 3R_2 \left( \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \right)^2$$

$$\text{On simplifying } P_c = \frac{3R_2 s^2 E_2^2}{R_2^2 + (sX_2)^2}$$

The ratio of  $P_2 : P_c : P_m = 1 : s : (1 - s)$

Where,  $P_2$  is the rotor input,

$P_c$  is the rotor copper losses,

$P_m$  is the mechanical power developed.

$$\frac{P_c}{P_m} = \frac{s}{1 - s}$$

$$\text{or } P_m = \frac{(1 - s)P_c}{s}$$

Substitute the value of  $P_c$  in above equation we get,

$$P_m = \frac{1}{s} \times \frac{(1 - s)3R_2 s^2 E_2^2}{R_2^2 + (sX_2)^2}$$

On simplifying we get,

$$P_m = \frac{(1 - s)3R_2 s E_2^2}{R_2^2 + (sX_2)^2}$$

The mechanical power developed  $P_m = T\omega$ ,

$$\omega = \frac{2\pi N}{60}$$

$$\text{or } P_m = T \frac{2\pi N}{60}$$

Substituting the value of  $P_m$

$$\frac{1}{s} \times \frac{(1-s) 3R_2 s^2 E_2^2}{R_2^2 + (sX_2)^2} = T \frac{2\pi N}{60}$$

$$\text{or } T = \frac{1}{s} \times \frac{(1-s) 3R_2 s^2 E_2^2}{R_2^2 + (sX_2)^2} \times \frac{60}{2\pi N}$$

We know that the rotor speed  $N = N_s(1-s)$

Substituting this value of rotor speed in above equation we get,

$$T = \frac{1}{s} \times \frac{(1-s) 3R_2 s^2 E_2^2}{R_2^2 + (sX_2)^2} \times \frac{60}{2\pi N_s(1-s)}$$

$N_s$  is speed in revolution per minute (rpm) and  $n_s$  is speed in revolution per sec (rps) and the relation between the two is

$$\frac{N_s}{60} = n_s$$

Substitute this value of  $N_s$  in above equation and simplifying it we get

$$\text{Torque, } T = \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{3}{2\pi N_s}$$

$$\text{or, } T = K s E_2^2 \frac{R_2}{R_2^2 + (sX_2)^2}$$

Comparing both the equations, we get, constant  $K = 3 / 2\pi n_s$ ,

- Starting torque is the torque produced by [induction motor](#) when it starts. We know that at the start the rotor speed,  $N$  is zero.

$$\text{So, slip } s = \frac{N_s - N}{N_s} \text{ becomes } 1$$

So, the equation of starting torque is easily obtained by simply putting the value of  $s = 1$  in the equation of torque of the three phase induction motor,

$$T = \frac{E_2^2 R_2}{R_2^2 + X_2^2} \times \frac{3}{2\pi n_s} N - m$$

The starting torque is also known as standstill torque.

- Maximum Torque Condition for Three-Phase Induction Motor**
- In the equation of torque,

$$T = \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{3}{2\pi n_s}$$

The rotor resistance, rotor inductive reactance and synchronous speed of induction motor remain constant. The supply voltage to the [three phase induction motor](#) is usually rated and remains constant, so the stator emf also remains the constant. We define the transformation ratio as the ratio of rotor emf to that of stator emf. So if stator emf remains constant, then rotor emf also remains constant.

If we want to find the maximum value of some quantity, then we have to differentiate that quantity concerning some variable parameter and then put it equal to zero. In this case, we have to find the condition for maximum torque, so we have to differentiate torque concerning some variable quantity which is the slip,  $s$  in this case as all other parameters in the equation of torque remains constant.

So, for torque to be maximum

$$\frac{dT}{ds} = 0$$

$$T = K s E_2^2 \frac{R_2}{R_2^2 + (sX_2)^2}$$

Now differentiate the above equation by using division rule of differentiation. On differentiating and after putting the terms equal to zero we get,

$$s^2 = \frac{R_2^2}{X_2^2}$$

Neglecting the negative value of slip we get

$$s^2 = \frac{R_2^2}{X_2^2}$$

So, when slip  $s = R_2 / X_2$ , the torque will be maximum and this slip is called maximum slip  $S_m$  and it is defined as the ratio of rotor resistance to that of rotor reactance.

NOTE: At starting  $S = 1$ , so the maximum starting torque occur when rotor resistance is equal to rotor reactance.

#### ○ Equation of Maximum Torque

○ The equation of torque is

$$T = \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

The torque will be maximum when slip  $s = R_2 / X_2$

Substituting the value of this slip in above equation we get the maximum value of torque as,

$$T_{max} = K \frac{E_2^2}{2X_2} \quad N - m$$

In order to increase the starting torque, extra [resistance](#) should be added to the rotor circuit at start and cut out gradually as motor speeds up.

Conclusion

From the above equation it is concluded that

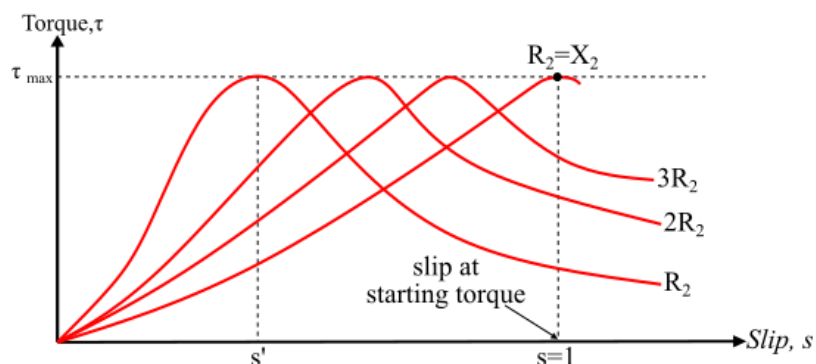
- The maximum torque is directly proportional to square of rotor induced emf at the standstill.
- The maximum torque is inversely proportional to rotor reactance.
- Notably, the maximum torque does not depend on the rotor resistance.
- The slip at which maximum torque occur depends upon rotor resistance,  $R_2$ . So, by varying the rotor resistance, maximum torque can be obtained at any required slip

The torque of a 3-phase induction motor under running conditions is given by,

$$\tau_r = \frac{KsE_2^2 R_2}{R_2^2 + (sX_2)^2} \dots (1)$$

From the eqn. (1), it can be seen that if  $R_2$  and  $X_2$  are kept constant, the torque depends upon the slip 's'. The torque-slip characteristics curve can be divided into three regions, viz.

- Low-slip region
- Medium-slip region
- High-slip region



#### Low-Slip Region

At synchronous speed, the slip  $s = 0$ , thus, the torque is 0. When the speed is very near to the synchronous speed, the slip is very low and the term  $(sX_2)^2$  is negligible in comparison with  $R_2$ . Therefore,

$$\tau \propto 2\tau \propto 2$$

If  $R_2$  is constant, then

$$\tau \propto s \dots (2) \quad \tau \propto s \dots (2)$$

Eqn. (2) shows that the torque is proportional to the slip. Hence, when the slip is small, **the torque-slip curve is straight line.**

Medium-Slip Region

When the slip increases, the term  $(sX_2)^2$  becomes large so that  $R_2^2$  may be neglected in comparison with  $(sX_2)^2$ . Therefore,

$$\tau \propto s(sX_2)^2 = 1sX_2^2 \quad \tau \propto s(sX_2)^2 = 1sX_2^2$$

If  $X_2$  is constant, then

$$\tau \propto 1s \dots (3) \quad \tau \propto 1s \dots (3)$$

Thus, the torque is inversely proportional to slip towards standstill conditions. Hence, for intermediate values of the slip, the torque-slip characteristics is represented by a **rectangular hyperbola**. The curve passes through the point of **maximum torque** when  $R_2 = sX_2$ .

The maximum torque developed by an induction motor is known as **pull-out torque** or **breakdown torque**. This breakdown torque is a measure of the short time overloading capability of the motor.

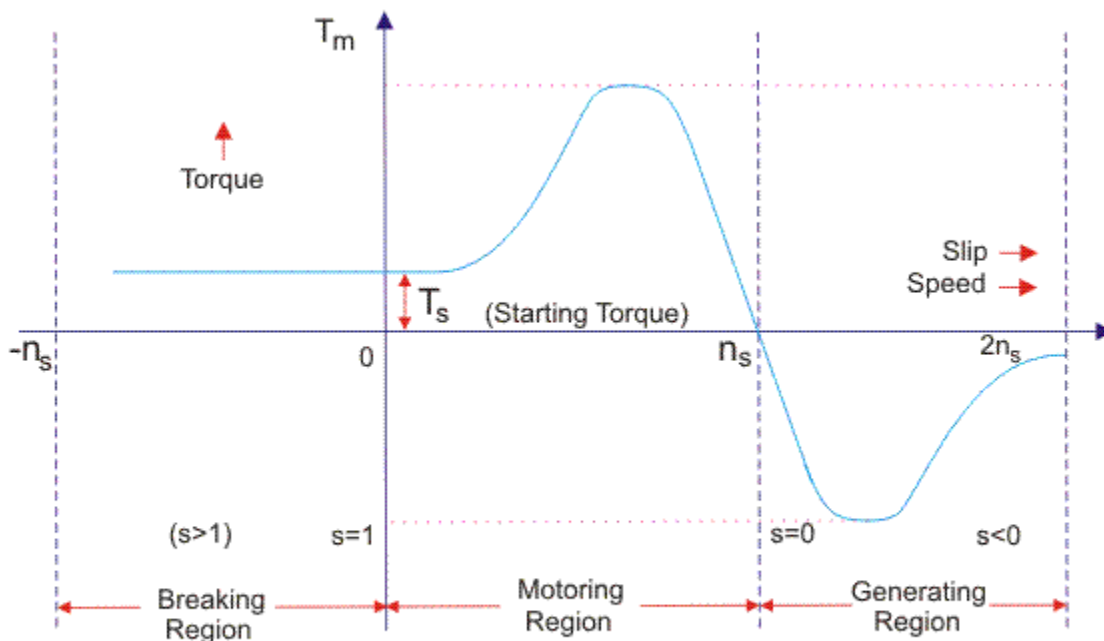
High-Slip Region

The torque decreases beyond the point of maximum torque. As a result of this, the motor slows down and eventually stops. The induction motor operates for the values of slip between  $s = 0$  and  $s = s_m$ , where  $s_m$  is the value of slip corresponding to maximum torque. For a typical 3-phase induction motor, the breakdown torque is 2 to 3 times of the full-load torque. Therefore, the motor can handle overloading for a short period of time without stalling.

### Torque Slip Characteristics of Three Phase Induction Motor

The torque slip curve of an [induction motor](#) shows how torque changes with slip. Slip is defined as the difference between synchronous speed and actual rotor speed, divided by synchronous speed. When speed changes, slip and the corresponding torque also change.

The curve can be described in three modes of operation-



### Torque Slip Curve for Three Phase Induction Motor

The torque-slip characteristic curve can be divided roughly into three regions:

- Low slip region
- Medium slip region
- High slip region

Motoring Mode

In this mode, power is supplied to the stator, and the motor rotates below synchronous speed. The motor torque ranges from zero to full load torque as the slip changes. Slip ranges from zero at no load to one at standstill. The curve shows that torque is directly proportional to slip.

That is, more is the slip, more will be the torque produced and vice-versa. The linear relationship simplifies the calculation of motor parameter to great extent.

### Generating Mode

In this mode of operation induction motor runs above the synchronous speed and it should be driven by a prime mover. The stator winding is connected to a three phase supply in which it supplies electrical energy. Actually, in this case, the torque and slip both are negative so the motor receives mechanical energy and delivers electrical energy. Induction motor is not much used as generator because it requires reactive power for its operation.

