LECTURE NOTES

Design of Machine Elements-1

B.Tech, 4th Semester, ME

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<u>COURSECONTENT</u> Design of Machine Element-1

B.Tech,4thSemester,ME

> Module-1

Mechanical Engineering design: Introduction to design procedure, Stages in design, Code and Standardization, Interchangeability, Preferred numbers, Fits and Tolerances, Factor of safety concept, Engineering materials: Ferrous, Non-ferrous, design requirements – Properties of Materials, Material selection, Use of Data books.

➢ Module-2

{Page No. 25} Machine Element Design: Design of Joints: Rivets, Welds and threaded fasteners based on different types of loading, Boiler joints, cotter joints and knuckle joints.

> Module-3

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Design of Keys, Shaft and Couplings: Classification of keys and pins, Design of keys and pins, Design of shafts: based on strength, torsional rigidity and fluctuating load, ASME code for shaft design, Design of couplings: Rigid coupling, Flexible coupling.

> Module-4

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Design of Mechanical Springs: Types of helical springs, Design of Helical springs, bulking of spring, spring surge, end condition of springs, Design of leaf springs: nipping Module-5

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Bearings: Types and selection of ball and roller bearings, Dynamic and static load ratings, Bearing life, Design of sliding contact bearings, Journal bearing, foot step bearing.

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Accreditation Board for Engineering and Technology (ABET)

Definition of Design

- *Engineering design* is the process of devising a system, component, or process to meet desired needs.
- It is a decision-making process (often iterative), in which the engineering sciences and mathematics are applied to convert resources optimally to meet a stated objective.
- Among the fundamental elements of the design process are the establishment of objectives and criteria, synthesis, analysis, construction, testing and evaluation.

Definition of Design

- *Mechanical design* means the design of components and systems of a mechanical nature—machines, products, structures, devices and instruments.
- For the most part mechanical design uses mathematics, materials, and the engineering-mechanics sciences.
- Additionally, it uses engineering graphics and the ability to communicate verbally to clearly express your ideas.
- *Mechanical engineering design* includes all mechanical design, but it is a broader study because it includes all the disciplines of mechanical engineering, such as the thermal fluids and heat transfer sciences too.
- Aside from the fundamental sciences which are required, the first studies in *mechanical engineering design* are in *mechanical design*, and that is the approach taken in this course.

Steps of the Design Process

1. Recognize the Need

- The first step is to establish the ultimate purpose of the project. Often, this is in the form of a general statement of the client's dissatisfaction with a current situation.
- example "There is too much damage to bumpers in low-speed collisions."
- This is a general statement that does not comment on the design approach to the problem. It does not say that the bumper should be stronger or more flexible.
- Recognition and phrasing of the need are often very creative acts because the need may only be a sensing that something is not right. For this reason, sensitive people are generally more creative.
- example the need to do something about a food packaging machine may be indicated by the noise level, variation in package weights, or by slight but perceptible variations in the quality of the packaging.



- This is one of the most critical steps of the design process.
- There is an iteration between the definition of the problem and the recognition of need. Often the true problem is not what it first seems.
- <u>The problem definition is more specific than recognizing the</u> <u>need</u>. For instance, if the need is for cleaner air, the problem might be that of reducing the dust discharge from power-plant stacks, or reducing the quantity of irritants from automotive exhausts, or means for quickly extinguishing forest fires.
- The problem definition must include all the specifications for the thing that is to be designed. **Anything which limits the designer's freedom of choice is a specification.**
- It is imperative to write a formal problem statement which expresses what the design is to accomplish

include:

objectives and goals (musts, must nots; wants, don't wants) constraints criteria used to evaluate the design

• Example: Mobile Vehicle

Design a vehicle which can maneuver in an indoor environment. The vehicle will be operated via remote control and must be able to:

- 1) Travel up to a speed of 7 ft/sec on a flat, horizontal, dry, bare concrete surface.
- 2) Climb 5" high stairs at speeds up to 2 ft/sec.
- 3) Carry a payload of at least 20 lbs.
- 4) Fit through doorways.
- 5) Cross obstacles up to 20" high and up to 24" across within 20 seconds.
- 6) Climb a slope of up to 30 degrees and cross side slopes up to 20 degrees.
- 7) Rotate with zero turning radius.
- 8) Travel in any direction.
- 9) Total vehicle weight should be less than 275 lbs.
- Design considerations (in no particular order)

strength	cost	flexibility
reliability	safety	control
thermal properties	weight	stiffness
corrosion	life	surface finish
wear	noise	lubrication
friction	styling	maintenance
ergonomics	shape	volume
utility	size	liability
manufacturability	speed	feedrate

ectetera

Note: Design considerations in bold might be pertinent to the design project in EML2322L.

3. Gathering of Information

- Often, either no information is easily found, or there is an abundance of information
- Never-ending process for the best design engineers
- Info sources:



textbooks trade journals & magazines technical reports from government sponsored R&D company catalogs, web pages and technical personnel handbooks company reports patents people

• Problems in gathering information:

LAZINESS

Where to find it?

How to get it?

How accurate & credible is the information?

How should the information be interpreted for my needs?

When do I have enough information?

What decisions result from the information?

PLAGIARISM (integrity = giving others credit for their ideas)



- analogy

obstacle avoidance similar to potential fields

- area thinking

improve an existing product by concentrating on one of its important characteristics (cost, performance, function, appearance, safety, etc.)

- brainstorming

group of people who are familiar with the general nature of the problem; everyone says what comes to mind rules: (1) no judgements; (2) the more unconventional the better; (3) the more ideas the better

- involvement

visualize yourself as being part of the mechanism

- functional synthesis

divide the system into subunits describe each subunit by a complete list of functional requirements list all the ways the functional requirements of each subunit can be realized study all combinations of partial solutions

Can Opener		
Part	Function	Realization
Subunit 1	1. Separate metal	1. Shearing
		2. Tearing
		3. Fatigue
		4. Melting
		5. Drawing thin
		6. Chemical erosion
Subunit 2	1. Apply power	1. Hand
		2. Electric motor
		3. Hot wire
		4. Hydraulic motor
		5. Flame
		6. Chemical reaction
		7. Mechanical vibration
		8. Laser
	2. Position	1. Bring can to opener
		2. Bring opener to can
		3. Have opener built on can

- try inversion

try reversing the ordering of things; i.e. an inversion is produced with an electric motor by holding the rotor stationary and permitting the field windings to rotate

- change the normal position and character of things if it operates horizontally, try operating it vertically. If it's round, try making it square. For example, doors hinged at top or bottom, a horizontal drill press, etc.

- talk it over

If the designer has followed these suggestions thus far, he/she is now quite familiar with the problem. Many solutions have been found but none are quite satisfying. Having worked to this point, the designer's mind is in a receptive condition and will instantly recognize a solution. The problem is to bridge a gap between two groups of ideas—to make an association of ideas. It is generally conceded that this association occurs by pure chance. This event is most likely to occur when the problem is being discussed with another person or group of persons.

5. Concept Selection

 form decision matrix to unbiasedly evaluate different ideas based on a weighted set of objectives the design team decides are important for the solving the problem



6. Communication of the Design

- The purpose of the design is to satisfy the needs of the client.
- Designer must provide oral presentations and written design reports.
- Continuous communication is important in order to avoid surprises.
- Many great designs and inventions have been lost simply because the originator was unable or unwilling to explain his/her accomplishments to others.
- There are only 3 forms of communication available to us: written, oral and graphical. The successful engineer will be technically competent and versatile in all three. Competency only comes from practice.
- Ability in writing can be acquired by writing letters, reports, memos, and papers. It doesn't matter whether the articles are published or reviewed—the practice is the important thing. Ability in speaking can be obtained in educational, fraternal, civic, church and professional activities. To acquire drawing ability, pencil sketching should be employed to illustrate every idea possible. **CAD work should complement this, not replace it.**
- Importance of sketches, drawings, visual aids, computer graphics and models in the communications process.



7. Detailed Design and Analysis

- The principal goal of your engineering studies is to enable you to create mathematical models which accurately simulate the real physical world.
- All real physical systems are complex. Creating a mathematical model of the system means we are simplifying the system to the point that it can be analyzed. The terms *rigid body* and *concentrated force* are examples. The rule in making such *assumptions*, is that, in creating the model, the model must be meaningful—i.e. a good and appropriate model given the design constraints involved.



- The nature of the problem, its economics, the computational facilities available and the ability and working time of the engineer, all play a key role in the formulation of the model.
- The designer's time investment typically increases exponentially with regard to model accuracy.



SYSTEM OF LIMITS, FITS, TOLERANCE AND GAUGING INTRODUCTION:

It is well known fact that no two things in the nature can be identical, they may be found to be closely similar. This is true of production of component parts in engineering also. We know that every process is a combination of three elements, man, machine and material. A change in any one of these will constitute a change in the process. All these elements are subjected to inherent and characteristic variations.

Generally, in engineering, any component manufactured is required to fit or to match with some other component.

If a machine is under control, i.e. no assignable causes of variation exist, and then the resultant frequency distribution of dimension produced will be roughly in the form of normal curve, i.e. 99.7% parts will be within $\pm 3\sigma$ limits of means setting



The value of σ depends upon the machine used to produce a component. If value of σ has to be used reduced, then precision machines have to be used produces the component having less variation in dimensions. It is thus important to note that the cost of production keeps on increasing tremendously for very precise tolerance as shown in above **fig**, as the tolerance approaches zero, the task of achieving it becomes enormous and finally impossible .in general, tolerance vs. fabrication cost is hyperbolic curve.

> LIMITS:

The maximum and minimum permissible sizes within which the actual size of a component lies are called limits.

- Limits are fixed with reference to the basic size of that dimension.
- Upper limit (The high limit) for that dimension is the largest size permitted and the low limit is the smallest size permitted for that dimension.

TERMINOLOGY

The terminology used in fits and tolerances is shown in Fig below. The important terms are



Basic size: It is the exact theoretical size arrived at by design. It is also called nominal size.

Actual size: The size of a part as may be found by measurement.

Maximum limit of size: The greater of the two limits of size.

Minimum limit of size: The smaller of the two limits of size.

Allowance: It is an intentional difference between maximum material limits of mating parts. It is a minimum clearance or maximum interference between mating parts.

Deviation: The algebraic difference between a size (actual, maximum, etc.) and the corresponding basic size.

Actual deviation: The algebraic difference between the actual size and the corresponding basic size.

Upper deviation: The algebraic difference between the maximum limit of size and the corresponding basic size.

Upper deviation of hole = ES (& art Superior)

Upper deviation of shaft = es

Lower deviation: The algebraic difference between the minimum limit of size and the corresponding basic size.

Lower deviation of hole = El (Ecart Inferior)

Lower deviation of shaft = ei

Upper deviation Lower deviation + Tolerance

Zero line: It is the line of zero deviation and represents the basic size.