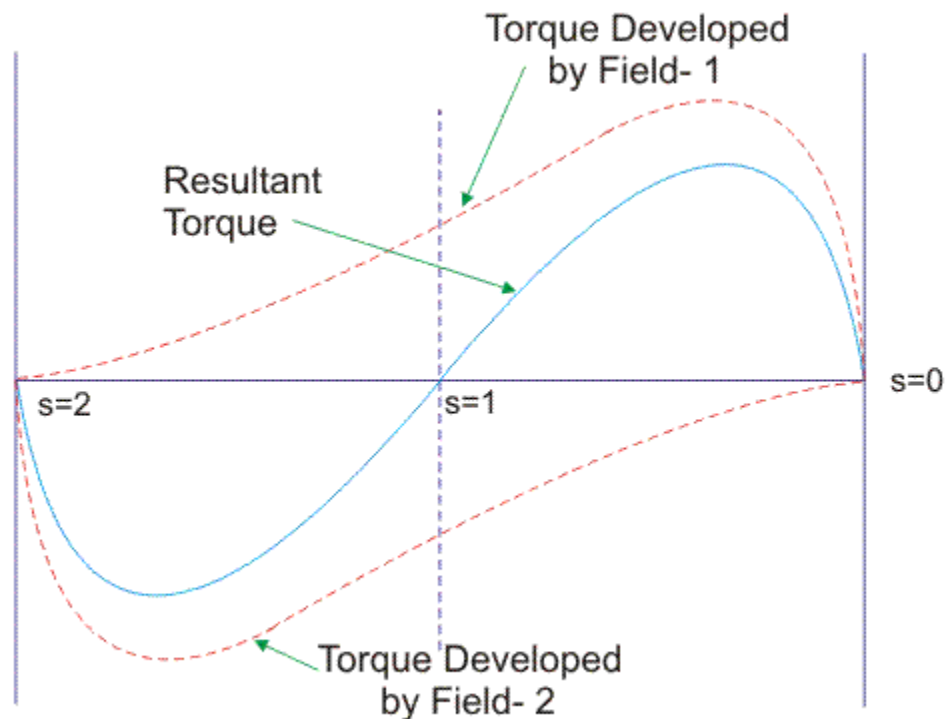
That is, reactive power should be supplied from outside and if it runs below the synchronous speed by any means, it consumes electrical energy rather than giving it at the output. So, as far as possible, [induction generators](#) are generally avoided

### Braking Mode

In braking mode, the motor's supply [voltage](#) polarity is reversed, causing it to rotate in the opposite direction and stop. This method, called plugging, is used to quickly stop the motor. The kinetic energy in the load is dissipated as heat, along with the power still received from the stator. Thus, the stator is disconnected before the motor enters braking mode to avoid excessive heat.

If load which the motor drives accelerates the motor in the same direction as the motor is rotating, the speed of the motor may increase more than synchronous speed. In this case, it acts as an [induction generator](#) which supplies electrical energy to the mains which tends to slow down the motor to its synchronous speed, in this case the motor stops. This type of breaking principle is called dynamic or regenerative breaking.

### Torque Slip Characteristics of Single Phase Induction Motor



### Torque Slip Characteristics of Single Phase Induction Motor

At a slip of one, forward and backward fields in a single-phase induction motor create equal but opposite torques, resulting in zero net torque, so the motor fails to start. Unlike [three phase induction motor](#), these motors are not self-starting and need an external method to provide starting torque. Increasing the forward speed decreases forward slip, increasing forward torque and decreasing reverse torque, thus starting the motor.

To start a [single phase induction motor](#), a difference in torque between the forward and backward fields is needed. If the forward field torque is greater, the motor rotates in the forward (anti-clockwise) direction. If the backward field torque is greater, the motor rotates in the backward (clockwise) direction.

### Effect of Change in Voltage and Frequency

Let us see the effect of change in voltage and frequency on the torque-slip characteristics of an induction motor. This effect can be studied by analyzing two cases as,

#### Case 1 : Halving the applied voltage, keeping frequency normal.

When the motor is running with slip  $s$ , the torque is,

$$T = \frac{k s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

Now standstill e.m.f.  $E_2$  is proportional to the supply voltage.

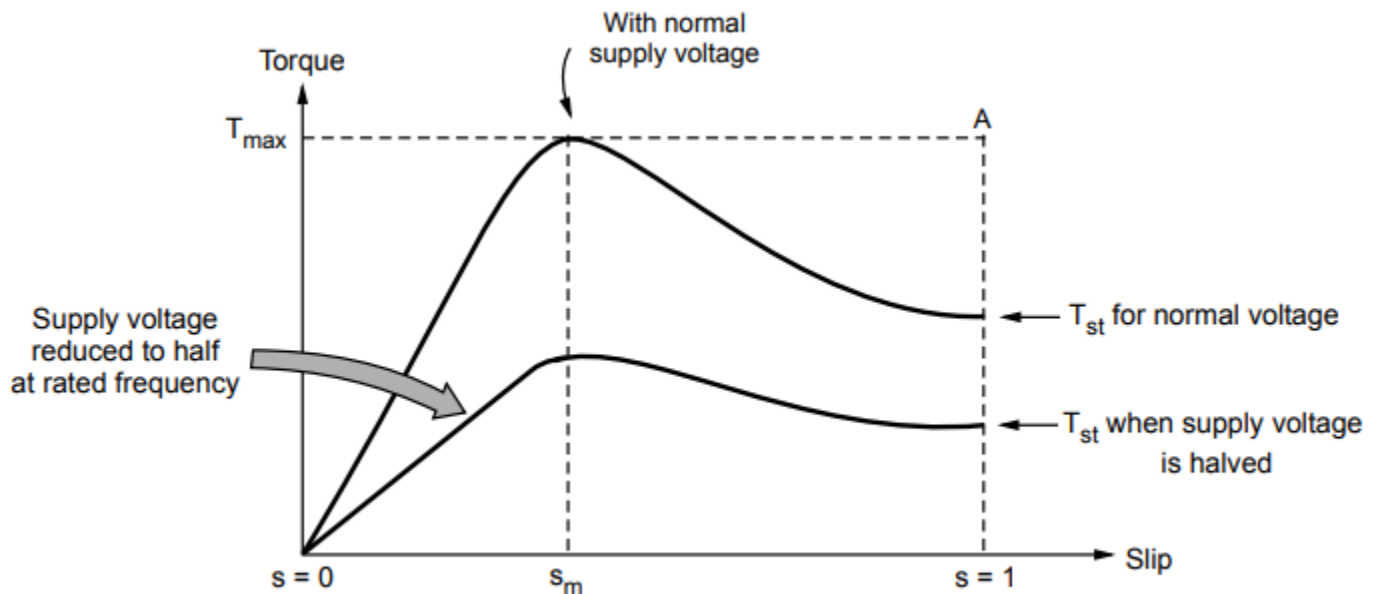
$$T = \frac{k' s V^2 R_2}{R_2^2 + (sX_2)^2} \quad \text{where } k' \text{ is another constant}$$

Generally on full load, slip  $s$  is very small hence  $(sX_2)^2 \ll R_2^2$  hence neglecting it

$$T = \frac{k' s V^2 R_2}{R_2^2} = \frac{k' s V^2}{R_2}$$

$$T \propto sV^2$$

If supply voltage is made half, then the torque will reduce by the factor  $(1/4)$ , in the running condition. The slip at which  $T_{\max}$  occurs remains same but the value of  $T_{\max}$  reduces. The corresponding speed-torque characteristics are shown in the Fig. 5.15.1.



**Fig. 5.15.1 Effect of change in supply voltage at normal frequency**

#### Case 2 : Halving both the applied voltage and frequency

For an induction motor, the air gap flux is given by,

$$\phi_g = \frac{1}{4.44k_1 T_{ph1}} \left( \frac{V}{f} \right)$$

Thus if  $f$  is changed, air gap flux also changes. This may result into saturation of stator and rotor cores. Such a saturation may leads to sharp increase in the current. But if  $(V/f)$  ratio is maintained constant, then air gap flux remains constant. Thus when both are halved air gap flux remains constant but as frequency is reduced, the shape of torque-slip characteristics remains same.

Let  $V_n$  = Normal voltage,  $f_n$  = Normal frequency

$V$  = New voltage,  $f$  = New frequency

$V/f = \text{constant}$  i.e.  $V_n/f_n = V/f$  i.e.  $V = f/f_n V_n$ .

As frequency is changed, the nominal rotor standstill reactance referred to stator also changes.

$X_{2n}$  = Nominal rotor reactance

$X_2$  = New rotor reactance

$X_2 = (f/f_n) X_{2n}$

As frequency changes,  $N_s$  changes hence,

$w_s = (f/f_n) w_{sn}$

where,  $w_{sn}$  = Nominal speed  $w_s$  = New speed

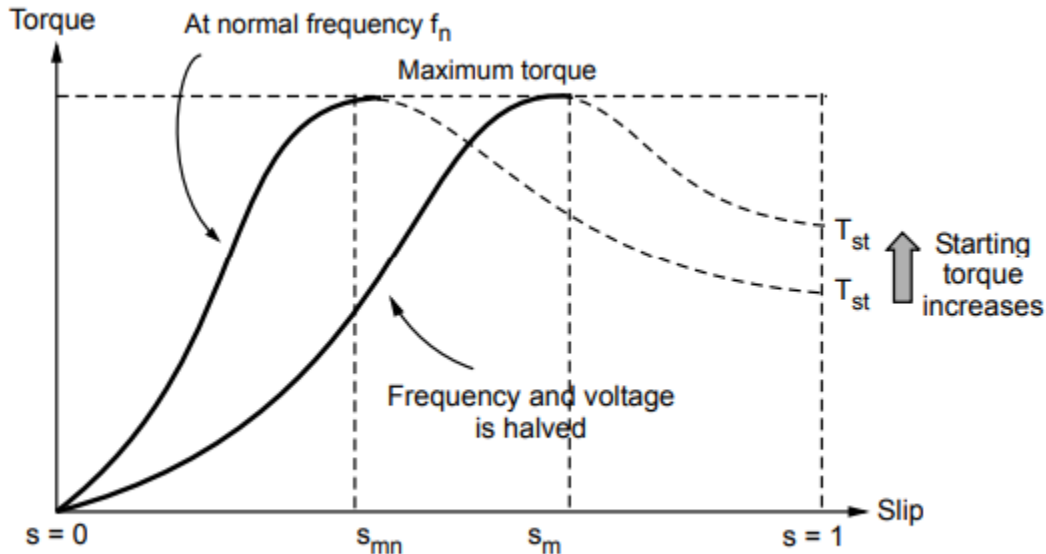
$s_m$  = Nominal slip at maximum torque =  $R_2/X_2$

$$\therefore s_m = \text{New slip at maximum torque} = \frac{R_2}{X_2}$$

$$= \frac{R_2}{\left(\frac{f}{f_n}\right) X_{2n}} = \left(\frac{f_n}{f}\right) \left(\frac{R_2}{X_{2n}}\right) = \left(\frac{f_n}{f}\right) s_{mn}$$

where,  $s_{mn}$  = Nominal slip at maximum torque

As frequency is halved with V/f constant, the maximum torque remains same but  $s_m$  increases while the starting torque increases. Hence the torque speed characteristics are as shown in the Fig. 5.15.2.



**Fig. 5.15.2 Effect of reducing  $f$ , maintaining  $V/f$  constant**

**Example 5.15.1** A 3-phase, 4 pole, 50 Hz squirrel cage induction motor has rotor leakage impedance of  $1 + j2 \Omega/\text{Ph}$ , standstill voltage of 100 V per phase driving a constant torque load at 0.03 slip, what is speed of the motor, if

i) Supply voltage is increased by 25 % and frequency is constant.

ii) Supply voltage is increased by 25 % and frequency is decreased by 25 %.

**Solution :**  $P = 4$ ,  $f = 50 \text{ Hz}$ ,  $R_2 = 1 \Omega$ ,  $X_2 = 2 \Omega$ ,  $E_{2ph} = 100 \text{ V}$ ,  $s_1 = 0.03$

**Solution :**  $P = 4$ ,  $f = 50 \text{ Hz}$ ,  $R_2 = 1 \Omega$ ,  $X_2 = 2 \Omega$ ,  $E_{2ph} = 100 \text{ V}$ ,  $s_1 = 0.03$

$$N_s = \frac{120 f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m. i.e. } n_s = \frac{N_s}{60} = 25 \text{ r.p.s.}$$

Now,

$$T = \frac{3}{2\pi n_s} \times \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

Initially with rated voltage the torque is,

$$T = \frac{3}{2\pi \times 25} \times \frac{0.03 \times 100^2 \times 1}{(1)^2 + (0.03 \times 2)^2} = 5.709 \text{ Nm}$$

Initially with rated voltage the torque is,

$$T = \frac{3}{2\pi \times 25} \times \frac{0.03 \times 100^2 \times 1}{(1)^2 + (0.03 \times 2)^2} = 5.709 \text{ Nm}$$

As voltage increases,  $E_2$  increases as  $E_2 \propto V$ , due to transformer action. i) Torque constant, frequency constant, supply voltage increased by 25 %.