Tolerance zone: It is the zone bounded by the two limits of size of the parts and defined by its magnitude, i.e. tolerance and by its position in relation to the zero line.

Fundamental deviation: That one of the two deviations which is conveniently chosen to define the position of the tolerance zone in relation to zero line, as shown in fig below.



Fig: Disposition of fundamental deviation and tolerance zone with respect to the zero line

Basic shaft: A shaft whose upper deviation is zero.

Basic hole: A hole whose, lower deviation of zero.

Clearance: It is the positive difference between the hole size and the shaft size.

Maximum clearance: The positive difference between the maximum size of a hole and the minimum size of a shaft.

Minimum clearance: The positive difference between the minimum size of a hole and the maximum size of a shaft.

> FITS

When two parts are to be assembled, the relation resulting from the difference between their sizes before assembly is called a fit. A fit may be defined as the degree of tightness and looseness between two mating parts.



(i) Clearance Fit:

This means there is a gap between the two mating parts. Let's see the following schematic representation of clearance fit. The diameter of the shaft is smaller than the diameter of the hole. There is a clearance between the shaft and the hole. Hence the shaft can easily slide into the hole.



Figure: Clearance fit

In clearance fit the difference between the maximum size of the hole and the minimum size of the shaft is known as the **Maximum clearance** and the difference between the minimum size of the hole and the maximum size of the shaft is known as the **Minimum clearance**.

Clearance fit can be sub-classified as follows:

Loose Fit: It is used between those mating parts where no precision is required. It provides minimum allowance and is used on loose pulleys, agricultural machineries etc.

Running Fit: For a running fit, the dimension of shaft should be smaller enough to maintain a film of oil for lubrication. It is used in bearing pair etc. An allowance 0.025 mm per 25 mm of diameter of boring may be used.

Slide Fit or Medium Fit: It is used on those mating parts where great precision is required. It provides medium allowance and is used in tool slides, slide valve, automobile parts, etc.

EXAMPLE:

Question: A spindle slides freely in a bush. The basic size of the fit is 50×10^{-3} mm. If the tolerances quoted are 0 +62 for the holes and -80 +180 for the shaft, find the upper limit and lower limit of the shaft and the minimum clearance.

Solution: Tolerances are given in units of one thousandth of millimeter, so the upper limit of the hole will be 50.062 mm and lower limit for the hole is the same as the basic size of 50.000 mm.

The shaft upper limit will be $(50.000 - 0.080) \times 10^{-3} = 49.92 \times 10^{-3} \text{ m}$

The shaft lower limit will be $(50.000 - 0.180) \times 10^{-3} = 49.82 \times 10^{-3} \text{ m}$

The minimum clearance or allowance is (50.000 - 49.920) $10^{-3} = 8 \times 10^{-3}$ mm

(ii) Interference Fit:

There is no gap between the faces and there will be an intersecting of material will occur. In the following schematic representation of the Interference fit. The diameter of the shaft is larger than the hole diameter. There will be the intersection of two mating components will be occurred. Hence the shaft will need additional force to fit into the hole.



Figure: Interference Fit

In Interference fit the difference between the <u>maximum size</u> of the shaft and the minimum size of the hole is known as the **Maximum Interference** and the difference between the minimum size of the shaft and the maximum size of the hole is known as the **Minimum Interference**.

The interference fit can be sub-classified as follows:

Shrink Fit or Heavy Force Fit: It refers to maximum negative allowance. In assembly of the hole and the shaft, the hole is expanded by heating and then rapidly cooled in its position. It is used in fitting of rims etc.

Medium Force Fit: These fits have medium negative allowance. Considerable pressure is required to assemble the hole and the shaft. It is used in car wheels, armature of dynamos etc.

Tight Fit or Press Fit: One part can be assembled into the other with a hand hammer or by light pressure. A slight negative allowance exists between two mating parts (more than wringing fit). It gives a semi-permanent fit and is used on a keyed pulley and shaft, rocker arm, etc.

EXAMPLE

A dowel pin is required to be inserted in a base. For this application H 7 fit for hole and a p 6 fit for the shaft are chosen. The tolerance quoted is 0 + 25 for the hole and 26 + 42 for the shaft. Find the upper and lower limits of the hole and also dowel pin, and the maximum interference between dowel pin and the hole. The basic size of the fit is 50×10^{-3} m.

Solution:

The upper limit for the hole will be $(50.000 + 0.025) \ge 10^{-3} = 50.025 \ge 10^{-3} = 50.025 \ge 10^{-3} = 50 \ge 10^{-3} = 50 \ge 10^{-3} =$

(iii) Transition Fit:

Transition fit is neither loose nor tight as like clearance fit and interference fit. The <u>tolerance</u> <u>zones</u> of the shaft and the hole will be overlapped between the interference and clearance fits. See the following schematic representation of the transition fit.



Figure: Transition Fit

Transition fit can be sub-classified as follows:

Push Fit: It refers to zero allowance and a light pressure (10 cating dowels, pins, etc.) is required in assembling the hole and the shaft. The moving parts show least vibration with this type of fit. It is also known as snug fit.

Force Fit or Shrink Fit: A force fit is used when the two mating parts are to be rigidly fixed so that one cannot move without the other. It either requires high pressure to force the shaft into the hole or the hole to be expanded by heating. It is used in railway wheels, etc.

Wringing Fit: A slight negative allowance exists between two mating parts in wringing fit. It requires pressure to force the shaft into the hole and gives a light assembly. It is used in fixing keys, pins, etc.

EXAMPLE:

For a particular application, an H 7 fit has been selected for the hole and a K 6 fit for the shaft. The tolerance quoted are 0 + 25 for the hole and 12 + 18 for the shaft. Find the upper limit and lower limit for the hole and also for bush. The basic size of fit is $50 \times 10 - 3$ m.

Solution:

The upper limit for the hole will be $(50.000 + 0.025) \times 10^{-3} = 50.025 \times 10^{-3} \text{ m}$ The lower limit for the hole will be $(50.000 + 0) \times 10^{-3} = 50.000 \times 10^{-3} \text{ m}$ The upper limit for the bush will be $(50.000 + 0.018) \times 10^{-3} = 50.018 \times 10^{-3} \text{ m}$ The lower limit for the bush will be $(50.000 + 0.002) \times 10^{-3} = 50.002 \times 10^{-3} \text{ m}$

SYSTEMS OF FITS:

A fit system is the systems of standard allowance to suit specific range of basic size. If these standard allowances are selected properly and assigned in mating parts ensures specific classes of fit.

There are two systems of fit for obtaining clearance, interference or transition fit. These are:

- 1. Hole basis system
- 2. Shaft basis system

1. Hole Basis System:

In the hole basis system, the size of the hole is kept constant and shaft sizes are varied to obtain various types of fits. In this system, lower deviation of hole is zero, i.e. the low limit of hole is same as basic size. The high limit of the hole and the two limits of size for the shaft are then varied to give desired type of fit. The hole basis system is commonly used because it is more convenient to make correct holes of fixed sizes, since the standard drills, taps, reamers and branches etc. are available for producing holes and their sizes are not adjustable. On the other hand, size of the shaft produced by turning, grinding, etc. can be very easily varied.



Fig: Hole basis system

2. Shaft Basis System:

In the shaft basis system, the size of the shaft is kept constant and different fits are obtained by varying the size of the hole. Shaft basis system is used when the ground bars or drawn bars are readily available. These bars do not require further machining and fits are obtained by varying the sizes of the hole. In this system, the upper deviation (fundamental deviation) of shaft is zero, i.e. the high limit of the shaft is same as basic size and the various fits are obtained by varying the low limit of shaft and both the limits of the hole.



Fig: Shaft Basis System

DIFFERENCE BETWEEN HOLE BASIS & SHAFT BASIS SYSTEM:

BASIS SYSTEM	SHAFT SYSTEM
1. Size of hole whose lower deviation is zero	1. Size of shaft whose upper deviation is zero
(H.hole) is assumed as the basic size.	(h-shaft) assumed as basic size.
2. Limits on the hole are kept constant and	2. Limits on the shaft are kept constant and
those of shafts are varied to obtain desired	those of holes are varied to have necessary fit.
type of fit.	
3. Hole basis system is preferred in mass	3. This system is not suitable for mass
production because it is convenient and less	production because it is inconvenient time
costly to make a hole of correct size due to	consuming and costly to make a hole of any
availability of standard drills reamers.	size w.r to field shaft size so as to obtain
	required fit.
4. It is much more easy to vary the shaft sizes	4. It is rather difficult to vary the hole sizes
according to the fit required.	according to the fit required.
5. It required less amount of capital and	5. It needs large amount of capital and storage
storage space for roofs needed to produce	space for large numbers of tools required to
shaft of different sizes.	produce holes of different sizes.
6. Changing of shafts can be easily and	6. Being internal measurement gauging of
conveniently done with suitable gap Gauges	holes can't be easily and conveniently done.

TOLERANCES:

Tolerance is a permissible limit and variation in dimensions or in physical parameters. It is possible to achieve dimensions and physical parameter exactly but it is very time consuming and economically unjustified or costly. It is quite often not necessary to achieve exact dimensions, in such functions or cases some permissible variation or tolerance is given.

The permissible variation in size or dimension is tolerance. The difference between the upper limit (high limit) and the lower limit of a dimension represents the margin for variation to workmanship, and is called a tolerance zone.

Tolerance can also be defined as the amount by which the job is allowed to go away from accuracy and perfectness without causing any functional trouble, when assembled with its mating part and put into actual service.



Fig: Tolerance

There are two ways of writing tolerances:

(a) Unilateral tolerance

(b) Bilateral tolerance.

Unilateral Tolerance:

In this system, the dimension of a part is allowed to vary only on one side of the basic size, i.e. tolerance lies only on one side of the basic size either above or below it (As shown in fig).



Fig: unilateral Tolerance

Examples of unilateral tolerance are :

$$25^{+0.02}_{+0.01}$$
, $25^{-0.02}_{-0.01}$, $25^{-0.01}_{-0.02}$, $25^{+0.0}_{-0.02}$ etc.

Unilateral system is preferred in interchangeable manufacture, especially when precision fits are required, because

(a) it is easy and simple to determine deviations,

(b) another advantage of this system is that "Go" Gauge ends can be standardized as the holes of different tolerance grades have the same lower limit and all the shafts have same upper limit, and (c) This form of tolerance greatly assists the operator, when machining of mating parts. The operator machines to the upper limit of shaft (lower limit for hole) knowing full well that he still has some margin left for machining before the parts are rejected.

Bilateral Tolerance:

In this system, the dimension of the part is allowed to vary on both the sides of the basic size, i.e. the limits of tolerance lie on either side of the basic size, but may not be necessarily equally disposing about it.



Fig: Bilateral Tolerance

Examples of bilateral tolerance are :

 $25^{\pm 0.02}, 25^{+0.02}_{-0.01}$ etc.

In this system, it is not possible to retain the same fit when tolerance is varied and the basic size of one or both of the mating parts are to be varied. This system is used in mass production when machine setting is done for the basic size.

EXAMPLE

A 50 mm diameter shaft is made to rotate in the bush. The tolerances for both shaft and bush are 0.050 mm. determine the dimension of the shaft and bush to give a maximum clearance of 0.075 mm with the hole basis system.

Solution: In the hole basis system, lower deviation of hole is zero, therefore low limit of hole = 50 mm.

High limit of hole = Low limit + Tolerance

= 50.00 + 0.050

 $= 50.050 \text{ mm} = 50.050 \text{ x} 10^{-3} \text{ m}$

High limit of shaft = Low limit of hole – Allowance

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= 50.00 - 0.075
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 $= 49.925 \text{ mm} = 49.925 \text{ x} 10^{-3} \text{ m}$

Low limit of the shaft = High limit – Tolerance

=49.925 - 0.050

 $= 49.875 \text{ mm} = 49.875 \text{ x} 10^{-3} \text{ m}$

The dimension of the system is shown in Figure

MODULE-II (DESIGN OF JOINTS)

Riveted Joint

Often small machine components are joined together to form a larger machine part. Design of joints is as important as that of machine components because a weak joint may spoil the utility of a carefully designed machine part. Mechanical joints are broadly classified into two classes viz., non-permanent joints and permanent joints. Non-permanent joints can be assembled and dissembled without damaging the components. Examples of such joints are threaded fasteners (like screw-joints), keys and couplings etc.

Permanent joints cannot be dissembled without damaging the components. These joints can be of two kinds depending upon the nature of force that holds the two parts. The force can be of mechanical origin, for example, riveted joints, joints formed by press or interference fit etc, where two components are joined by applying mechanical force. The components can also be joined by molecular force, for example, welded joints, brazed joints, joints with adhesives etc. Not until long ago riveted joints were very often used to join structural members permanently. However, significant improvement in welding and bolted joints has curtained the use of these joints. Even then, rivets are used in structures, ship body, bridge, tanks and shells, where high joint strength is required.

Rivets and riveting

A Rivet is a short cylindrical rod having a head and a tapered tail. The main body of the rivet is called shank (see figure 2.1).



Fig. 2.1 Rivets and its parts

According to Indian standard specifications rivet heads are of various types. Rivets heads for general purposes are specified by Indian standards IS: 2155-1982 (below 12 mm diameter) and IS: 1929-1982 (from 12 mm to 48 mm diameter). Rivet heads used for boiler works are specified by IS: 1928-1978. To get dimensions of the heads see any machine design handbook.

Riveting is an operation whereby two plates are joined with the help of a rivet. Adequate mechanical force is applied to make the joint strong and leak proof. Smooth holes are drilled (or punched and reamed) in two plates to be joined and the rivet is inserted. Holding, then, the head by means of a backing up bar as shown in figure 2.2, necessary force is applied at the tail end with a die until the tail deforms plastically to the required shape. Depending upon whether the rivet is initially heated or not, the riveting operation can be of two types: (a) cold riveting is done at ambient temperature and (b) hot riveting rivets are initially heated before applying force. After riveting is done, the joint is heat-treated by quenching and tempering. In order to ensure leak-proofness of the joints, when it is required, additional operation like caulking is done.



Fig. 2.2 Riveting operation

Types of rivet joints

Riveted joints are mainly of two types

- 1. Lap joints
- 2. Butt joints

Lap joints

The plates that are to be joined are brought face to face such that an overlap exists, as shown in figure 10.1.3. Rivets are inserted on the overlapping portion. Single or multiple rows of rivets are used to give strength to the joint. Depending upon the number of rows the riveted joints may be classified as single riveted lap joint, double or triple riveted lap joint etc. When multiple joints are used, the arrangement of rivets between two neighbouring rows may be of two kinds. In chain riveting the adjacent rows have rivets in the same transverse line. In zig-zag riveting, on the other hand, the adjacent rows of rivets are staggered.



Fig. 2.3 Lap joint

But joints

In this type of joint, the plates are brought to each other without forming any overlap. Riveted joints are formed between each of the plates and one or two cover plates. Depending upon the number of cover plates the butt joints may be single strap or double strap butt joints. A single strap butt joint is shown in figure 2.4. Like lap joints, the arrangement of the rivets may be of various kinds, namely, single row, double or triple chain or zigzag.



Fig. 2.4 Butt joint

Important terms used in rivet joints

Few parameters, which are required to specify arrangement of rivets in a riveted joint are as follows:

- *a) Pitch:* This is the distance between two centers of the consecutive rivets in a single row. (usual symbol *p*)
- b) Back Pitch: This is the shortest distance between two successive rows in a multiple riveted joint. (usual symbol b_p)
- *c) Diagonal pitch:* This is the distance between the centers of rivets in adjacent rows of zigzag riveted joint. (usual symbol p_d)
- *d) Margin or marginal pitch:* This is the distance between the centre of the rivet hole to the nearest edge of the plate. (usual symbol *m*)

These parameters are shown in figure 2.5.



Fig. 2.5 Important design parameters of riveted joints

Modes of failure of rivet joints

(1) *Tearing of the plate at the edge*: Figure 2.6 shows the nature of failure due to tearing of the plate at the edge.

Such a failure occurs due to insufficient margin. This type of failure can be avoided by keeping margin, m = 1.5d, where d is the diameter of the rivet.



Fig. 2.6 Tearing of the plate at the edge

(2) *Tearing of the plate across a row of rivets*: In this, the main plate or cover plates may tear-off across a row of rivets, as shown in Fig. 2.7. Considering one pitch length,

Tearing strength per pitch length, $F_t = \sigma_t (p-d)t$ (2.1)

Where, σ_t = permissible tensile stress for the plate material; p = pitch; d = diameter of the rivet; t = thickness of the plate.



Fig. 2.7 Tearing of the plate across a row of rivets

(3) *Shearing of rivets*: Rivets are in single shear (Fig. 2.8*a*)in lap joints and in double shear in double strap butt joints (Fig. 2.8*b*). Considering one pitch length,



(a)

Fig. 2.8 Shearing of rivets

Shearing resistance per pitch length, $F_s = \frac{\pi d^2}{4} n\tau$ in single shear (2.2)

$$= 2 \times \frac{\pi d^2}{4} \times n\tau \text{ in double shear}$$
(2.3)

Where, n = number of rivets per pitch length

(4) *Crushing of rivets (plates)*: When the joint is loaded, compressive stress is induced over the contact area between rivet and the plate (Fig. 2.9).



Fig. 2.9 Crushing of a rivets

The contact area is given by the projected area of the contact. Considering one pitch length, Crushing resistance per pitch length, $F_c = ndt\sigma_c$ (2.4)

Where, n = number of rivets per pitch length; $\sigma_c =$ permissible compressive stress.

Note: Number of rivets under crushing is equal to the number of rivets under shear.

Efficiency of a riveted joint

The efficiency of a riveted joint is defined as the ratio of the strength of the joint (least of calculated resistances) to the strength of the solid plate.

Efficiency of a riveted joint,
$$\eta = \frac{F_t, F_s, orF_c(least)}{pt\sigma_t}$$
 (2.5)

Where, ' $pt\sigma_t$ ' is the strength of the solid plate per pitch length.

Design of boiler joints

In general, for longitudinal joint, butt joint is adopted, while for circumferential joint, lap joint is preferred.

Design of longitudinal butt joint

1. *Thickness of the plate*: The thickness of the boiler shell is determined, by using thin cylinder formula, i.e.

$$t = \frac{P_i d_i}{2\sigma_i \eta} + 1mm \tag{2.6}$$

Where, 1 mm is the allowance for corrosion; P_i = internal steam pressure; d_i = internal diameter of the boiler shell; σ_t = permissible stress of the shell material.

2. *Diameter of rivets*: The diameter of the rivets may be determined from the empirical relation, $d = 6\sqrt{t}$ for $t \ge 8$ mm

Note: (1) The diameter of rivet should not be less than the plate thickness.

- (2) If the plate thickness is less than 8 mm, the diameter of the rivet is determined by equating the shearing resistance of the rivet to its crushing resistance.
- 3. *Pitch of the rivets*: The pitch of the rivets may be obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. However, it should be noted that,
 - (i) the pitch of the rivets should not be less than 2d.
 - (ii) the maximum value of the pitch, for a longitudinal joint is given by,

 $P_{\text{max}} = ct + 41.28mm$ where 'c' is a constant.

Note: If the pitch of the rivets obtained by equating the tearing resistance to the shearing resistance is more than P_{max} , then the value of P_{max} can be adopted.

- 4. *Row (transverse) pitch:*
- (i) For equal number of rivets in more than one row for lap joint or butt joint, the row pitch should not be less than, 0.33p + 0.67d for zig-zag riveting and 2*d*, for chain riveting.
- (ii) For joints in which the number of rivets in the rows is half the number of rivets in the inner rows, and if the inner rows are chain riveted, the distance between the outer row and the next row, should not be less than, 0.33p + 0.67d or 2d, whichever is greater. The distance between the rows in which there are full number of rivets, should not be less than 2d.
- (iii) For joints in which the number of rivets in outer row is half the number of rivets in inner rows, and if the inner rows are zig-zag riveted, the distance between the outer row and the next row, should not be less than, 0.2p + 1.15d. The distance between the rows in which there are full number of rivets (zig-zag), should not be less than, 0.165p + 0.67d.

Note: p is the pitch of the rivets in the outer row.

5. *Thickness of butt straps*: The thickness of butt strap(s) is given by, (in no case it should not be less than 10 mm).

 $t_1 = 1.125t$, for ordinary single butt strap (chain riveting)

=1.125t $\left(\frac{p-d}{p-2d}\right)$, for a single butt strap, with alternate rivets in the outer rows

omitted

= 0.625t, for ordinary double straps of equal width (chain riveting)

$$= 0.625t \left(\frac{p-d}{p-2d}\right)$$
, for double straps of equal width, with alternate rivets in the

outer rows omitted

When two unequal widths of butt straps are employed, the thickness of butt straps are given by, $t_1 = 0.75t$, for wide strap on the inside and $t_2 = 0.625t$, for narrow strap on the outside.

Note: The thickness of butt strap, in no case, shall be less than 10 mm.

6. *Margin*: The margin '*m*' is generally followed as 1.5*d*.

Design of circumferential lap joint

- 1. *Diameter of rivets*: It is usual practice to adopt the rivet diameter and plate thickness, same as those used for longitudinal joint.
- 2. Number of rivets: The rivets are in single shear, since lap joint is used for circumferential joint.