$E_2$  increases by 25 % =  $100 + 0.25 \times 100 = 125 \text{ V}$

$$T = \frac{3}{2\pi \times 25} \times \frac{s' \times 125^2 \times 1}{(1)^2 + (s' \times 2)^2} = 5.709$$

$$4(s')^2 - 52.271s' + 1 = 0$$

$$s' = 13.048, 0.0191 \quad \dots \text{Neglecting higher value}$$

$$s' = 0.0191 \text{ i.e. } 1.91 \%$$

$$\therefore N' = N_s(1 - s') = 1500(1 - 0.0191) = \mathbf{1471.35 \text{ r.p.m.}}$$

ii) Torque constant, frequency decreased by 25 %, supply voltage increased by 25 %.

$$\therefore f' = f - 0.25 f = 50 - 0.25 \times 50 = 37.5 \text{ Hz}$$

$$\therefore N'_s = \frac{120f'}{P} = 1125 \text{ r.p.m.}, n'_s = \frac{N'_s}{60} = 18.75 \text{ r.p.s}$$

$$\therefore T = \frac{3}{2\pi \times 18.75} \times \frac{s'' \times 125^2 \times 1}{(1)^2 + (s'' \times 2)^2} = 5.709$$

$$\therefore 4(s'')^2 - 69.6947s'' + 1 = 0$$

$$\therefore s'' = 17.4, 0.01436 \quad \dots \text{Neglecting higher value}$$

$$\therefore s'' = 0.01436, \text{ i.e. } 1.436 \%$$

$$\therefore N'' = N'_s(1 - s'') = 1125(1 - 0.01436) = \mathbf{1108.845 \text{ r.p.m.}}$$

**Example 5.15.2** A 3-phase induction motor has operating p.f. of 0.85 at full load speed of 960 r.p.m. and at 400 V supply voltage. In case the supply voltage falls to 380 V, find the operating p.f. at the same full load torque.

**Solution :**

**Solution :**  $N_1 = 960$  r.p.m.,  $V_1 = 400$  V,  $V_2 = 380$  V,  $\cos \phi_1 = 0.85$ ,  $N_s = 1000$  r.p.m. as  $N_1 = 960$  r.p.m. on full load.

$$\therefore s_1 = \frac{N_s - N_1}{N_s} = 0.04$$

$$\cos \phi_1 = \frac{R_2}{\sqrt{R_2^2 + (s_1 X_2)^2}} = 0.85$$

$$\therefore 1.38408 R_2^2 = R_2^2 + (0.04 X_2)^2 \quad \text{i.e.} \quad 0.38408 R_2^2 = 1.6 \times 10^{-3} X_2^2$$

$$\therefore \frac{R_2}{X_2} = 0.06454 \quad \dots (1)$$

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} \propto \frac{s V^2 R_2}{R_2^2 + (s X_2)^2} \quad \dots E_2 \propto V$$

$$\therefore \frac{T_1}{T_2} = \frac{s_1 V_1^2 R_2}{R_2^2 + (s_1 X_2)^2} \times \frac{R_2^2 + (s_2 X_2)^2}{s_2 V_2^2 R_2} = 1 \quad \dots T_1 = T_2 \text{ given}$$

$$\therefore s_1 V_1^2 [R_2^2 + (s_2 X_2)^2] = s_2 V_2^2 [R_2^2 + (s_1 X_2)^2]$$

Dividing both sides by  $(X_2)^2$ ,

$$\therefore s_1 V_1^2 \left[ \left( \frac{R_2}{X_2} \right)^2 + s_2^2 \right] = s_2 V_2^2 \left[ \left( \frac{R_2}{X_2} \right)^2 + s_1^2 \right]$$

$$\therefore 0.04 \times (400)^2 [(0.06454)^2 + s_2^2] = s_2 \times 380^2 [(0.06454)^2 + (0.04)^2]$$

$$\therefore 6400 s_2^2 - 832.5237 s_2 + 26.65863 = 0$$

$$\therefore s_2 = 0.0569, 0.07309$$

Choosing lower value as it is going to occur first,

$$s_2 = 0.0569$$

$$\begin{aligned} \therefore \cos \phi_2 &= \frac{R_2}{\sqrt{R_2^2 + (s_2 X_2)^2}} = \frac{\frac{R_2}{X_2}}{\sqrt{\left( \frac{R_2}{X_2} \right)^2 + s_2^2}} \\ &= \frac{0.06454}{\sqrt{(0.06454)^2 + (0.0569)^2}} = 0.7501 \text{ lagging} \quad \dots \text{New p.f.} \end{aligned}$$

#### Effect of Change in Rotor Resistance

It is known that in slip ring induction motor, externally resistance can be added in the rotor. Let us see the effect of change in rotor resistance on the torque produced.

Let  $R_2$  = Rotor resistance per phase

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Corresponding torque,

Now externally resistance is added in each phase of rotor through slip rings.

Let  $R_2$  = New rotor resistance per phase

$$T' \propto \frac{s E_2^2 R'_2}{R'^2_2 + (sX_2)^2}$$

Corresponding torque,

Similarly the starting torque at  $s = 1$  for  $R_2$  and  $R'_2$  can be written as

$$T_{st} \propto \frac{E_2^2 R_2}{R_2^2 + X_2^2} \quad \text{and} \quad T'_{st} \propto \frac{E_2^2 R'_2}{R'^2_2 + X_2^2}$$

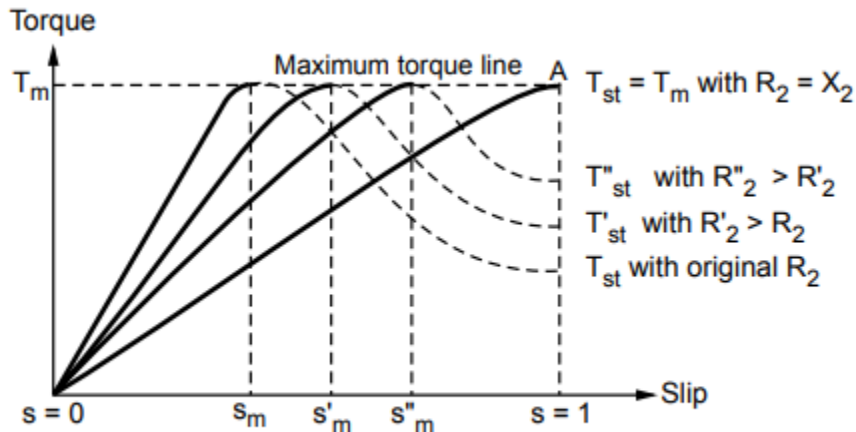
$$\text{Maximum torque} \quad T_m \propto \frac{E_2^2}{2X_2}$$

**Key Point** It can be observed that  $T_m$  is independent of  $R_2$  hence whatever may be the rotor resistance, maximum torque produced never changes but the slip and speed at which it occurs depends on  $R_2$ .

For  $R_2$   $s_m = R_2 / X_2$  where  $T_m$  occurs.

For  $R'_2$   $s'_m = R'_2 / X_2$  where  $T_m$  occurs.

As  $R_2 > R'_2$ , the slip  $s'_m > s_m$ . Due to this, we get a new torque-slip characteristics for rotor resistance  $R'_2$ . This new characteristics is parallel to the characteristics for  $R_2$  with same  $T_m$  but occurring at  $s'_m$ . The effect of change in rotor resistance on torque-slip characteristics is shown in the Fig. 5.14.1



**Fig. 5.14.1 Effect of rotor resistance on torque-slip curves**

It can be seen that the starting torque  $T_{st}$  for  $R'_2$  is more than  $T_{st}$  for  $R_2$ . Thus by changing rotor resistance the starting torque can be controlled.

If now resistance is further added to rotor to get resistance as  $R'_2$  and so on, it can be seen that  $T_m$  remains same but slip at which it occurs increases to  $s'_m$  and so on. Similarly starting torque also increases to  $T'_{st}$  and so on.

If maximum torque  $T_m$  is required at start then  $s_m = 1$  as at start slip is always unity, so

$$s_m = \frac{R_2}{X_2} = 1$$

$$R_2 = X_2 \quad \dots \quad \text{Condition for getting } T_{st} = T_m$$

**Key Point** Thus by adding external resistance to rotor till it becomes equal to  $X_2$ , the maximum torque can be achieved at start.

It is represented by point A in the Fig. 5.14.1

**Example 5.14.1** A 6 pole, 50 Hz, 3-phase induction motor has a rotor resistance of 0.25  $\Omega$  per phase and a maximum torque of 10 N-m at 875 r.p.m. Calculate 1) The torque when the slip is 5 % and 2) The resistance to be added to the rotor circuit to obtain 60 % of the maximum torque at starting. Explain why two values are obtained for this resistance. Which value will be used ? The stator impedance is assumed to be negligible. AU : May-08, Marks 12

**Solution :**

**Solution :**  $P = 6$ ,  $f = 50$  Hz,  $R_2 = 0.25 \Omega$ ,  $T_{\max} = 10$  Nm,  $N_m = 875$  r.p.m.

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ r.p.m.}$$

$$s_m = \frac{N_s - N_m}{N_s} = \frac{1000 - 875}{1000} = 0.125 \quad \text{i.e. } 12.5 \%$$

But  $s_m = \frac{R_2}{X_2} \quad \text{i.e. } 0.125 = \frac{0.25}{X_2} \quad \text{i.e. } X_2 = \frac{0.25}{0.125} = 2 \Omega$

$$1) \quad T \propto \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

$$\therefore \frac{T_1}{T_{\max}} = \frac{s_1 E_2^2 R_2}{R_2^2 + (s_1 X_2)^2} \times \frac{R_2^2 + (s_m X_2)^2}{s_m E_2^2 R_2} \quad \dots s_1 = 5 \% = 0.05$$

$$\therefore \frac{T_1}{10} = \frac{0.05[(0.25)^2 + (0.125 \times 2)^2]}{0.125[(0.25)^2 + (0.05 \times 2)^2]}$$

$$\therefore T_1 = 6.8965 \text{ Nm} \quad \dots \text{Torque at slip } 5 \%$$

$$2) \quad T_{\text{st}} = \frac{E_2^2 R_2}{R_2^2 + X_2^2} \quad \text{as } s = 1 \text{ at start}$$

Now  $R_x$  is added to the rotor to make its resistance  $R'_2$  and  $T_{\text{st}} = 60 \%$  of  $T_{\max}$

$$\therefore \frac{T_{\text{st}}}{T_{\max}} = 0.6 = \frac{E_2^2 R'_2}{(R'_2)^2 + (X_2)^2} \times \frac{[R_2^2 + (s_m X_2)^2]}{s_m E_2^2 R_2}$$

$$\therefore 0.6 = \frac{R'_2[(0.25)^2 + (0.125 \times 2)^2]}{0.125[(R'_2)^2 + (2)^2] \times 0.25}$$

$$\therefore 0.01875 [(R'_2)^2 + 4] = 0.125 R'_2 \quad \text{i.e. } 0.01875 (R'_2)^2 - 0.125 R'_2 + 0.075 = 0$$

$$\therefore R'_2 = 6, \quad 0.6666$$

But  $R'_2 = R_2 + R_x \quad \text{i.e. } 0.666 = 0.25 + R_x$

$$\therefore R_x = 0.4166 \Omega$$

Mathematically there are two values of this resistance, one for motoring action and other for generating action. The higher of the two must be eliminated as it can produce large rotor copper losses and it gives absurd values for the slip at which maximum torque occurs. Hence smaller of the two is to be used.

**Example 5.14.2** An 8 pole, 50 Hz, 3 phase induction motor is running at 4% slip when delivering full load torque. It has standstill rotor resistance of 0.1  $\Omega$  and reactance of 0.6  $\Omega$  per phase. Calculate the speed of the motor if an additional resistance of 0.5  $\Omega$  per phase is inserted in the rotor circuit. Assume full load torque remains constant. AU : Dec.-08, Marks 8

**Solution :**  $P = 8$ ,  $f = 50$  Hz,  $s = 4\% = 0.04$ ,  $R_2 = 0.1 \Omega$ ,  $X_2 = 0.6 \Omega$

**Solution :**  $P = 8$ ,  $f = 50$  Hz,  $s = 4\% = 0.04$ ,  $R_2 = 0.1 \Omega$ ,  $X_2 = 0.6 \Omega$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{8} = 750 \text{ r.p.m.}$$

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

$$\text{New } R'_2 = R_2 + R_{ex} = 0.1 + 0.5 = 0.6 \Omega$$

The corresponding new slip be  $s'$ .

$$\therefore \frac{T_1}{T_2} = \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{(R'_2)^2 + (s' X_2)^2}{s' E_2^2 R'_2} = 1 \quad \dots \text{Torque constant}$$

$$\therefore s R_2 [(R'_2)^2 + (s' X_2)^2] = s' R'_2 [R_2^2 + (s X_2)^2]$$

$$\therefore 0.04 \times 0.1 [(0.6)^2 + (0.6s')^2] = s' \times 0.6 [(0.1)^2 + (0.04 \times 0.6)^2]$$

$$\therefore 0.36(s')^2 - 1.5864 s' + 0.36 = 0$$

$$\text{Solving,} \quad s' = 0.24 \quad \dots \text{Neglecting value greater than 1}$$

Hence the new speed of the motor is,

$$N' = N_s(1-s') = 750 (1-0.24) = 570 \text{ r.p.m}$$

**Example 5.14.3** A 40 kW, 3-phase, slip-ring induction motor of negligible stator impedance runs at a speed of 0.96 times synchronous speed at rated torque. The slip at maximum torque is four times the full-load value. If the rotor resistance of the motor is increased by 5 times, determine : a) The speed, power output and rotor copper loss at rated torque, b) The speed corresponding to maximum torque.