Total number of rivets to be used for the joint,

$$N = \frac{\text{steam load}}{\text{Shear strength of one}} = (\frac{\pi d_i^2}{4} \times p_i) / (\frac{\pi d^2}{4} \tau) = \left(\frac{d_i}{d}\right)^2 \times \frac{p_i}{\tau}$$

Where, d_i = inner diameter of boiler; d = rivet diameter; τ = allowable shear strength of rivet material

3. *Pitch of rivets*: In general, the efficiency of the circumferential joint may be taken as 50% of the tearing efficiency of the longitudinal joint. If intermediate circumferential joints are used, the strength of the seam should not be less than 62% of the strength of the undrilled plate. Knowing the (tearing) efficiency of the circumferential joint, the pitch of the rivets can be obtained from,

Efficiency,
$$\eta = \frac{(p-d)t\sigma_t}{pt\sigma_t} = \frac{p-d}{p}$$

4. *Number of rows*: Number of rivets per row,
$$n = \frac{\pi(d_i + t)}{p}$$

Number of rows, $Z = \frac{\text{Total number of rivets}}{\text{No. of rivets per row}}$
- 5. *Selection of the type of joint*: After determining the number of rows, the type of joint (single riveted, double riveted etc.) may be decided.
- 6. *Row (back) pitch and margin*: The proportions suggested for longitudinal joint, may be followed for the circumferential joint as well.



	1	
Equating equations (i) and (ii) we have, $141.37d^2 = 1800d$		
d = 12.73 mm		
The nearest standard diameter of the rivet recommended, $d = 14 \text{ mm}$		
Pitch of the rivets: The pitch of the rivets may be obtained by equating the		
tearing resistance of the plate to the shearing resistance of the rivets.		
Tearing resistance, $F_t = \sigma_t (p-d)t = (p-14) \times 6 \times 80 \text{ N}$ (iii)	Page-66,	
Shearing resistance, $F_s = n \times \frac{\pi d^2}{4} \times \tau = 3 \times \frac{\pi 14^2}{4} \times 60 = 27708.8 \text{ N}$ (iv)	Eq. 5.31b	
Equating equations (i) and (ii) we have, $(p-14) \times 480 = 27708.8$		
<i>p</i> = 71.73 mm, say 72 mm.		
Distance between the rows of rivets, p_b (or p_t) = 0.33 p + 0.67 d = 33.14 mm , say		
34 mm.		
Mode of failure of the joint:		
Tearing efficiency = $\frac{p-d}{p} = \frac{72-14}{72} = 0.801 = 80.1\%$		
Crushing efficiency = $\frac{ndt\sigma_c}{pt\sigma_t} = \frac{3 \times 14 \times 6 \times 100}{72 \times 6 \times 80} = 0.802 = 80.2\%$		
The lowest efficiency indicates the mode of failure of the joint. In the present		
case, the joint will fail by crushing of the rivets.		
Example problem – 2 : A double riveted, zig-zag butt joint, in which the pitch	Reference	
of the rivets in the outer row is twice that in the inner rows; connects two 16		
mm plates with two cover plates, each 12 mm thick. Determine the diameter of		
the rivets and pitch of the rivets and pitch of the rivets, if the working stresses		
are: $\sigma_t = 100$ MPa, $\sigma_c = 150$ MPa, and $\tau = 75$ MPa.		
Solution:	Design	
Diameter of the rivet:	Data Book	
Diameter of the rivet, $d = 6\sqrt{t} = 24mm$	by K. Mahadevan	
Pitch of the rivets:	& K. B.	
Let p_0 = pitch of the rivets in the outer row		
$P_{\rm i}$ = pitch of the rivets in the outer row		

The pitch of the rivets may be obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. Referring Fig. 2.11, since the pitch in the outer row is twice the pitch of inner row; for one pitch length in the outer row, there are three rivets, which are under double shear. Fig. 2.11- Double riveted, double strap, zig-zag butt joint Tearing resistance, $F_t = (p - d)t\sigma_t = (p_0 - 14) \times 16 \times 100 = (p_o - 24) \times 1600 \text{ N}$ (i) Shearing resistance, $F_s = n \times 1.875 \times \frac{\pi d^2}{4} \tau$, assuming that the rivets under double shear are 1.875 times as strong as those under single shear = $3 \times 1.875 \times \frac{\pi}{4} \times 24^2 \times 75 = 190851.8 \text{ N}$ (ii) Equating equations (i) and (ii) we have, $(p_o - 24) \times 1600 = 190851.8$ $p_{\rm o} = 143.3$, say 144 mm Pitch of rivets in the inner row, $p_i = \frac{p_o}{2} = \frac{144}{2} = 72 \text{ mm}$ Page-66, Distance between the rows of rivets: Eq. 5.33a For zig-zag riveting, the row (back) pitch, pb $\geq 0.2 p_o + 1.15 d \geq$ $0.2 \times 144 + 1.15 \times 24 \ge 56.4$ mm. A back/row pitch of 60 mm may be recommended.

Welded joints

Welded joints and their advantages:

Welding is a very commonly used permanent joining process. Thanks to great advancement in welding technology, it has secured a prominent place in manufacturing machine components. A welded joint has following advantages:

- (i) Compared to other type of joints, the welded joint has higher efficiency. An efficiency > 95 % is easily possible.
- (ii) Since the added material is minimum, the joint has lighter weight.
- (iii) Welded joints have smooth appearances.
- (iv) Due to flexibility in the welding procedure, alteration and addition are possible.
- (v) It is less expensive.
- (vi) Forming a joint in difficult locations is possible through welding.

The advantages have made welding suitable for joining components in various machines and structures.

Types of welded joints

Welded joints are primarily of two kinds

a) *Lap or fillet joint*: obtained by overlapping the plates and welding their edges. The fillet joints may be single transverse fillet, double transverse fillet or parallel fillet joints (see figure 2.12).



Single transverse lap joint







Fig. 2.12 – Different types of lap joint

b) *Butt joints*: formed by placing the plates edge to edge and welding them. Grooves are sometimes cut (for thick plates) on the edges before welding. According to the shape of the grooves, the butt joints may be of different types, e.g.,

- Square butt joint
- Single V-butt joint, double V-butt joint
- Single U-butt joint, double U-butt joint
- Single J-butt joint, double J-butt joint
- Single bevel-butt joint, double bevel butt joint

These are schematically shown in figure 2.13.



Double – V butt joint



Strength of welds: in-plane loading

There are different forms of welded joints, subjected to in-plane loading under tension.

1. Transverse fillet weld

Figure 2.14*a* shows a double transverse fillet weld under tension. It is assumed that the section of the weld is an isosceles right angled triangle, ABC, i.e. 45° fillet weld (Fig. 2.14b).



Fig. 2.14 – Double transverse fillet weld

The length of each side (AB=BC) is known as leg length or size of the weld. The minimum cross-sectional dimension, BD (at 45° from the plate surface or edge) is termed as throat thickness. Transverse fillet welds are assumed to fail in tension across the throat.

Let

t = thickness of the plate or size of the weld l = length of the weld

 $\sigma_{\rm t}$ = allowable tensile stress for the weld material

From the geometry of Fig. 2.14b,

Throat thickness,
$$BD(=h) = t\sin 45^\circ = \frac{t}{\sqrt{2}}$$

Resisting throat area = $hl = \frac{tl}{\sqrt{2}}$

Tensile strength of the joint = $\frac{tl}{\sqrt{2}}\sigma_t$, for single fillet

$$=\frac{2tl}{\sqrt{2}}\sigma_t=\sqrt{2}tl\sigma_t, \text{ for double filler}$$

2. Parallel fillet weld

Figure 2.15*a* shows a double parallel fillet weld under tension. Parallel fillet welds are assumed to fail in shear across the throat.



(a) Double parallel fillet weld (b) Combination of transverse and parallel fillet weld

Fig. 2.15

Let τ = allowable shear stress for the weld material

Resisting throat area = $\frac{tl}{\sqrt{2}}$ Shear strength of the joint = $\frac{tl}{\sqrt{2}}\tau$, for single fille (Tensile strength)

$$= \frac{2tl}{\sqrt{2}}\tau = \sqrt{2}tl\tau$$
, for double fillet

3. Butt weld

Fig. 2.15a shows a single V-butt joint under tension.



(a) Single-V butt joint
 (b) Double-V butt joint
 Fig. 2.15 – Butt joints under tension

In case of single V-butt weld, the throat thickness of the weld is considered to be equal to the plate thickness, *t*. Hence, tensile strength of the joint = $tl\sigma_t$

Where, l =length of the weld = width of the plate.

Figure 2.15b shows a double V-butt joint under tension.

Let h_1 = throat thickness at the top

 h_2 = throat thickness at the bottom

Then tensile strength of the joint = $(h_1 + h_2)l\sigma_t$

4. Fillet welds under torsion

Circular fillet weld: Figure 2.16 shows a circular shaft, connected to a plate, by a fillet weld of leg length, *t* and subjected to torque, *T*. The shear stress in

the weld, in a horizontal plane, coinciding with the pate surface

is given by,

Where,

$$J = \pi t d \left(\frac{d}{2}\right)^2$$
$$\tau = \frac{T \times d/2}{\pi t d \left(\frac{d}{2}\right)^2} = \frac{2T}{\pi t d^2}$$

 $\tau = \frac{T \times d/2}{I}$



Fig. 2.16

The maximum value of the shear stress occurs in the weld throat, the length of which is $\frac{i}{\sqrt{2}}$

Therefore,

$$\tau_{\rm max} = \frac{2T\sqrt{2}}{\pi d^2} = \frac{2.83T}{\pi d^2}$$

Long adjacent fillet welds: Fig 2.17 shows a vertical plate, connected to a horizontal plate by two identical fillet welds, and subjected to torque, *T* about the vertical axis of the joint.

Let l =length of the joint

T =leg length of the weld

The effect of the applied torque is to produce shear stress, varying from zero at the axis and maximum at the plate ends (This is similar to the variation of normal stress over the depth of a beam, subjected to bending).

The torsional shear stress, induced at the plate ends, and in a **Fig. 2.17** horizontal plane, coinciding with the top surface of the horizontal plane, is given by,

$$\tau_{\max} = \frac{3T\sqrt{2}}{tl^2} = \frac{4.2T}{tl^2}$$

5. Fillet welds under bending moment

Annular fillet weld: Figure 2.18 shows one example of an annular fillet weld, subjected to bending moment, M. To determine the maximum bending stress induced in the joint; let us consider a small element of the weld, at an angle, θ , subtending an angle, $d\theta$ at the centre of the shaft.