**Solution :**

**Solution :**  $P_{out} = 40 \text{ kW}$ ,  $N = 0.96 N_s$  on full load,  $s_m = 4 s_{fl}$

$$s_{fl} = \frac{N_s - N}{N_s} = \frac{N_s - 0.96 N_s}{N_s} = 0.04 \quad \text{i.e.} \quad 4 \%$$

$$\therefore s_m = 4 s_{fl} = 4 \times 0.04 = 0.16 \quad \text{i.e.} \quad 16 \%$$

a)  $R'_2 = 5 R_2$  ... Rotor resistance increased.

Due to this,  $s_m$  changes to  $s'_m$  though magnitude of  $T_{max}$  remains the same.

$$s'_m = \frac{R'_2}{X_2} \quad \text{and} \quad s_m = \frac{R_2}{X_2} \quad \text{i.e.} \quad \frac{s'_m}{s_m} = \frac{R'_2}{R_2} = 5$$

$$\therefore s'_m = 5 s_m = 5 \times 0.16 = 0.8 \quad \text{... New } s'_m$$

New full load slip will be  $s'_{fl}$

$$\therefore \frac{T_{F.L.}}{T_m} = \frac{2 s'_{fl} s'_m}{(s'_m)^2 + (s'_{fl})^2} \quad \text{and} \quad \frac{T_{F.L.}}{T_m} \text{ value remains same as before.}$$

$$\therefore \frac{T_{F.L.}}{T_m} = \frac{2 s_{fl} s_m}{(s_m)^2 + (s_{fl})^2} \quad \text{... Original value}$$

$$\therefore \frac{2 \times 0.04 \times 0.16}{(0.16)^2 + (0.04)^2} = \frac{2 \times s'_{fl} \times 0.8}{(0.8)^2 + (s'_{fl})^2}$$

$$\therefore (s'_{fl})^2 + (0.8)^2 = 3.4 s'_{fl} \quad \text{i.e.} \quad (s'_{fl})^2 - 3.4 s'_{fl} + 0.64 = 0$$

Solving,  $s'_{fl} = 0.2, 3.2$  ... Neglect higher value

$\therefore$  New full load slip  $s'_{fl} = 0.2$  i.e. 20 % with  $R'_2 = 5 R_2$

New speed  $N' = N_s(1 - s'_{fl}) = N_s(1 - 0.2) = 0.8 N_s \text{ r. p. m.}$

$$T_{F.L.} = \frac{P_{out}}{\left(\frac{2\pi N}{60}\right)} = \frac{40 \times 10^3}{\left(\frac{2\pi \times 0.96 N_s}{60}\right)} = \frac{397.887 \times 10^3}{N_s} \text{ Nm}$$

This torque must remain same on full load.

$$\therefore T_{F.L.} = \frac{P'_{out}}{\left(\frac{2\pi N'}{60}\right)} \quad \text{i.e.} \quad \frac{397.887 \times 10^3}{N_s} = \frac{P'_{out}}{\left(\frac{2\pi \times 0.8 N_s}{60}\right)}$$

$$\therefore P'_{out} = 33.33 \text{ kW} \quad \text{... New power output}$$

As  $1 : s : 1 - s$  is  $P_2 : P_c : P_m$

Neglecting friction loss,  $P_m = P'_{out}$

$$\therefore \frac{P_c}{P'_{out}} = \frac{s'_{fl}}{1 - s'_{fl}} \quad \text{i.e.} \quad P_c = \frac{33.33 \times 10^3 \times 0.2}{(1 - 0.2)} = 8.333 \text{ kW}$$

These are copper losses at rated torque with  $R'_2$ .

b)  $N'_m = N_s(1 - s'_m) = N_s(1 - 0.8) = 0.2 N_s \text{ r. p. m.}$

This is the speed corresponding to  $T_{max}$  with  $R'_2$ .

## Losses and Efficiency of Induction Motor

There are two types of losses occur in [three phase induction motor](#). These losses are,

1. Constant or fixed losses,
2. Variable losses.

### Constant or Fixed Losses

Constant losses are those losses which are considered to remain constant over normal working range of [induction motor](#). The fixed losses can be easily obtained by performing no-load test on the three phase induction motor. These losses are further classified as-

1. Iron or core losses,
2. Mechanical losses,
3. Brush friction losses.

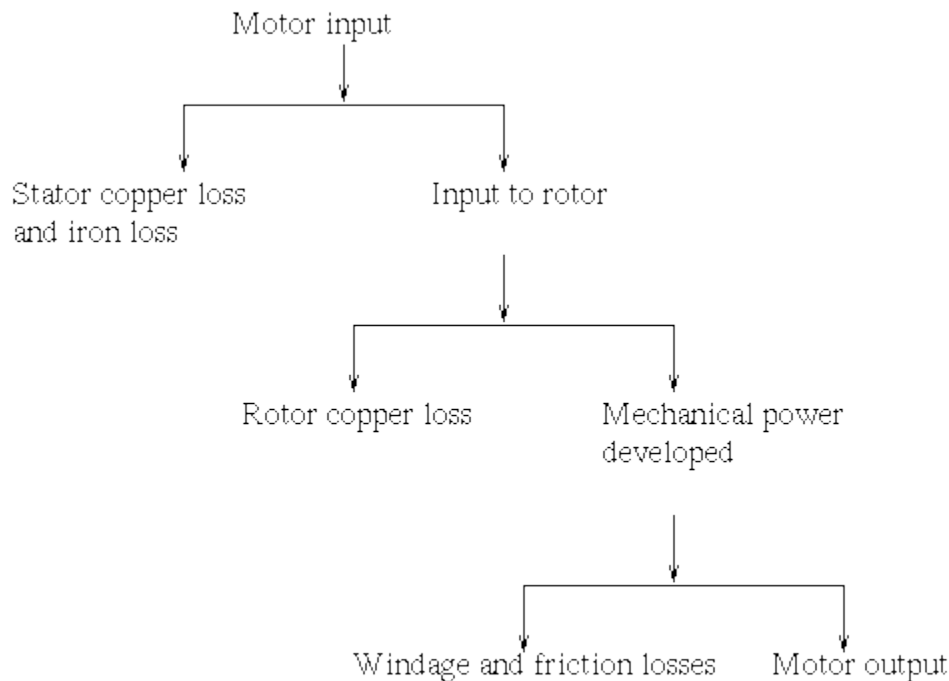
### Iron or Core Losses

Iron or core losses are divided into hysteresis and [eddy current losses](#). Eddy current losses are reduced by laminating the core, which increases [resistance](#) and decreases [eddy currents](#). Hysteresis losses are minimized using high-grade silicon steel. Core losses depend on supply voltage frequency. The stator frequency is the supply frequency, while the rotor frequency is slip times the supply frequency, usually much lower. For a 50 Hz stator frequency, the rotor frequency is about 1.5 Hz due to a typical slip of 3%. Thus, rotor core loss is usually negligible compared to stator core loss during operation.

### Mechanical and Brush Friction Losses

Mechanical losses occur at the bearings, and brush friction losses occur in wound rotor induction motors. These losses are minimal at start-up but increase with speed. In [three phase induction motors](#), the speed generally stays constant, so these losses also remain nearly constant.

### Variable Losses



Variable losses, also known as copper losses, occur due to the [current](#) in the stator and rotor windings. As the load changes, the current and thus these losses change. These losses are determined by performing a blocked rotor test on a three-phase induction motor. The main function of an [induction motor](#) is to convert [electrical power](#) into mechanical power, which involves different stages of power flow.

This power flowing through different stages is shown by power flow diagram. As we all know the input to the [three phase induction motor](#) is three phase supply. So, the three phase supply is given to the stator of three phase induction motor.

Let,  $P_{in}$  = electrical power supplied to the stator of three phase induction motor,

$V_L$  = line [voltage](#) supplied to the stator of three phase induction motor,

$I_L$  = line current,

$\cos\phi$  = [power factor](#) of the three phase induction motor.

Electrical power input to the stator,  $P_{in} = \sqrt{3}V_L I_L \cos\phi$

A part of this power input is used to supply stator losses which are stator iron loss and stator copper loss. The remaining power i.e (input electrical power – stator losses) are supplied to rotor as rotor input.

So, rotor input  $P_2 = P_{in} - \text{stator losses (stator copper loss and stator iron loss)}$ .

Now, the rotor has to convert this rotor input into mechanical energy but this complete input cannot be converted into mechanical output as it has to supply rotor losses. As explained earlier the rotor losses are of two types rotor iron loss and rotor copper loss. Since the iron loss depends upon the rotor frequency, which is very small when the rotor rotates, so it is usually neglected. So, the rotor has only rotor copper loss. Therefore the rotor input has to supply these rotor copper losses. After supplying the rotor copper losses, the remaining part of Rotor input,  $P_2$  is converted into mechanical power,  $P_m$ .

Let  $P_c$  be the rotor copper loss,

$I_2$  be the rotor current under running condition,

$R_2$  is the rotor resistance,

$P_m$  is the gross mechanical power developed.

$$P_c = 3I_2^2 R_2$$

$$P_m = P_2 - P_c$$

Now this mechanical power developed is given to the load by the shaft but there occur some mechanical losses like friction and windage losses. So, the gross mechanical power developed has to be supplied to these losses. Therefore the net output power developed at the shaft, which is finally given to the load is  $P_{out}$ .

$$P_{out} = P_m - \text{Mechanical losses (friction and windage losses)}.$$

$P_{out}$  is called the shaft power or useful power.

### Efficiency of Three Phase Induction Motor

Efficiency is defined as the ratio of the output to that of input,

$$\text{Efficiency, } \eta = \frac{\text{output}}{\text{input}}$$

Rotor efficiency of the three phase induction motor ,

$$= \frac{\text{rotor output}}{\text{rotor input}}$$

= Gross mechanical power developed / rotor input

$$= \frac{P_m}{P_2}$$

Three phase induction motor efficiency,

$$= \frac{\text{power developed at shaft}}{\text{electrical input to the motor}}$$

Three phase induction motor efficiency

$$\eta = \frac{P_{out}}{P_{in}}$$

### Complete Equivalent Circuit of 3 Phase Induction Motor

The complete equivalent circuit refers to the exact equivalent model obtained by transferring the rotor parameters to the stator side using the turns ratio. This yields a circuit identical to a two-winding transformer incorporating all stator and transformed rotor variables.

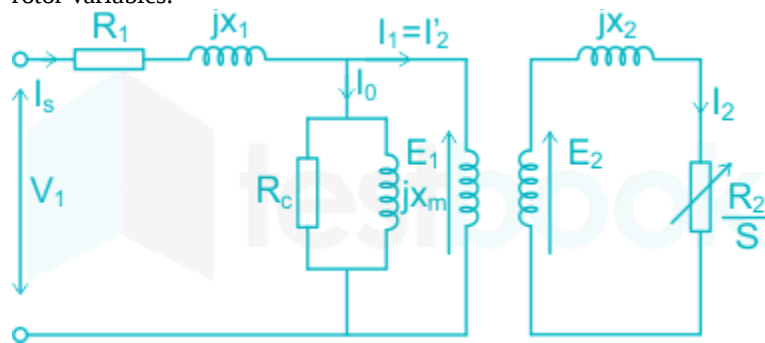


Fig- Equivalent circuit diagram of 3 phase induction

motor

Here,  $R_1$  is the winding resistance of the stator.

$X_1$  is the inductance of the stator winding.

$R_c$  is the core loss component.

$X_M$  is the magnetizing reactance of the winding.

$R_2/s$  is the power of the rotor, which includes output mechanical power and copper loss of the rotor.