Area of the element = $r.d\theta.t$ Where, t = size of the weld Normal force acting on the weld element,

$$dF = r \times d\theta \times t \times \sigma_t$$



Since the normal stress in the element is proportional to the distance from the neutral plane,

$$\frac{\sigma_{t\max}}{r} = \frac{\sigma_t}{r\sin\theta}$$

Where,

 σ = normal (bending) stress induced in the weld element

 $\sigma_{\text{tmax}} =$ maximum bending stress

$$\sigma_t = \sigma_{t \max} \sin \theta$$

Moment due to the force, dF about the neutral plane,

$$= dF \times r \sin \theta$$
$$= r \cdot d\theta \cdot t \cdot \sigma_t \cdot r din\theta$$
$$= r \cdot d\theta \cdot t \cdot \sigma_{t \max} \cdot \sin \theta \cdot r \sin \theta$$



$$= r^2 t \boldsymbol{\sigma}_{t \max} \cdot \sin^2 \boldsymbol{\theta} \cdot d\boldsymbol{\theta}$$

Total resisting moment offered by weld = $\int_{0}^{2\pi} r^{2} t \sigma_{t \max} \cdot \sin^{2} \theta \cdot d\theta$

 $= r^2 t \sigma_{t \max} \pi$ = external moment, *M*

Therefore, $M = r^2 t \sigma_{t \max} \pi$

or

$$\sigma_{t\max} = \frac{M}{r^2 t\pi}$$

Considering the throat area, for evaluation of the stress,

$$\sigma_{t \max} = \frac{M}{\left(\frac{d}{2}\right)^2 \times \frac{t}{\sqrt{2}} \times \pi} = \frac{5.66M}{d^2 \pi t}$$

Parallel fillet weld:

Figure 2.19 shows a double parallel fillet weld, subjected to bending moment, *M*. The joint is symmetric about the neutral plane.

Area resisting bending on the tensile (compressive) side,

= throat area =
$$\frac{t}{\sqrt{2}} \times l$$

Where, t = size of the weld

Assuming the moment arm equal to (b+t),

Resisting moment = $\frac{t}{\sqrt{2}} \times l(b+t)\sigma_t$

b = thickness of the plate

Therefore,

Where.

$$M = \frac{t}{\sqrt{2}} \times l(b+t)\sigma_t$$

And

6. Welded joints under eccentric loading

 $\sigma_t = \frac{\sqrt{2}M}{tl(h+t)}$

Case – I

Figure 2.20 shows a T-joint, with double parallel fillet weld, subjected to an eccentric load, F at a distance, e

Let

t = size of the weld l = length of the weld and b = thickness of the plate



Fig. 2.19

To analyse the effect of the eccentric load, F, introduce two equal and opposite forces, $F_1 - F_2$ such that $F_1 = F_2 = F$, as shown in Fig. 2.20.



Fig. 2.20

The effect of F_1 (=F) is to produce transverse shear stress, given by,

$$\tau = \frac{F}{2 \times \frac{t}{\sqrt{2}} \times l} = \frac{F}{\sqrt{2}tl}$$

Where, $\frac{t}{\sqrt{2}}$ = throat thickness (*h*)

The effect of $F - F_2 (F - F)$ is to produce bending moment, M, given by, Fe. Bending stress induced due to M is,

$$\sigma_{b} = \frac{\sqrt{2}M}{tl(b+t)} = \frac{\sqrt{2}Fe}{tl(b+t)}$$

The resultant (maximum) normal stress is given by,

$$\sigma_{\max} = \sqrt{\sigma_b^2 + \tau^2} = \frac{F}{tl(b+t)} \times \sqrt{2e^2 + \frac{(b+t)^2}{2}}$$

Case – II

Figure 2.21 shows a T-joint with double parallel fillet weld, loaded eccentrically, but very much different from that of the joint as shown in Fig. 2.20.

Let F = load; e = eccentricity; t = leg length; l = length of the weldSimilar to previous case, to analyse the effect of the eccentric load, F, introduce two equal and opposite forces, $F_1 - F_2$ such that $F_1 = F_2 = F$, as shown in Fig. 2.21.

The effect of $F_1 = F$ is to produce transverse shear stress, given by,

$$\tau = \frac{F}{2 \times \frac{t}{\sqrt{2}} \times l} = \frac{F}{\sqrt{2}tl}$$



Where, $\frac{t}{\sqrt{2}}$ = throat thickness

The effect of $F - F_2 (F - F)$ is to produce bending moment, M, given by, Fe. Bending stress induced due to M is,

$$\sigma_b (= \sigma_t = \sigma_c) = \frac{M}{Z}$$

$$Z = 2 \times \frac{1}{6} \left(\frac{t}{\sqrt{2}}\right) l^2 = \frac{\sqrt{2}}{6} t l^2 = \frac{t l^2}{4.242}$$

$$\sigma_b = \frac{4.242 Fe}{t l^2}$$

Where,

The resultant (maximum) normal stress is given by,

$$\sigma_{\max} = \sqrt{\sigma_b^2 + \tau^2} = \frac{0.707F}{tl} \times \sqrt{1 + \left(\frac{6e}{l}\right)^2}$$

Case – III

A more general case of eccentric loading is shown in Fig. 2.22. Here, the fillet welds are subjected to the action of a load, *F* acting at a distance, *e* from the centre of gravity of the weld system. To understand the effect of eccentric load, *F*; introduce two equal and opposite forces, $F_1 - F_2$ (=*F*) and passing through *G*, the centre of gravity of the weld system, as shown.



Fig. 2.22

The effect of $F_1(=F)$ is to produce direct or primary shear

stress, τ_1 , and the effect of $F - F_2$ (=*F*-*F*) is to produce twisting moment, *Fe*; resulting in secondary shear stress, τ_2 in the welds.

Primary shear stress,
$$\tau_1 = \frac{\sqrt{2}F}{tl}$$

t = size of the weld

Where,

 $l = \text{total length of the weld} \approx a + 2b$

Considering bending action, the shear stress induced is proportional to the distance of the weld section from G. Obviously, it is maximum at the corners of the weld.

Let τ_2 = maximum secondary shear stress at, say corner, A. Then from Fig. 2.22,

$$\frac{\tau}{r} = \frac{\tau_2}{GA} = \frac{\tau_2}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}}$$
$$\tau = 2\tau_2 \times \frac{r}{\sqrt{a^2 + b^2}}$$

Where, τ is the secondary shear stress at distance, *r* from *G*.

The moment of the shear force on the weld element of area, dA and at distance, r from G is,

$$dM = \tau \cdot dA \cdot r = \frac{2\tau_2 r^2 \cdot dA}{\sqrt{a^2 + b^2}}$$

Total resisting moment due to the welds AB, BC, CD shall be equal to the external (applied) twisting moment, *Fe*.

$$Fe = \frac{2\tau_2}{\sqrt{a^2 + b^2}} \left[\int_A^B r^2 \cdot dA + \int_B^C r^2 \cdot dA + \int_C^D r^2 \cdot dA \right] = \frac{2\tau_2}{\sqrt{a^2 + b^2}} \cdot J$$

Where $J = \sum r^2 \cdot dA$ = polar moment of inertia of the throat area about *G*.

$$\tau_2 = \frac{Fe \times \sqrt{a^2 + b^2}}{2I_G} = \frac{Fe}{J} \times r_{\max}$$

The resultant stress, au_{\max} is obtained by adding au_1 and au_2 vectorially. Thus,

$$\tau_{\rm max} = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2\cos\theta}$$

Where θ is the angle between primary and secondary shear loads, and is obtained from,

$$\cos\theta = \frac{b}{\sqrt{a^2 + b^2}}$$

Example Problem-1 : Figure 2.23 shows a cylindrical rod of 50 mm diameter,	Reference
welded to a flat plate. The cylindrical fillet weld is loaded eccentrically, by a	
force of 10 kN acting at 200 mm from the welded end. If the size of the weld is	
20 mm, determine the maximum normal stress in the weld.	
t t	Design Data Book
Solution : Let h = throat thickness = $\frac{t}{\sqrt{2}}$	Data Book
	by K.
Referring Fig. 2.23, let us introduce two equal and opposite forces, $F_1 - F_2$ and	Mahadevan



$$x = \frac{l^2}{2l+b} = \frac{150^2}{2 \times 150 + 200} = 45 \,\mathrm{mm}$$

Eccentrically, e = 400 + (150 - x) = 400 + (150 - 45) = 505 mm

Polar moment of inertia of the weld throat about G,

$$J = \frac{t}{\sqrt{2}} \left[\frac{(b+2l)^3}{12} - \frac{l^2(b+l)^2}{b+2l} \right]$$
$$= \frac{t}{\sqrt{2}} \left[\frac{(200+2\times150)^3}{12} - \frac{150^2\times(200+150)^2}{(200+2\times150)} \right] = 3468.3 \times 10^3 t \text{ mm}^4$$

Maximum radius of the weld, $GA = r_{max} =$

$$\sqrt{AB^2 + AC^2} = \sqrt{100^2 + 105^2} = 145 \text{ mm}$$

 $\cos \theta = \frac{GB}{GA} = \frac{105}{145} = 0.724$

Throat area of the weld, $A = (b + 2l) \times \frac{t}{\sqrt{2}} = 353.6t \text{ mm}^2$

Referring Fig. 2.24b, let us introduce two equal and opposite forces, $F_1 - F_2$ through *G*, and parallel to *F* such that $F_1 = F_2 = F$.

The effect of $F_1 (= F)$ is to produce primary shear stress.

Primary shear stress, $\tau_1 = \frac{F}{A} = \frac{50 \times 1000}{353.6t} = \frac{141.4}{t} \text{ N/mm}^2$

The effect of $F - F_2$ (=F-F) is to produce moment, Fe; inducing secondary shear stress. Maximum secondary shear stress,

$$\tau_2 = \frac{Fe}{J} \times r_{\text{max}} = \frac{50 \times 1000 \times 505}{3468.3 \times 10^3 \times t} \times 145 = \frac{1055.6}{t}$$

Resultant (maximum) shear stress,

$$\tau_{\max} = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2\cos\theta}$$

= $\sqrt{\left(\frac{141.6}{t}\right)^2 + \left(\frac{1055.6}{t}\right)^2 + 2 \times \frac{141.6}{t} \times \frac{1055.6}{t} \times 0.724}$
= $\frac{1162.1}{t}$
 $80 = \frac{1162.1}{t}$
 $t = 14.5 \text{ mm}$

Design of Bolted Joints

Threaded fasteners

Bolts, screws and studs are the most common types of threaded fasteners. They are used in both permanent and removable joints.

Bolts: They are basically threaded fasteners normally used with nuts.

Screws: They engage either with a preformed or a self made internal thread.

Studs: They are externally threaded headless fasteners. One end usually meets a tapped component and the other with a standard nut.

There are different forms of bolt and screw heads for a different usage. These include bolt heads of square, hexagonal or eye shape and screw heads of hexagonal, Fillister, button head, counter sunk or Phillips type. These are shown in Figs. 2.25 and 2.26.









Flat or Countersunk Head

Round or Button Head

Fig. 2.25 – Types of screw heads



Fig. 2.26 – Types of bolt heads

Tapping screws

These are one piece fasteners which cut or form a mating thread when driven into a preformed hole. These allow rapid installation since nuts are not used.

There are two types of tapping screws. They are known as **thread forming** which displaces or forms the adjacent materials and **thread cutting** which have cutting edges and chip cavities which create a mating thread.

Set Screws

These are semi permanent fasteners which hold collars, pulleys, gears etc on a shaft. Different heads and point styles are available. Some of them are shown in **Fig. 2.27**.



Fig. 2.27 – Different types of set screws

Thread forms

Basically when a helical groove is cut or generated over a cylindrical or conical section, threads are formed. When a point moves parallel to the axis of a rotating cylinder or cone held between centers, a helix is generated. Screw threads formed in this way have two functions to perform in general: (a) to transmit power – Square, ACME, Buttress, Knuckle types of thread forms are useful for this purpose. (b) to secure one member to another- V-threads are most useful for this purpose.

Some standard forms are shown in Fig. 2.28.

V-threads are generally used for securing because they do not shake loose due to the wedging action provided by the thread. Square threads give higher efficiency due to a low friction. This is demonstrated in **Fig. 2.29**.



Fig. 2.28 – Different types of thread forms



Fig. 2.29 - Loading on square and V-threads

Design of bolted joints

Stresses in screw fastenings

It is necessary to determine the stresses in screw fastening due to both static and dynamic loading in order to determine their dimensions. In order to design for static loading both initial tightening and external loadings need be known.

4.4.1.1 Initial tightening load

When a nut is tightened over a screw following stresses are induced:

- (a) Tensile stresses due to stretching of the bolt
- (b) Torsional shear stress due to frictional resistance at the threads.
- (c) Shear stress across threads
- (d) Compressive or crushing stress on the threads
- (e) Bending stress if the surfaces under the bolt head or nut are not perfectly normal to the bolt axis.

(a) Tensile stress

Since none of the above mentioned stresses can be accurately determined bolts are usually designed on the basis of direct tensile stress with a large factor of safety. The initial tension in the bolt may be estimated by an empirical relation P1=284 d kN, where the nominal bolt diameter d is given in mm. The relation is used for making the joint leak proof. If leak proofing is not required half of the above estimated load may be used. However, since

initial stress is inversely proportional to square of the diameter $\sigma = \frac{284d}{\frac{\pi}{4}d^2}$, bolts of smaller

diameter such as M16 or M8 may fail during initial tightening. In such cases torque wrenches must be used to apply known load. The torque in wrenches is given by $T = C P_1 d$ where, *C* is a constant depending on coefficient of friction at the mating surfaces, P is tightening up load and d is the bolt diameter.

(b) Torsional shear stress

This is given by $\tau = \frac{16T}{\pi d_c^3}$ where T is the torque and d_c the core diameter. We may

relate torque T to the tightening load P_1 in a power screw configuration (**figure-2.30**) and taking collar friction into account we may write

$$T = P_1 \frac{d_m}{2} \left(\frac{1 + \mu \pi d_m \sec \alpha}{\pi d_m - \mu L \sec \alpha} \right) + \frac{P_1 \mu_c d_{cm}}{2}$$

where $d_{\rm m}$ and $d_{\rm cm}$ dare the mean thread diameter and mean collar diameter respectively, and μ and $\mu_{\rm c}$ are the coefficients of thread and collar friction respectively and α is the semi-thread angle. If we consider that $d_{cm} = \frac{d_m + 1.5d_m}{2}$, then we may write $T = C P_1 d_{\rm m}$ where *C* is a constant for a given arrangement. As discussed earlier, similar equations are used to find the torque in a wrench.



Fig. 2.30 – A typical power screw configuration

(c) Shear stress across the threads

This is given by $\tau = \frac{3P}{\pi d_c bn}$ where d_c is the core diameter and b is the base width of

the thread and n is the number of threads sharing the load.

(d) Crushing stress on threads

This is given by $\sigma_c = \frac{P}{\frac{\pi}{4}(d_o^2 - d_c^2)n}$ where d_o and d_c are the outside and core diameters

as shown in Fig. 2.30.

(e) Bending stress

If the underside of the bolt and the bolted part are not parallel as shown in Fig. 2.31, the bolt may be subjected to bending and the bending stress may be given by $\sigma_B = \frac{xE}{2L}$ where *x* is the difference in height between the extreme corners of the nut or bolt head, *L* is length of the bolt head shank and *E* is the young's modulus.



Fig. 2.31 - Development of bending stress in a bolt

Combined effect of initial tightening load and external load

When a bolt is subjected to both initial tightening and external loads i.e. when a preloaded bolt is in tension or compression the resultant load on the bolt will depend on the relative elastic yielding of the bolt and the connected members.



Fig. 2.32 - A bolted joint subjected to both initial tightening and external load

This situation may occur in steam engine cylinder cover joint for example. In this case the bolts are initially tightened and then the steam pressure applies a tensile load on the bolts. This is shown in Fig. 2.32a and b.

Initially due to preloading the bolt is elongated and the connected members are compressed. When the external load P is applied, the bolt deformation increases and the compression of the connected members decrease. Here, P_1 and P_2 in Fig. 2.32a are the tensile loads on the bolt due to initial tightening and external load respectively. The increase in bolt deformation is given by $\delta_B = \frac{P_b}{K_b}$ and decrease in member compression is $\delta_C = \frac{P_c}{K_c}$ where, P_b is the share of P_2 in bolt, and P_c is the share of P_2 in members, K_b and K_c are the stiffnesses of bolt and members. If the parts are not separated then $\delta_b = \delta_c$ and this gives, $\frac{P_b}{K_b} = \frac{P_c}{K_c}$. Therefore, the resultant load on bolt is P+KP. Sometimes connected members may be more yielding than the bolt and this may occurs when a soft gasket is placed between the surfaces. Under these circumstances $K_b >> K_c$ or $\frac{K_c}{K_b} << 1$ and this gives $K \approx 1$. Therefore the total load P = P + P. Normally, K has a value around 0.25 or 0.5 for a hard conper gasket with long

 $P = P_1 + P_2$ Normally *K* has a value around 0.25 or 0.5 for a hard copper gasket with long through bolts. On the other hand if $K_c >> K_b$, *K* approaches zero and the total load *P* equals the initial tightening load. This may occur when there is no soft gasket and metal to metal contact occurs. This is not desirable. Some typical values of the constant *K* are given in Table 2.1.

Type of joint	K
Metal to metal contact with through bolt	0-0.1
Hard copper gasket with long through bolt	0.25-0.5
Soft copper gasket with through bolts	0.75
Soft packing with through bolts	0.75-1.00
Soft packing with studs	1.00

Table 2.1

Cotter Joint

A cotter is a flat wedge-shaped piece of steel as shown in Fig. 2.33. This is used to connect rigidly two rods which transmit motion in the axial direction, without rotation. These joints may be subjected to tensile or compressive forces along the axes of the rods. Examples of cotter joint connections are: connection of piston rod to the crosshead of a steam engine, valve rod and its stem etc.

A typical cotter joint is as shown in Fig. 2.34. One of the rods has a socket end into which the other rod is inserted and the cotter is driven into a slot, made in both the socket and the rod. The cotter tapers in width (usually 1:24) on one side only and when this is driven in, the rod is forced into the socket. However, if the taper is provided on both the edges it must be less than the sum of the friction angles for both the edges to make it self locking i.e. $\alpha_1 + \alpha_2 < \phi_1 + \phi_2$ where α_1 , α_2 are the angles of taper on the rod edge and socket edge of the cotter respectively and ϕ_1 ,



Fig. 2.33

 ϕ_2 are the corresponding angles of friction. This also means that if taper is given on one side only then $\alpha < \phi_1 + \phi_2$ for self locking. Clearances between the cotter and slots in the rod end and socket allows the driven cotter to draw together the two parts of the joint until the socket end comes in contact with the cotter on the rod end.



Fig. 2.34 – Cross-sectional views of a typical cotter joint



Fig. 2.35 – An isometric view of a typical cotter joint

Design of a cotter joint:

If the allowable stress in tension, compression and shear for the socket, rod and cotter be σ_t , σ_c , and τ respectively, assuming that they are all made of the same material, we may write the following failure criteria:

P

1. Tension failure of the rod at diameter d (Fig. 2.36)

$$\frac{\pi}{4}d^2\sigma_t = P$$

2. Tension failure of the rod across slot (Fig. 2.37)

$$\left(\frac{\pi}{4}d^2 - d_1t\right)\sigma_t = P$$

3. Tension failure of the socket across slot (Fig. 2.38)

$$\left(\frac{\pi}{4}(d_{2}^{2}-d_{1}^{2})-(d_{2}-d_{1})t\right)\sigma_{t}=F$$



4. Shear failure of cotter (Fig. 2.39)

 $2bt\tau = P$

Fig. 2.36

Fig. 2.37

5. Shear failure of the rod end (Fig. 2.40)

$$2l_1d_1\tau = P$$

- 6. Shear failure of socket end (Fig. 2.41) $2l(d_2 - d_1)\tau = P$
- Crushing failure of rod or cotter (Fig. 2.42) P/2, 7.

$$d_1 t \sigma_c = P$$

8. Crushing failure of socket or rod (Fig. 2.43)

$$(d_3 - d_1)t\sigma_c = P$$

9. Crushing failure of collar (Fig. 2.44)

$$\left(\frac{\pi}{4}\left(d_4^2-d_1^2\right)\right)\sigma_c=P$$

10. Shear failure of collar (Fig. 2.45)

$$\pi d_1 t_1 \tau = P$$

P/

Fig. 2.42

Fig. 2.43

Fig. 2.45

$$d_4^2 - d_1^2 \bigg) \sigma_c = P$$

Fig. 2.44

 $1^{\iota}1$

Cotters may bend when driven into position. When this occurs, the bending moment cannot be correctly estimated since the pressure distribution is not known. However, if we assume a triangular pressure distribution over the rod, as shown in Fig. 2.46 (a), we may approximate the loading as shown in Fig. 2.46 (b)



Fig. 2.40

Fig. 2.41

P

Fig. 2.46

This gives maximum bending moment = $\frac{P}{2}\left(\frac{d_3 - d_1}{6} + \frac{d_1}{4}\right)$ and the bending stress,

$$\sigma_{b} = \frac{\frac{P}{2} \left(\frac{d_{3} - d_{1}}{6} + \frac{d_{1}}{4} \right) \frac{b}{2}}{\frac{tb^{3}}{12}} = \frac{3P \left(\frac{d_{3} - d_{1}}{6} + \frac{d_{1}}{4} \right)}{tb^{2}}$$

Tightening of cotter introduces initial stresses which are again difficult to estimate. Sometimes therefore it is necessary to use empirical proportions to design the joint. Some typical proportions are given below:

$$d_1 = 1.21d$$
; $d_2 = 1.75d$; $d_3 = 2.4d$; $d_4 = 1.5d$; $t = 0.31d$; $b = 1.6d$; $l = l_1 = 0.75d$;
 $t_1 = 0.45d$; $s =$ clearance.