The complete equivalent circuit of an induction motor includes all the parameters that affect the motor's performance:

- Stator [resistance](#) ( $R_1$ ) and leakage reactance ( $X_1$ ): Represent the resistance and reactance of the stator windings.
- Rotor resistance ( $R_2$ ) and leakage reactance ( $X_2$ ): Represent the resistance and reactance of the rotor windings, referred to as the stator side.
- Magnetizing branch ( $R_m$  and  $X_m$ ): Represents the core losses and magnetizing inductance.
- Rotor slip ( $s$ ): Indicates the difference between synchronous speed and actual rotor speed.

An [induction motor](#) is a well-known device that operates on the same principle as a transformer, earning it the nickname "rotating transformer." When an EMF is applied to its stator, electromagnetic induction induces a voltage in its rotor. Thus, an induction motor can be considered a transformer with a rotating secondary. In this analogy, the transformer's primary winding corresponds to the induction motor's stator winding, and the secondary winding corresponds to the rotor.

An induction motor always runs below the synchronous or full load speed. The relative difference between the synchronous speed and the rotor's actual speed is known as slip, denoted by  $S$ .

$$S = \frac{N_s - N}{N_s} \text{ Where,}$$

$N_s$  is synchronous speed of rotation which is given by

$$N_s = \frac{120f}{P}$$

Where,  $f$  is the frequency of the supply voltage.

$P$  is the number of poles of the machine.

The rotor impedance is given by the equation below:

$$Z_{2S} = R_2 + jX_{2S} \text{ or}$$

$$Z_{2S} = R_2 + jsX_{20}$$

Per phase rotor current is given by

$$I_{2S} = \frac{E_{2S}}{Z_{2S}}$$

$$I_{2S} = \frac{sE_{20}}{R_2 + jsX_{20}} \dots \dots (5)$$

Here,  $I_2$  is the slip frequency current produced by a slip [frequency](#) induced voltage  $sE_{20}$  acting in the rotor circuit having an impedance per phase of  $(R_2 + jsX_{20})$ .

Now, dividing the equation (5) by slip  $s$  we get the following equation:

$$I_{2s} = \frac{E_{20}}{\frac{R_2}{s} + jX_{20}} \dots\dots (6)$$

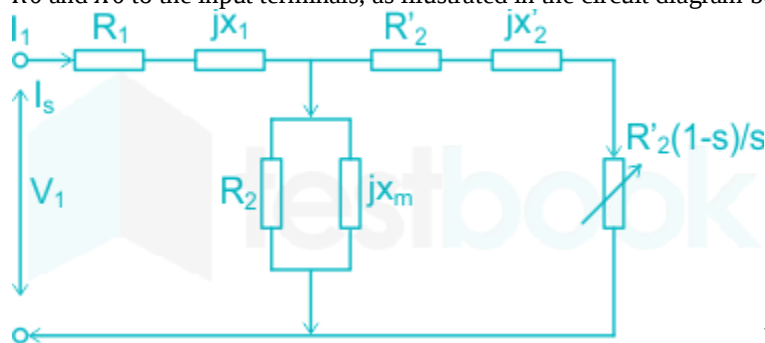
The rotor resistance  $R_2$  is constant, while the leakage reactance is variable, denoted as  $sX_2''$ . Similarly, the rotor circuit below includes a constant leakage reactance  $X_2''$  and a variable resistance  $R_2/s$ .

Equation (6) describes the secondary circuit of an imaginary transformer with a constant voltage ratio and the same frequency on both sides. This hypothetical stationary rotor carries the same current as the actual rotating rotor, allowing the transfer of the secondary rotor impedance to the primary stator side.

### Approximate Equivalent Circuit of 3 Phase Induction Motor

In practical analysis, the approximate equivalent circuit simplifies the complete model by neglecting certain elements to make calculations easier:

- Neglecting stator resistance ( $R_1$ ) or rotor reactance ( $X_2$ ): Simplifies the circuit, focusing mainly on the magnetizing and core loss components.
  - Combining series elements: Some series resistances and reactances may be combined for a more straightforward representation.
- The [equivalent circuit](#) is further simplified by transferring the shunt impedance branches  $R_0$  and  $X_0$  to the input terminals, as illustrated in the circuit diagram below:



**Fig - Approx Equivalent Circuit**

The approximate circuit is based on the assumption that  $V_1 = E_1 = E_2'$ . In this circuit, the only component that varies with slip is the resistance, while all other quantities remain constant. At any given slip  $s$ , the impedance beyond AA' is given by the following equations:

$$Z_{AA'} = \left( R_1 + \frac{R'_2}{s} \right) + j(X_1 + X'_2) \dots\dots (7)$$

$$I'_2 = \frac{V_1}{Z_{AA'}} \dots\dots (8)$$

Putting the value of  $Z_{AA'}$  from equation (7) in equation (8) we get,

$$I'_2 = \frac{V_1}{\left( R_1 + \frac{R'_2}{s} + j(X_1 + X'_2) \right)}$$

$$T_d = \frac{P_g}{\omega_s} \quad \text{or}$$

$$T_d = \frac{V_1^2 (R'_2/s)}{\omega_s \left[ \left( R_1 + \frac{R'_2}{s} \right)^2 + (X_1 + X'_2)^2 \right]} \dots\dots\dots (16)$$

The equation above represents the torque equation of an induction motor. The approximate equivalent circuit model is the standard used for all performance calculations of an induction motor.

### Phasor Diagram of Three Phase Induction Motor

In a 3-phase induction motor, the stator winding is connected to 3-phase supply and the rotor winding is short-circuited. The energy is transferred magnetically from the stator winding to the short-circuited, rotor winding. Therefore, an induction motor may be considered to be a transformer with a rotating secondary (short-circuited). The stator winding corresponds to transformer primary and the rotor winding corresponds to transformer secondary. In view of the similarity of the flux and voltage conditions to those in a transformer, one can expect that the equivalent circuit of an induction motor will be similar to that of a transformer. Fig. 3.8 shows the equivalent circuit per phase for an induction motor. Let discuss the stator and rotor circuits separately.

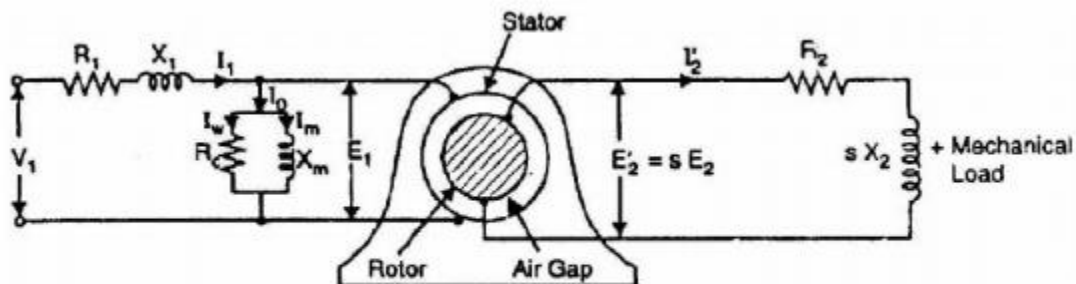


Fig. 3.8

**Stator circuit.** In the stator, the events are very similar to those in the transformer primary. The applied voltage per phase to the stator is  $V_1$  and  $R_1$  and  $X_1$  are the stator resistance and leakage reactance per phase respectively. The applied voltage  $V_1$  produces a magnetic flux which links the stator winding (i.e., primary) as well as the rotor winding (i.e., secondary). As a result, self-induced e.m.f.  $E_1$  is induced in the stator winding and mutually induced e.m.f.  $E'_2 (= s E_2 = s K E_2$  where  $K$  is transformation ratio) is induced in the rotor winding. The flow of stator current  $I_1$  causes voltage drops in  $R_1$  and  $X_1$ .

$$V_1 = -E_1 + I_1(R_1 + jX_1) \dots\dots \text{phasor sum}$$

When the motor is at no-load, the stator winding draws a current  $I_0$ . It has two components viz.,

(i) which supplies the no-load motor losses and (ii) magnetizing component  $I_m$  which sets up magnetic flux in the core and the air gap. The parallel combination of  $R_c$  and  $X_m$ , therefore, represents the no-load motor losses and the production of magnetic flux respectively.

$$I_0 = I_w + I_m$$

**Rotor circuit.** Here  $R_2$  and  $X_2$  represent the rotor resistance and standstill rotor reactance per phase respectively. At any slip  $s$ , the rotor reactance will be  $sX_2$ . The induced voltage/phase in the rotor is  $E'_2 = sE_2 = sK E_1$ . Since the rotor winding is short-circuited, the whole of e.m.f.  $E'_2$  is used up in circulating the rotor current  $I'_2$ .

$$E'_2 = I'_2 (R_2 + jsX_2)$$

The rotor current  $I'_2$  is reflected as  $I''_2 (= K I'_2)$  in the stator. The phasor sum of  $I''_2$  and  $I_0$  gives the stator current  $I_1$ .

It is important to note that input to the primary and output from the secondary of a transformer are electrical. However, in an induction motor, the inputs to the stator and rotor are electrical but the output from the rotor is mechanical. To facilitate calculations, it is desirable and necessary to replace the mechanical load by an equivalent electrical load. We then have the transformer equivalent circuit of the induction motor.

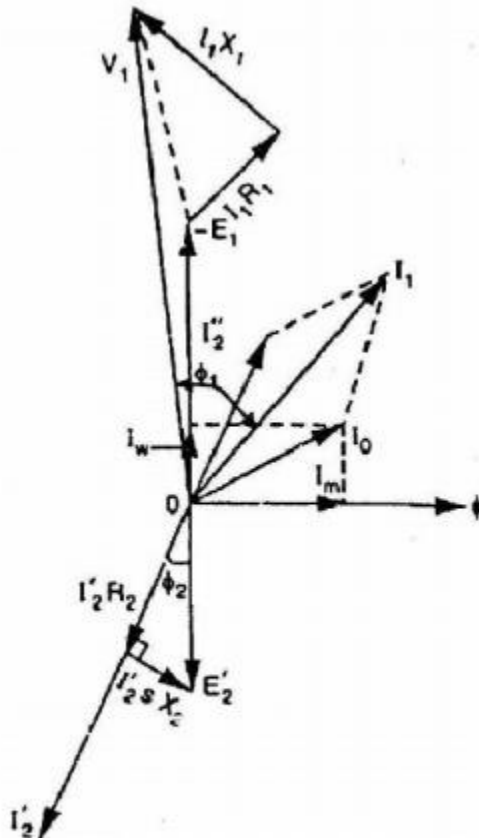


Fig: 3.9

It may be noted that even though the frequencies of stator and rotor currents are different, yet the magnetic fields due to them rotate at synchronous speed  $N_s$ . The stator currents produce a magnetic flux which rotates at a speed  $N_s$ . At slip  $s$ , the speed of rotation of the rotor field relative to the rotor surface in the direction of rotation of the rotor is

$$= \frac{120 f'}{P} = \frac{120 s f}{P} = s N_s$$

But the rotor is revolving at a speed of  $N$  relative to the stator core. Therefore, the speed of rotor field relative to stator core

$$= s N_s + N = (N_s - N) + N = N_s$$

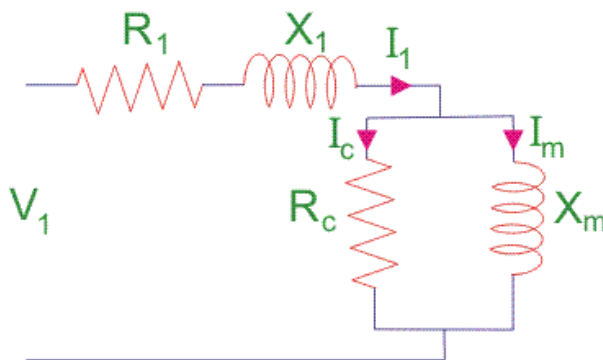
Thus no matter what the value of slip  $s$ , the stator and rotor magnetic fields are synchronous with each other when seen by an observer stationed in space. Consequently, the 3-phase induction motor can be regarded as being equivalent to a transformer having an air-gap separating the iron portions of the magnetic circuit carrying the primary and secondary windings. Fig. 3.9 shows the phasordiagram of induction motor.

#### Theory of No Load Test of Induction Motor

The impedance of the motor's magnetizing path is high, which limits current flow. Thus, a small current is applied, causing a minor drop in stator impedance and ensuring the rated voltage across the magnetizing branch. This drop and the power loss due to stator resistance are negligible compared to the applied voltage. Hence, it is assumed that all drawn power converts into core loss. The air gap in the magnetizing branch in an [induction motor](#) increases the exciting current, making the no-load stator  $I^2R$  loss recognizable.

It's crucial to ensure the current does not exceed its rated value to prevent the rotor from accelerating beyond safe limits.

The test is conducted at poly-phase voltages and rated frequency applied to the stator terminals. After the motor runs for a while and the bearings are fully lubricated, readings of applied voltage, input current, and input power are taken. Rotational loss is calculated by subtracting the stator  $I^2R$  losses from the input power.



#### Calculation of No Load Test of Induction Motor

Let the total input power supplied to [induction motor](#) be  $W_0$  watts.

$$W_0 = \sqrt{3} V_1 I_0 \cos \Phi_0$$

Where,

$V_1$  = line voltage

$I_0$  = No load input current

Rotational loss =  $W_0 - S_1$

Where,

$S_1$  = stator winding loss =  $N_{ph} I^2 R_1$

$N_{ph}$  = Number phase

The various losses like winding loss, core loss, and rotational loss are fixed losses which can be calculated by

Stator winding loss =  $3 I_0^2 R_1$

Where,

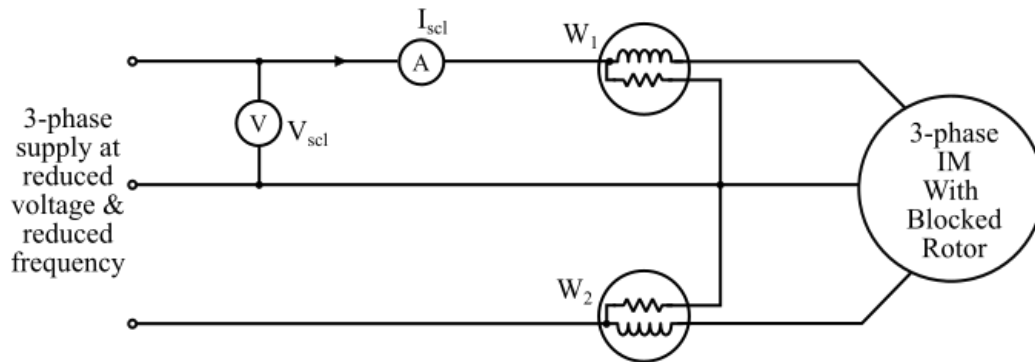
$I_0$  = No load input current

$R_1$  = Resistance of the motor

Core loss =  $3 G_0 V^2$

### Induction Motor Blocked Rotor Test

The figure shows the circuit diagram for the blocked rotor test of an induction motor. The blocked rotor test enables us to determine the efficiency and the circuit parameters of the equivalent circuit of a 3-phase induction motor.



In the blocked rotor test, the shaft of the motor is locked so that it cannot rotate and the rotor winding is short circuited. In a slip-ring induction motor, the rotor winding is short-circuited through the slip-rings while in a squirrel cage induction motor, the rotor bars are permanently short-circuited with the help of end rings.

In the blocked rotor test, a reduced voltage at reduced frequency is applied to the stator of the induction motor through a 3-phase autotransformer so that the rated current flows in the stator winding. The readings obtained are given as follows –

- Total input power on short circuit is measured by the two wattmeter method and is given by the algebraic sum of the two wattmeter readings. Here, a reduced voltage is applied to the stator and rotation of the rotor is not allowed, thus, the core and mechanical losses are negligible. Therefore, the total input power in the blocked rotor test is equal to the sum of stator copper losses and rotor copper losses for all the 3-phases.
- The ammeter reads the value of line current ( $I_{sc1}$ ) with blocked rotor which corresponds to the short circuit condition.
- The voltmeter reads the value of reduced line voltage with blocked rotor ( $V_{sc1}$ ).

Therefore, the input power under blocked rotor condition is given by,

$$P_{sc} = \sqrt{3} V_{sc} I_{sc} \cos \varphi_{sc}$$

Where,  $\cos \varphi_{sc}$  is the power factor under blocked rotor condition.

The equivalent resistance of the motor referred to the stator is

$$R_{e1} = \frac{P_{sc.ph}}{I_{sc.ph}^2}$$

Where,

- $P_{sc.ph}$  = Per Phase Power Under Blocked Rotor Condition.
- $I_{sc.ph}$  = Per Phase Current Under Locked Rotor Condition.

Equivalent impedance of the motor referred to the stator is

$$Z_{e1} = \frac{P_{sc.ph}}{I_{sc.ph}^2}$$

And the equivalent reactance of the motor referred to the stator is

$$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$

### Construction of Circle Diagram

1. Draw horizontal axis OX and vertical axis OY. Here the vertical axis represents the voltage reference.
2. With suitable scale, draw phasor OA with length corresponding to  $I_0$  at an angle  $\phi_0$  from the vertical axis. Draw a horizontal line AB.
3. Draw OS equal to  $I_{SV}$  at an angle  $\phi_{sc}$  and join AS.
4. Draw the perpendicular bisector to AS to meet the horizontal line AB at C.
5. With C as centre, draw a semi circle passing through A and S. This forms the circle diagram which is the locus of the input current.
6. From point S, draw a vertical line SL to meet the line AB.
7. Fix the point K as below.

For wound rotor machines where equivalent rotor resistance  $R_2'$  can be found out:

Divide SL at point K so that SK: KL = equivalent rotor resistance : stator resistance.

For squirrel cage rotor machines:

Find Stator copper loss using  $I_{SV}$  and stator winding resistance  $R_1$ .

Rotor copper loss = total copper loss – stator copper loss.

Divide SL at point K so that SK : KL = rotor copper loss : stator copper loss

*Note:* If data for separating stator copper loss and rotor copper loss is not available then assume that stator copper loss is equal to rotor copper loss. So divide SL at point K so that SK = KL.

8. For a given operating point P, draw a vertical line PEFGD as shown.

Then, PD = input power, PE = output power, EF = rotor copper loss, FG = stator copper loss, GD = constant loss (iron loss + mechanical loss)

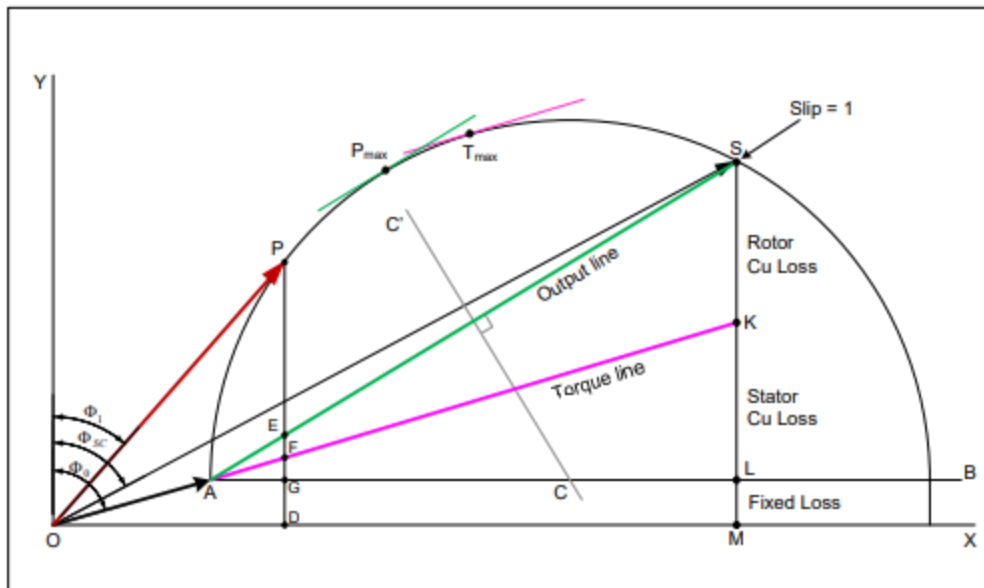
9. Efficiency of the machine at the operating point P,  $\eta = \frac{PE}{PD}$

10. Power factor of the machine at operating point P =  $\cos\Phi_1$

11. Slip of the machine at the operating point P,  $s = \frac{EF}{PF}$

12. Starting torque at rated voltage (in syn. watts) = SK

13. To find the operating points corresponding to maximum power and maximum torque, draw tangents to the circle diagram parallel to the output line and torque line respectively. The points at which these tangents touch the circle are respectively the maximum power point ( $T_{max}$ ) and maximum torque point ( $P_{max}$ )



# Module V

## Starting and speed control of three-phase induction motors

### DOL Starter (Direct On Line Starter) Diagram And Working Principle

A **DOL starter** (also known as a **direct on line starter** or **across the line starter**) is a method of starting a [3 phase induction motor](#). In a DOL Starter, an [induction motor](#) is connected directly across its 3-phase supply, and the DOL starter applies the full line voltage to the motor terminals.

Even with direct connection to the power supply, the [motor](#) remains protected. A DOL motor starter includes protective devices and, in some models, condition monitoring features. Below is a wiring diagram of a DOL starter: Since the DOL starter connects the motor directly to the main supply line, the motor draws a very high [inrush current](#) compared to the full load [current](#) of the motor (up to 5-8 times higher). The value of this large current decreases as the motor reaches its rated speed.

A direct on line starter can only be used in circumstances when the high inrush current of the motor does not cause an excessive voltage drop in the supply circuit. If a high voltage drop needs to be avoided, a [star delta starter](#) should be used instead. Direct on line starters are commonly used to start small motors, especially [3 phase squirrel cage induction motors](#).

$$I_a = \frac{(V - E)}{R_a}$$

As we know, the equation for armature current in the motor. The value of back emf (E) depends upon speed (N), i.e. E is directly proportional to N. At start-up, the back electromotive force (E) is zero, resulting in a very high initial current. Small motors with longer axial lengths and smaller diameters accelerate quickly due to this effect.

Hence, speed increases and thus the value of armature current decreases rapidly. Therefore, small rating motors smoothly run when it is connected directly to a 3-phase supply.

If we connect a large motor directly across 3-phase line, it would not run smoothly and will be damaged, because it does not get accelerated as fast as a smaller motor since it has short axial length and larger diameter more massive rotor. However, for large-rated motors, we can use an oil-immersed DOL starter.

#### Stator Resistance Starting Method

In this method, external resistors are connected in series with each phase of the stator winding during startup. These resistors cause a voltage drop across them, resulting in a reduced voltage being applied to the motor terminals. Consequently, the starting current is lowered. As the motor accelerates, these external resistors are gradually removed from the stator circuit in steps. Once the motor reaches its rated speed, the resistors are completely eliminated, and the full line voltage is applied directly across the motor terminals.

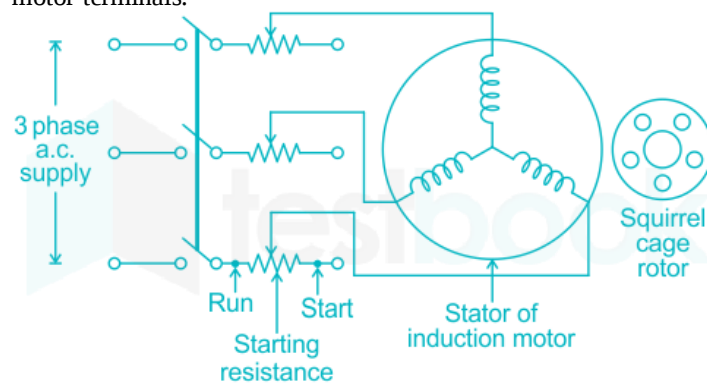


Fig- Circuit Diagram of Stator Resistance Starting

#### Method

This method has two drawbacks. First, the reduced voltage during startup decreases the starting torque, thereby extending the acceleration time. Second, a significant amount of [power](#) is lost in the starting resistors.

#### Advantages

- Reduced inrush current
- Lower starting torque, reducing mechanical stress

#### Disadvantages

- Power loss in resistors
- Reduced efficiency during start-up
- Requires additional components

#### Autotransformer Starting Method

An [autotransformer](#) is used to step down the supply voltage during startup, via tapping arrangements. This limits the starting current. Once the motor picks up sufficient speed, it is reconnected directly to the supply voltage. This method provides a controlled and gradual increase in voltage, thereby limiting the starting current and torque.

Auto-transformers, also known as auto-starters, can be used for both star-connected and delta-connected squirrel cage motors. Essentially, they are three-phase step-down transformers with various taps that allow the motor to start at different voltage levels, such as 50%, 65%, or 80% of the line voltage. With auto-transformer starting, the current drawn from the supply line is always less than the motor current by a factor corresponding to the transformation ratio.