Design of a cotter joint:

Knuckle Joint

A knuckle joint (as shown in Fig. 2.47) is used to connect two rods under tensile load. This joint permits angular misalignment of the rods and may take compressive load if it is guided.



Fig. 2.47 – A typical knuckle joint

These joints are used for different types of connections e.g. tie rods, tension links in bridge structure. In this, one of the rods has an eye at the rod end and the other one is forked with eyes at both the legs. A pin (knuckle pin) is inserted through the rod-end eye and forkend eyes and is secured by a collar and a split pin. Normally, empirical relations are available to find different dimensions of the joint and they are safe from design point of view. The proportions are given in the Fig. 2.47.

> d = diameter of rod $d_1 = d$ t = 1.25d $d_2 = 2d$ $t_1 = 0.75d$ $d_3 = 1.5d$ $t_2 = 0.5d$

Mean diameter of the split pin = 0.25 d

However, failures analysis may be carried out for checking. The analyses are shown below assuming the same materials for the rods and pins and the yield stresses in tension, compression and shear are given by σ_t , σ_c and τ .

1. Failure of rod in tension:

$$\frac{\pi}{4}d^2\sigma_t = P$$

2. Failure of knuckle pin in double shear:

$$2\frac{\pi}{4}d_1^2\tau = P$$

3. Failure of knuckle pin in bending (if the pin is loose in the fork):

Assuming a triangular pressure distribution on the pin, the loading on the pin is shown in Fig. 2.48.

Equating the maximum bending stress to tensile or compressive yield stress we have,

$$\sigma_t = \frac{16P\left(\frac{t_1}{3} + \frac{t}{4}\right)}{\pi d_1^3}$$

4. Failure of rod eye in shear:

$$(d_2 - d_1)t\tau = P$$



5. Failure of rod eye in crushing:

$$d_1 t \sigma_c = P$$

6. Failure of rod eye in tension:

$$(d_2 - d_1)t\sigma_t = P$$
 Fig. 2.48

7. Failure of forked end in shear:

$$2(d_2 - d_1)t_1\tau = P$$

8. Failure of forked end in tension:

$$2(d_2 - d_1)t_1\sigma_t = P$$

9. Failure of forked end in crushing:

$$2d_1t_1\sigma_c = P$$

The design may be carried out using the empirical proportions and then the analytical relations may be used as checks. For example using the 2nd equation we have, $\tau = \frac{2P}{\pi d_1^2}$. We

may now put value of d_1 from empirical relation and then find $F.S. = \frac{\tau_y}{\tau}$ which should be more than one.

Example Problem-1 : Design a typical cotter joint to transmit a load of 50 kN		
in tension or compression. Consider that the rod, socket and cotter are all made		
of a material with the following allowable stresses: Allowable tensile stress σ_y		
= 150 MPa; Allowable crushing stress $\sigma_c = 110$ MPa; Allowable shear stress τ_y		
= 110 MPa.		
Solution:		
Axial load, $P = \frac{\pi}{4} d^2 \sigma_y$. On substitution this gives d=20 mm. In general		
standard shaft size in mm are:		
6 mm to 22 mm diameter 2 mm	in increment	
25 mm to 60 mm diameter 5 mm	in increment	

60 mm to 110 mm diameter	10 mm in increment	
110 mm to 140 mm diameter	15 mm in increment	
140 mm to 160 mm diameter	20 mm in increment	
500 mm to 600 mm diameter	30 mm in increment	
We therefore choose a suitable rod size to be 25	mm.	
For tension failure across $\operatorname{slot}\left(\frac{\pi}{4}d^2 - d_1t\right)\sigma_t =$	P . This gives $d_1 t = 1.58 \times 10^{-4}$	Fig. 2.37
m ² . From empirical relations we may take $t = 0$.	$4d$ i.e. 10 mm and this gives d_1	
= 15.8 mm. Maintaining the proportion let $d_1 = 1$.2 $d = 30$ mm.	
The tensile failure of socket across slot, $\left(\frac{\pi}{4}\left(d_2^2\right)\right)$	$-d_1^2 - (d_2 - d_1)t \sigma_t = P$. This	Fig. 2.38
gives $d2 = 37$ mm. Let $d2 = 40$ mm.		
For shear failure of cotter $2bt\tau = P$. On substituti	ion this gives $b = 22.72$ mm.	Fig. 2.39
Let $b = 25$ mm.		
For shear failure of rod end $2l_1d_1\tau = P$ and this g	gives $l_1 = 7.57$ mm. Let $l_1 =$	Fig. 2.40
10 mm.		
For shear failure of socket end $2l(d_2 - d_1)\tau = P$.	This gives $l = 22.72$ mm. Let	Fig. 2.41
L = 25 mm.		0
For crushing failure of socket or $rod(d_3 - d_1)t$	$t\sigma_c = P$. This gives d3 = 75.5	Fig. 2.43
mm. Let $d3 = 77$ mm.		
For crushing failure of collar $\left(\frac{\pi}{4}\left(d_4^2-d_1^2\right)\right)\sigma_c$	= P. On substitution this gives	Fig. 2.44
$d_4 = 38.4$ mm. Let $d_4 = 40$ mm.		
For shear failure of collar $\pi d_1 t_1 \tau = P$ which give	s $t_1 = 4.8$ mm. Let $t_1 = 5$	Fig. 2.45
mm.		
Therefore the final chosen values of dimensions	are:	
$d = 25 \text{ mm}; d_1 = 30 \text{ mm}; d_2 = 40 \text{ mm}; d_3 = 77 \text{ mm}; d_4 = 40 \text{ mm}; t = 10 \text{ mm};$		
$t_1 = 5 \text{ mm}; l = 25 \text{ mm}; l_1 = 10 \text{ mm}; b = 27 \text{ mm}.$		

MODULE-III

SHAFTS AND COUPLINGS

Introduction on Key

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

The following types of keys are important from the subject point of view:

- 1. Sunk keys
- 2. Saddle keys
- 3. Tangent keys
- 4. Round keys and
- 5. Splines.

We shall now discuss the above types of keys, in detail, in the following pages.

<u>Sunk Keys</u>

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types:

1. Rectangular sunk key. A rectangular sunk key is shown in Fig. 2.1. The usual proportions of this key are

Width of key, w = d / 4 and thickness of key, t = 2w / 3 = d / 6

where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.



Fig 2.1 Rectangle Sunk Key.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 471]

2.Square sunk key. The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e.

w = t = d / 4

3. Parallel sunk key. The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. Gib-head key. It is a rectangular sunk key with a head at one end known as gib head. It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. 2.2 (a) and its use in shown in Fig. 2.2 (b).



Fig 2.2 Gib-head key.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 471]

The usual proportions of the gib head key are

Width, w = d / 4

and thickness at large end, t = 2w / 3 = d / 6

5. Feather key. A key attached to one member of a pair and which permits relative axial movement is known as feather key. It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.



Fig 2.3 Feather key.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 472]

The feather key may be screwed to the shaft as shown in Fig. 2.3 (a) or it may have double gib heads as shown in Fig. 2.3 (b). The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

6. Woodruff key. The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. 2.4. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.



Fig 2.4 Woodruff key.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 473]

The main advantages of a woodruff key are as follows:

1. It accommodates itself to any taper in the hub or boss of the mating piece.

2. It is useful on tapering shaft ends. Its extra depth in the shaft *prevents any tendency to turn over in its keyway.

The disadvantages are

- 1. The depth of the keyway weakens the shaft.
- 2. It cannot be used as a feather.

Saddle keys

The saddle keys are of the following two types

- 1. Flat saddle key, and
- 2. Hollow saddle key.

1. Flat saddle key: A flat saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. 2.5. It is likely to slip round the shaft under load. Therefore, it is used for comparatively light loads.



Fig 2.5 Saddle key.

Fig 2.6 Tangent key.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 473]

2. Hollow saddle key: A hollow saddle key is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

Tangent Keys

The tangent keys are fitted in pair at right angles as shown in Fig. 2.6. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

Round Keys

The round keys, as shown in Fig. 2.7(a), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.



Fig 2.7 Round Keys

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 474]

Sometimes the tapered pin, as shown in Fig. 2.7 (b), is held in place by the friction between the pin and the reamed tapered holes.

Splines

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as splined shafts as shown in Fig. 2.8. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway. The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.



Fig 2.8 Splines.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 474]

Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key:

1. Forces (F1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.



Fig 2.9 Forces acting on a sunk key.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 475]

The forces acting on a key for a clockwise torque being transmitted from a shaft to

a hub are shown in Fig. 2.9. In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig. 2.10.

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

 τ and σ_c = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

 $F = Area resisting shearing \times Shear stress = 1 \times w \times \tau$

 \therefore Torque transmitted by the shaft,

$$T = F \times \frac{d}{2}$$
$$T = 1 \times w \times \tau \times \frac{d}{2} \qquad \dots (i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

F = Area resisting crushing × Crushing stress = $1 \times \frac{t}{2} \times \sigma_c$

 \therefore Torque transmitted by the shaft,

$$T = F \times \frac{d}{2}$$
$$T = 1 \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \qquad \dots (ii)$$

The key is equally strong in shearing and crushing, if

$$1 \times w \times \tau \times \frac{d}{2} = 1 \times \frac{t}{2} \times \sigma_{c} \times \frac{d}{2} \qquad \dots [\text{Equating equations (i) and (ii)}]$$
$$\frac{w}{t} = \frac{\sigma_{c}}{2\tau} \qquad \dots (\text{iii})$$

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore, from equation (iii), we have w = t. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = 1 \times w \times \tau \times \frac{d}{2} \qquad \qquad \dots (iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3$$

... (Taking τ_1 = Shear stress for the shaft material)

From equations (iv) and (v), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau}$$

$$l = \frac{\pi d}{2} \times \frac{\tau_1}{\tau}$$

$$l = 1.571d \times \frac{\tau_1}{\tau} \qquad \dots \text{ (Taking w = d/4) } \dots \text{(vi)}$$

When the key material is same as that of the shaft, then

$$\tau = \tau_1$$
.
 $\therefore l = 1.571 \text{ d}$... [From equation (vi)]

Effect of Keyways

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$$e = 1 - 0.2 \left(\frac{w}{d}\right) - 1.1 \left(\frac{h}{d}\right)$$

where e =Shaft strength factor.