At startup, the switch is in the "start" position, applying a reduced voltage (selected via a tap) across the stator. As the motor reaches an appropriate speed, typically around 80% of its rated speed, the auto-transformer automatically disconnects from the circuit, and the switch moves to the "run" position. The switch that changes the connection from start to run can be either air-break (for small motors) or oil-immersed (for large motors). Additionally, auto-starters are equipped with provisions for no-voltage and overload protection, often incorporating time delay circuits.

#### Advantages

- Reduced inrush current
- Smooth acceleration
- Higher efficiency compared to stator resistance starting

#### Disadvantages

- More complex and expensive than D.O.L. starting
- Requires additional space for the autotransformer

#### Star-Delta Starting

The stator winding is initially connected in star configuration to avail reduced phase voltage during startup. Once the rated speed is achieved, apply the full line voltage. It is reconfigured to delta for full voltage operation. A TPDT (triple pole double throw) switch is used for star-delta changeover. In star-delta starting, the motor windings are initially connected in a star configuration, reducing the voltage applied to each winding to  $1/\sqrt{3}$  (58%) of the line voltage.

The starting torque will be  $1/3$  times that will be for delta-connected winding. Hence a star-delta starter is equivalent to an auto-transformer of ratio  $1/(\sqrt{3})$  or 58% reduced voltage.

- The star delta starter method of starting three three-phase induction motors is very common and widely used methods.
- In this method, at starting motor will be in star connection and runs at delta connected stator windings.

#### Delta starting

$$I_{st}(\text{phase}) = \frac{V_1}{Z_{01}}$$

$$\sqrt{3}$$

$$I_{st(\text{phase})} = I_{st}(\text{line}) = \sqrt{3} \frac{V_1}{Z_{01}}$$

#### Star connected starting

$$I_{st(\text{line})} = I_{st}(\text{phase})$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The ratio of the current of star connection and delta connection is  $1/3$

Therefore starting current is reduced to  $1/3$ .

The ratio of the current of star connection and delta

connection is  $1/3$

Therefore starting current is reduced to  $1/3$ .

#### Advantages

- Reduced starting current
- Lower starting torque
- Cost-effective for motors with star-delta capability

#### Disadvantages

- Lower starting torque may not be sufficient for high-load applications
- Transition from star to delta can cause mechanical and electrical transients

#### Rotor-Resistance Starting Method

Rotor [resistance](#) starting is used for wound rotor or slip ring induction motors. External resistors are connected to the rotor circuit during start-up, increasing the rotor resistance and reducing the starting current. Initially, the full starting resistance is connected, which reduces the supply current to the stator. As the rotor begins to rotate, the resistances in the rotor circuit are gradually removed as the motor's speed increases. When the motor reaches its rated full load speed, the starting resistances are completely cut out, and the slip rings are short-circuited.

#### Advantages

- High starting torque
- Smooth acceleration
- Adjustable starting characteristics

#### Disadvantages

- Power loss in external resistors
- More complex and expensive
- Requires maintenance of slip rings and brushes

#### Starting of Slip-Ring Induction Motors

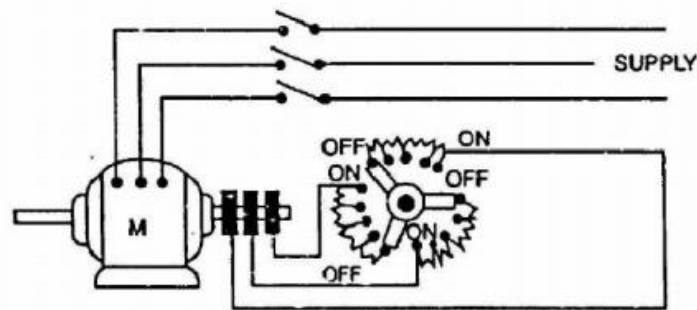


Fig: 3.27

#### Starting of Slip-Ring Induction Motors

Slip-ring motors are invariably started by rotor resistance starting. In this method, a variable star-connected rheostat is connected in the rotor circuit through slip rings and full voltage is applied to the stator winding as shown in Fig: 3.27.

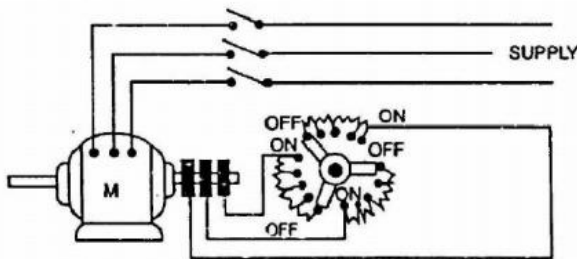


Fig: 3.27

(i) At starting, the handle of rheostat is set in the OFF position so that maximum resistance is placed in each phase of the rotor circuit. This reduces the starting current and at the same time starting torque is increased.

(ii) As the motor picks up speed, the handle of rheostat is gradually moved in clockwise direction and cuts out the external resistance in each phase of the rotor circuit. When the motor attains normal speed, the change-over switch is in the ON position and the whole external resistance is cut out from the rotor circuit.

#### V/f Control Method

##### Speed control of induction motor by changing supply frequency:

By varying the supply voltage and frequency proportionally keeping the V/f ratio constant using a variable frequency drive, smooth speed control can be achieved. This is a commonly used method.

##### Speed Control of Slip Ring Induction Motor

Speed control of slip ring induction motor involves varying the rotor [resistance](#) to change the slip and consequently the motor speed. Slip is defined as  $(\text{Synchronous speed} - \text{Actual motor speed}) / \text{Synchronous speed}$ .

Methodology:

- In no-load condition, minimum external resistance is connected resulting in maximum slip and minimum speed.
- As the load increases, additional external resistance is inserted in steps to maintain the speed.

### **Voltage Control Method**

The voltage control method adjusts the speed of an induction motor by varying the voltage supplied to the motor.

This method is simple and economical for small motors and is primarily used where a broad range of speed control is not required. It operates under the principle that the torque produced in an induction motor is directly proportional to the square of the applied voltage.

Reducing the supply voltage decreases the [magnetic flux](#), thereby lowering the torque and speed. However, this method is not efficient for large motors or applications requiring precise speed control due to its limited range and impact on torque.

### **Frequency Control Method**

#### **Speed control of 3 phase induction motor using vfd:**

The frequency control method varies the speed of an induction motor by changing the frequency of the electrical power supplied to the motor while keeping the supply voltage constant.

This method provides a wide range of precise speed control, making it suitable for applications requiring variable speeds and high efficiency. As the speed of an induction motor is directly proportional to the supply frequency, adjusting the frequency allows for smooth control over the motor's speed without significantly affecting the torque.

This control strategy is commonly implemented using electronic variable-frequency drives (VFDs), offering flexibility and energy efficiency in motor operations.

### **Stator Resistance Method**

#### **Speed control of induction motor by stator voltage control:**

The stator resistance method controls the speed of an induction motor by adding external resistance in series with the stator winding.

This method primarily affects the starting torque and speed of the motor by introducing voltage drop across the added resistance, which changes the torque-speed characteristic of the motor.

While it's an uncomplicated and low-cost method for speed control, particularly for slip-ring induction motors, its application is limited due to reduced efficiency and significant power loss in the resistance.

### **Rotor Resistance Control Method**

#### **Speed control of slip ring induction motor by rotor resistance:**

In slip-ring induction motors, the rotor resistance control method adjusts the motor's speed by adding external resistance in series with the rotor windings.

This method effectively controls the speed at constant torque, making it beneficial for applications requiring high starting torque and smooth speed control under varying load conditions.

Since increasing the rotor resistance increases the slip, the motor can operate at lower speeds without losing torque. However, similar to the stator resistance method, this approach results in power losses in the external resistors, affecting overall efficiency.

### **Slip Power Recovery Method**

The slip power recovery method enhances the efficiency of controlling the speed of slip-ring induction motors by recovering the slip power, which would otherwise be lost.

This method involves collecting the power from the rotor circuit through slip rings and either feeding it back to the supply or using it for another purpose. It employs sophisticated power electronic devices to convert the collected power into an appropriate form.

This method provides significant energy savings and improved speed control range without compromising torque, making it ideal for high-power applications.

### **Crawling of Induction Motor**

It has been observed that squirrel cage type induction motor has a tendency to run at very low speed compared to its synchronous speed, this phenomenon is known as crawling. The resultant speed is nearly  $1/7^{\text{th}}$  of its synchronous speed. Now the question arises why this happens? This action is due to the fact that [harmonics](#) fluxes produced in the gap of the stator winding of odd harmonics like  $3^{\text{rd}}$ ,  $5^{\text{th}}$ ,  $7^{\text{th}}$  etc. These harmonics create additional torque fields in addition to the synchronous torque.

The torque produced by these harmonics rotates in the forward or backward direction at  $N_s/3$ ,  $N_s/5$ ,  $N_s/7$  speed respectively. Here we consider only  $5^{\text{th}}$  and  $7^{\text{th}}$  harmonics and rest are neglected. The torque produced by the  $5^{\text{th}}$  harmonic rotates in the backward direction. This torque produced by fifth harmonic which works as a braking action is small in quantity, so it can be neglected. Now the seventh harmonic produces a forward rotating torque at synchronous speed  $N_s/7$ . Hence, the net forward torque is equal to the sum of the torque produced by  $7^{\text{th}}$  harmonic and fundamental torque. The torque produced by  $7^{\text{th}}$  harmonic reaches its maximum positive value just below  $1/7$  of  $N_s$  and at this point slip is high. At this stage motor does not reach up to

its normal speed and continue to rotate at a speed which is much lower than its normal speed. This causes crawling of the motor at just below  $1/7$  synchronous speed and creates the racket. The other speed at which motor crawls is  $1/13$  of synchronous speed.

### **Cogging of Induction Motor**

Cogging is a characteristic of [induction motor](#) that occurs when the motor fails to start, sometimes due to low supply [voltage](#). However, the primary cause is the locking of the stator slots with the rotor slots, preventing the motor from turning.

In induction motors, both the stator and rotor contain a series of slots. When these slots align perfectly due to matching numbers, the magnetic path's reluctance drops to its minimum, which can prevent the motor from starting.

Cogging in induction motors is defined as the locking of the stator and rotor slots, preventing the motor from starting. Another cause of cogging is torque modulation, which happens when the supply voltage's harmonic frequencies align with the slot frequencies, enhancing the locking effect. This issue is also referred to as magnetic teeth locking.

Methods to overcome Cogging

This problem can be easily solved by adopting several measures. These solutions are as follows:

- The number of slots in rotor should not be equal to the number of slots in the stator.
- Skewing of the rotor slots, that means the stack of the rotor is arranged in such a way that it angled with the axis of the rotation.
- Single Phase Induction Motor: Working, Construction, Types & Equivalent Circuit

### **What is a Single Phase Motor?**

This is a broader term that encompasses all motor types that operate on a single-phase power supply. This category includes various types of motors, such as single-phase induction motors, shaded pole motors, split-phase motors, capacitor-start motors, and others. Each of these types uses a different method to start and run the motor. In a three-phase motor, the rotating magnetic field is naturally created by the three-phase power supply. However, single-phase power does not inherently produce a rotating magnetic field. Therefore, single phase motors need additional mechanisms to start.

### **What is Single Phase Induction Motor?**

A single-phase induction motor consists of a single-phase winding which is mounted on the stator of the motor and a cage winding placed on the rotor. A pulsating [magnetic field](#) is produced when the stator winding of the single-phase induction motor is energized by a single phase supply. It consists of two main parts - a stator having one or more electromagnetic coils supplied with single-phase AC [power](#) and a rotor attached to the output shaft. The rotor is made up of bars short-circuited using rings at both ends, similar to a squirrel cage rotor of a three phase induction motor. The single phase supply produces a pulsating rotating magnetic field in the stator due to which current is induced in the rotor coils. However, due to the pulsating nature of the field, the rotor fails to produce [torque](#) for self-starting. Various techniques are therefore used to derive a quasi-rotating magnetic field for enabling self-starting capability in single phase induction motors. This is also known as a single phase motor.

### **Working Principle of Single Phase Induction Motor**

The working principle of a single phase induction motor is based on [electromagnetic induction](#). When the stator winding is connected to a single-phase AC supply, it produces a pulsating magnetic field. The word Pulsating means that the field builds up in one direction falls to zero and then builds up in the opposite direction. Under these conditions, the rotor of an induction motor does not rotate. Hence, a single phase induction motor is not self-starting. It requires some special starting means. If the 1-phase stator winding is excited and the rotor of the motor is rotated by an auxiliary means the starting device is removed, and the motor continues to rotate in the direction in which it is started.

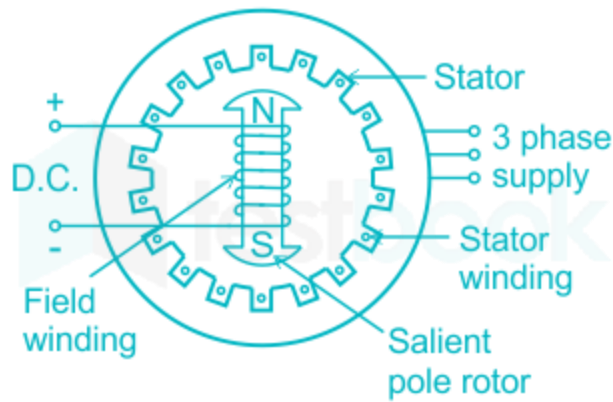
The performance of the single-phase induction motor can be understood through two theoretical frameworks: the Double Revolving Field Theory and the Cross Field Theory. These theories are parallel in their explanations of how torque is produced once the rotor begins to rotate. A single-phase induction motor includes a single-phase winding on the stator and a cage winding on the rotor.

When this stator winding is powered by a single-phase electrical supply, it generates a pulsating magnetic field. This term "pulsating" describes how the magnetic field intensifies in one direction, diminishes to zero, and then strengthens in the reverse direction. Due to this behavior, the rotor of a single-phase induction motor does not start spinning on its own. Thus, single-phase induction motors are not self-starting and require additional starting mechanisms to operate.

### **Construction of a Single Phase Induction Motor**

The construction of a single phase induction motor involves two main parts:

1. **Stator:** The stationary part that creates the magnetic field. It consists of a laminated iron core with slots that house the stator windings.
2. **Rotor:** The rotating part that is placed inside the stator. It is typically a squirrel cage rotor made of aluminum or copper bars short-circuited by end rings.



**Fig- Diagram of Single phase induction motor diagram**

### **Double-revolving Field Theory**

When a single-phase supply is connected to the stator winding, it produces two equal magnetic fields rotating in opposite directions with the same amplitude. The fields neutralize each other, producing a pulsating magnetic field. This pulsating field fails to produce the torque required for starting the motor.

When the rotor is at rest, it experiences alternate pushing and pulling from the pulsating magnetic fields without any resultant force. As it starts rotating even slightly, it enters a zone of leading field first and then lags behind the following field. This makes the magnetic fields cut the rotor conductors sequentially, generating [EMF](#) and current in them. The interaction between the stator and rotor magnetic fields produces a starting torque enabling self-starting.

### **Why Single Phase Induction Motor is not Self Starting?**

As discussed above, the pulsating magnetic field produced by a single phase supply fails to develop torque for self-starting of the motor. During each half cycle, the fields alternatively accelerate and decelerate the rotor alternatively without providing any resultant rotational force.

If the rotor is at rest, it experiences equal and opposite torques successively without any net starting torque. Even a small residual rotating magnetic field is sufficient to get the motor into the accelerating mode. Methods like split phase, shaded pole, and capacitor start are used to introduce a phase difference between two fields, transforming the pulsating field into a quasi-rotating one for enabling self-starting.

### **Equivalent Circuit of Single Phase Induction Motor**

The dynamic behavior of single phase induction motors can be studied using an approximate equivalent circuit model. It consists of a stator resistance ( $R_s$ ), stator reactance ( $X_s$ ), rotor resistance ( $R_r$ ), rotor reactance ( $X_r$ ), and magnetizing reactance ( $X_m$ ).

### **Fig- Equivalent Circuit of Single Phase Induction Motor**

The equivalent circuit takes the stator and rotor as separate circuits magnetically coupled through the magnetizing branch. The behavior is analogous to a [transformer](#) with the stator represented by the primary and the rotor by the secondary. It is a suitable model for performance evaluation based on circuit parameters and determining slip for various loading conditions.

### **Types of Single Phase Induction Motor**

Some commonly used types of single phase motors include:

- **Split Phase Motor:** Has two windings with different resistances to induce a phase difference for starting.
- **Shaded Pole Motor:** Uses conductor shading on poles to delay [magnetic flux](#) for one portion, introducing a phase lag.
- **Capacitor Start Motor:** Employs a starting capacitor in series with auxiliary winding to improve starting torque.
- **Permanent Split Capacitor Motor:** Has a permanent capacitor for both starting and running.
- **Capacitor Start Capacitor Run Motor:** Provides maximum starting torque using dual [capacitors](#) for start and run modes.

### **Application of Single Phase Induction Motor**

Due to their simple and rugged construction with self-starting capability, single phase induction motors find application in various household, commercial, and small industrial applications requiring a power rating of up to 1 HP (0.75 kW).

### **Some common applications include:**

- Household appliances like fans, blowers, mixers, grinders, etc.
- Commercial refrigerators, water pumps, air conditioners, etc.
- Light industrial tools for metalworking, woodworking, etc.
- Agricultural applications like water pumps, threshing machines, etc.
- CNC machines, lathe machines, and drilling machines for small industries.

The single phase induction motor is well-suited for applications requiring frequent starts/stops with reasonable starting torque in the 1/6 to 1/4 HP power range.

### **Advantages of Single Phase Induction Motor**

- Economical and inexpensive due to simple construction

- Self-starting capability enables direct connection to single phase supply
- Rugged and maintenance-free squirrel cage rotor construction
- Compact and lightweight motor suitable for domestic and light applications
- Easy speed control through external resistors in starters
- Widely available with standard designs and ratings

#### **Disadvantages of Single Phase Induction Motor**

- Develops only about 60-65% of the maximum torque of a 3 phase motor
- Low power factor of around 0.4 to 0.5 compared to near unity in 3 phase motors
- Produces increased vibrations and noise due to torque pulsations
- Efficiency is lower by about 5% compared to an equivalent 3 phase motor
- Requires special circuits for self-starting, increasing cost and size
- Has 20-30% higher current drawn from the supply mains during start-up

#### **Difference Between 3-Phase and Single Phase Induction Motor**

The key differences between a 3-phase and 1-phase induction motor are:

| <b>Features</b>     | <b>3-Phase Induction Motor</b>                            | <b>1-Phase Induction Motor</b>                                                     |
|---------------------|-----------------------------------------------------------|------------------------------------------------------------------------------------|
| <b>Power Supply</b> | Uses a balanced 3-phase AC supply.                        | Uses a single-phase AC supply.                                                     |
| <b>Construction</b> | Has one set of 3-phase windings.                          | Uses two windings, one for starting and one for running.                           |
| <b>Starting</b>     | Self-starting without auxiliary circuits.                 | Requires auxiliary circuits like a capacitor for self-starting.                    |
| <b>Power Factor</b> | Typically close to unity (1).                             | Ranges from 0.4 to 0.5 due to non-sinusoidal current.                              |
| <b>Torque</b>       | Develops higher torque, and more effective load handling. | Torque is about 60-65% of a 3-phase motor at rated load.                           |
| <b>Efficiency</b>   | Higher efficiency, typically 95-97%.                      | Lower efficiency, typically 90-92% due to increased losses.                        |
| <b>Applications</b> | Suited for industrial applications.                       | Commonly used in household applications, especially where less than 1HP is needed. |

#### **Definition of a Capacitor Start Capacitor Run Motor**

A capacitor start capacitor run motor is a type of single-phase induction motor that incorporates both a start capacitor and a run capacitor. These capacitors are used to create a phase shift in the motor's windings, improving its starting torque and running efficiency.

The capacitor start capacitor run motor features a cage rotor, with its stator comprising two windings: the Main Winding and the Auxiliary Winding. These windings are spatially displaced by 90 degrees. This motor employs two capacitors: the starting capacitor, which is utilized during startup to provide high initial torque, and the run [capacitor](#), which is engaged for continuous operation, ensuring efficient and stable performance.

Key Components:

- **Start Capacitor:** Provides a high starting torque by creating a large phase shift during the start-up.
- **Run Capacitor:** Maintains the phase shift during normal operation, ensuring smooth and efficient running.

#### **Working of a Capacitor Start Capacitor Run Motor**

The working principle of the capacitor start capacitor run motor relies on creating a rotating [magnetic field](#) using phase correction provided by the capacitors.

At startup, the starting capacitor (Cs) connected in series with the auxiliary winding generates a leading current which is 90° ahead of the main winding current. This phase difference is essential for producing the rotating magnetic field needed to develop torque and get the rotor rotating.

As the motor reaches nearly synchronous speed, the starting capacitor is cut off automatically through a centrifugal switch. Now only the running capacitor (Cr) connected in parallel to the auxiliary winding provides the necessary 90° lagging current for continued rotation.

The running capacitor being permanently connected improves the motor's [power factor](#) during steady-state operation. The continuous phase correction supplied by it allows smooth, vibration-free running of the motor.

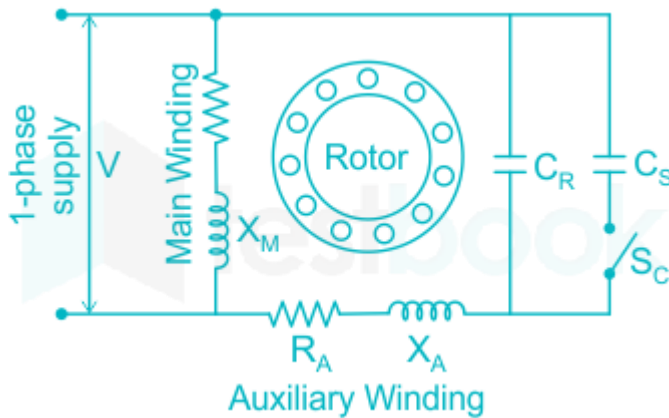


Fig- Circuit diagram of capacitor start capacitor run motor

This unique two-capacitor design enables the motor to produce high starting torque for quick acceleration and also high operating efficiency through a good power factor - making it suitable for heavy-duty applications requiring frequent starts.

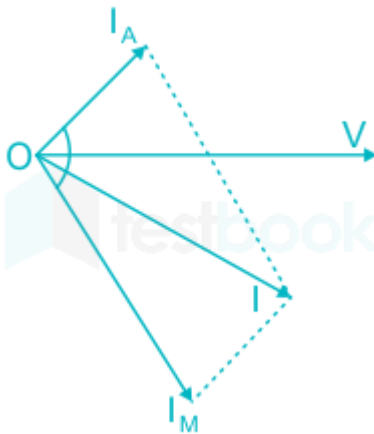


Fig- Phasor diagram of capacitor start capacitor run motor

#### Advantages of Capacitor Start Capacitor Run Motor

Some of the key advantages of this special electric motor include:

- High starting/breakaway torque for easy starts under full load conditions
- Improved power factor and higher efficiency due to permanent phase correction from the running capacitor
- Constant torque characteristic leads to smooth, pulsation-free operation
- Compact and lightweight design suitable for integration with different machine tools
- Quiet running performance is useful in settings requiring low noise levels
- Long mechanical life due to negligible starting and stopping stresses on components
- Operates at higher efficiencies compared to conventional induction motors
- Maintains uniform speed under varying loads
- Self-starting without the need for external starters
- Low cost of installation and maintenance
- Protection from overloads and current surges

#### Disadvantages of Capacitor Start Capacitor Run Motor

Some potential downsides of this motor type are:

- Higher initial cost compared to shaded-pole or split-phase induction motors
- Prone to failure of capacitors requiring periodic replacement
- Complex internal construction with additional switching components
- Difficulty in reversing the direction of rotation without rewiring
- Lower accelerating torque than a split-phase motor of same rating
- Requires careful sizing of capacitors for optimal performance
- Higher losses due to frictional dissipation in capacitor switch

Thus while offering distinct advantages, the capacitor start capacitor run motor demands more careful sizing, installation practices, and regular maintenance for reliable long-term operation.

#### Applications of Capacitor Start Capacitor Run Motor

Given its beneficial performance traits, the capacitor start capacitor run induction motor finds usage across many industries for driving applications involving frequent starts, high starting loads, or the need for smooth, constant speed operation. Some common applications include:

- Pumps (vane, centrifugal, gear) used in irrigation, water supply, sewage, etc.
- Fans and blowers in ventilation, and air conditioning plants
- Screw compressors in refrigeration, air conditioning units
- Hoists, cranes, and elevators for frequent load handling
- Conveyors and material handling systems in process plants
- Machine tools like lathes, mills, and drilling machines.
- Rolling mills and processing lines in manufacturing plants
- Welding machines, grinders, mixers and centrifuges
- Housed applications like washing machines, dishwashers, blenders, etc.
- Medical equipment like X-Ray machines, ventilators, dialysis units

Thus industries where above mentioned applications dominate widely use this special motor to achieve reliable operation and energy efficiency.