It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,
d = Diameter of shaft, and
h = Depth of keyway =
$$\frac{\text{Thickness of key (t)}}{2}$$

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation. In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_{θ} as given by the following relation:

$$k_{\theta} = 1 + 0.4 \; (\frac{w}{d}) + 0.7(\frac{h}{d})$$

where

 $k_{\theta} = Reduction$ factor for angular twist.

Problem 2.1

A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.

Given Data:

$$P = 15 \text{ kW} = 15 \times 10^3 \text{ W}$$

N = 960 r.p.m.

d = 40 mm l = 75 mm $\tau = 56 \text{ MPa} = 56 \text{ N/mm}^2$ $\sigma_c = 112 \text{ MPa} = 112 \text{ N/mm}^2$

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 960} = 149 \text{ N-m}$$
$$T = 149 \times 10^3 \text{ N-mm}$$

w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^{3} = 1 \times w \times \tau \times \frac{d}{2}$$
$$= 75 \times w \times 56 \times \frac{40}{2}$$
$$= 84 \times 10^{3} w$$
$$\therefore w = 149 \times 10^{3} / 84 \times 10^{3}$$
$$\therefore w = 1.8 mm$$

This width of keyway is too small. The width of keyway should be at least d / 4.

$$w = \frac{d}{4} = \frac{40}{4}$$
$$w = 10 \text{ mm.}$$

Since $\sigma_c = 2\tau$, therefore a square key of w = 10 mm and t = 10 mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2 \left(\frac{w}{d}\right) - 1.1 \left(\frac{h}{d}\right)$$

$$e = 1 - 0.2 \left(\frac{w}{d}\right) - 1.1 \left(\frac{t}{2d}\right)$$

$$e = 1 - 0.2 \left(\frac{10}{20}\right) - 1.1 \left(\frac{10}{2 \times 40}\right)$$

$$e = 0.8125$$

 \therefore Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e$$
$$= \frac{\pi}{16} \times 56 \times 40^3 \times 0.8125$$

Let

and shear strength of the key

$$= 1 \times w \times \tau \times \frac{d}{2}$$
$$= 75 \times 10 \times 56 \times \frac{40}{2}$$
$$= 840000 \text{ N}$$
$$\therefore \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840000}{571844}$$

$$\therefore \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = 1.47$$

Introduction

Couplings are used to connect two rotating shafts to transmit torque from one to the other. For example coupling is used to connect the output shaft of an electric motor to the input shaft of a hydraulic pump.

Types of Shafts Couplings

Rigid Couplings

Rigid Couplings are used to connect two shafts which are perfectly aligned. These are simple and inexpensive.

Rigid Couplings are of following types:

- 1. Sleeve or Muff Coupling
- 2. Clamp or Split-muff or Compression Coupling
- 3. Flange Coupling

Flexible Couplings

Flexible couplings are used to connect two shafts having lateral or angular misalignment. Flexible elements provided in flexible coupling absorb shocks and vibrations.

Flexible Couplings are of following types:

- 1. Bushed pin type Coupling
- 2. Universal Coupling
- 3. Oldham Coupling

Introduction

Assembly of muff coupling is shown in Figure 16.1. Sleeve, a hollow cylinder, is fitted on the ends of input and output shaft with the help of a sunk key. Torque is transmitted from input shaft to the sleeve through key and from the sleeve to the output shaft through the key again. It is simple to design and manufacture but difficult to assemble and dismantle. It requires more axial space and has small radial dimensions. Sleeve is made of cast iron and for it a larger factor of safety of 6-8 is used on the ultimate strength. Standard proportions used for sleeve are:

Outer diameter of the sleeve,	D = 2d + 13
Length of the sleeve,	L = 3.5d

where d is the diameter of the shaft.

So the muff coupling has three main components: shafts, sleeve



Shafts are designed on the basis of torsional shear stress induced because of the torque to be transmitted. Shear stress induced in shaft for transmitting torque, T is given by,

$$\tau = \frac{T r}{J} \le [\tau]$$

Where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation

r = Distance from neutral axis to the outer most fibre = d/2

So dimensions of the shaft can be determined from above relation for a known value of allowable shear stress, $[\tau]$.

Sleeve Design

As discussed earlier, following relations are used to calculate the dimensions.

$$D = 2d + 13$$
 $L = 3.5d$

Then the torsional shear stress in the sleeve is checked considering it as a hollow shaft.

Shear stress, $\tau = \frac{T r}{J} \leq [\tau]$

where, T = Twisting moment (or torque) to be transmitted

J = Polar moment of inertia about the axis of rotation

r = Distance from neutral axis to the outer most fibre = D/2

Design of Key

Cross-section of the key is taken from the table corresponding to the shaft diameter or relations (square key) or and (for rectangular key) are used to find the cross-section, where w is width and h is the height of the key.

Length of key in each shaft, .

The keys are then checked in shear and crushing.

Shear stress,	$\tau = \frac{P}{wl} \le [\tau]$
Crushing stress,	$\sigma_{crushing} = \frac{P}{l \ h/2} \le [\sigma_c]$

Clamp Coupling

Introduction

Clamp coupling is also known as split-muff coupling or compression coupling. In this coupling, sleeve or muff is made in two halves, which are split along the plane passing through the axes of the shafts. These two halves are clamped together with the help of bolts, which are placed in recesses made in the sleeve halves. Dynamic balancing of clamp coupling is difficult making it unsuitable for high speed applications. It is also unsuitable for shock loads. Assembly and dismantling is easier for this coupling. Figure 16.2 shows Clamp Coupling assembly.

A small clearance is provided between the two halves of the sleeve along the parting line and the force due to clamping of bolts creates frictional force between the surface of the shafts and inner surface of sleeve halves. Torque is transmitted by means of this frictional force and through the key, from the input shaft to the sleeve and from sleeve to the output shaft. It is not possible to find out the exact percentage of torque transmitted by friction and by the key. Therefore, for designing the bolts it is assumed that whole of the torque is transmitted by friction and while designing the key, it is assumed that whole of the torque is transmitted by it.

Design is similar to the design of muff coupling and an additional calculation is required for designing the bolts.



Design of Clamp Coupling is similar to the design of muff coupling and an additional calculation is required for designing the bolts.

Design of Shafts

Same as discussed in sleeve coupling.

Sleeve Design

Same as discussed in Sleeve Coupling

Design of Key

Same as discussed in Sleeve Coupling

Design of Bolts

Bolts are designed assuming that whole of the torque is transmitted by friction between sleeve and shafts.

Let $[\sigma_t]$ = permissible tensile stress of bolts

 d_c = core diameter of bolts

n = number of bolts

Clamping force of each bolt,

 $P_b = \frac{\pi}{4} d_c^2 [\sigma_t]$

Forces acting on Bolts

Assuming that half of the bolts apply clamping force on one shaft and half of the bolts on the other. Clamping force on each

shaft, $N = \frac{\pi}{4} d_c^2 [\sigma_t] \frac{n}{2}$

Frictional Torque, $T_f = \mu N d$

where, m = coefficient of friction between shafts and sleeve.

Above two relations can be used to find the core diameter of the bolts by equating to the total torque transmitted.



Flange Coupling

Introduction

Flange coupling consists of two flanges keyed to the shafts. The flanges are connected together by means of bolts arranged on a

circle concentric to shaft. Power is transmitted from driving shaft to flange on driving shaft through key, from flange on driving shaft to the flange on driven shaft through bolts and then to the driven shaft through key again. Projection is provided on one of the flanges and a corresponding recess is provided in the other for proper alignment. Flange coupling is of two types – unprotected and protected. These are shown in Figure 16.3. If in case failure of bolts occurs during the operation, the bolts may hit the operator in case of unprotected flange coupling. To avoid this, protective circumferential flanges are provided in the protected type flange coupling.

Flange of a protected type flange coupling has three distinct regions – inner hub, flanges and protective circumferential flanges. Following standard proportions are used in the design of flange coupling:

Outer diameter of hub,	D = 2 d
Pitch circle diameter of bolts,	$D_1 = 3 d$
Outer diameter of flange,	$D_2 = 4 \ d$
Length of the hub,	L = 1.5 d
Thickness of flange,	$t_f=0.5\ d$
Thickness of protective circumferential flange,	$t_p = 0.25 d$
where d is the diameter of shafts to be coupled.	

Design of Shafts

Same as discussed in sleeve coupling.

Design of Hub

Hub is designed considering it as a hollow shaft, with inner diameter equal to diameter of shafts and outer diameter double of that. It is checked for torsional shear stress.

Shear stress,
$$\tau = \frac{T r}{J} \le [\tau]$$

Where T = Twisting moment (or torque) to be transmitted

J = Polar moment of inertia about the axis of rotation

r = Distance from neutral axis to the outer most fibre = D/2

Design of Key

In this case two separate keys are used for the two shafts. Key is designed as discussed earlier. In this case, length of key, (length of the hub)

Design of Flange

The flange is subjected to shear at the junction of the hub as it transmits torque through the bolts. Area resisting shear

where, is the thickness of the flange.

If T is the torque to be
transmitted, tangential force,
$$F = \frac{T}{d/2}$$
Shear
stress, $\tau = \frac{F}{\pi D t_f} \leq [\tau]$

Design of Bolts

Due to transmission of torque, force acts perpendicular to the bolt axes and the bolts are subjected to shear and crushing stresses. Let n be the total number of bolts.

Force acting on each bolt,

$$F_b = \frac{T}{n D_1/2}$$

where D_1 is the pitch circle diameter of bolts.

Area re shear	isting $=\frac{\pi}{4}d_c^2$
where, bolts	$c_c = core diameter of$
Shear stress,	$\tau = \frac{F_b}{\frac{\pi}{4}d_c^2} \le [\tau]$
Area ur	der crushing

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Crushing stress,	$\sigma_{crushing} = \frac{F_b}{d_c t_f} \leq [\sigma_c]$

References:

- 1. Design of Machine Elements by VB Bhandari
- 2. Analysis and Design of Machine Elements by VK Jadon
- 3. A Text Book of Machine Design by RS Khurmi

MODULE-IV

Spring is defined as an elastic machine element(flexible element) that deflects under the action of load and returns to its original shape when load is removed.

IMPORTANT FUNCTIONS AND APPLICATIONS OF SPRING

- 1. Springs are used to absorb shocks and vibrations
 - eg: vehicle suspension springs, railway buffers to control energy, buffer springs in elevators and vibration mounts for machinery.
- 2. Measuring forces :Spring balances, gages
- 3. Storing of energy in clocks ,toys ,movie cameras, circuit breakers ,starters
- 4. Springs are used to apply force and control motion.

TYPES OF SPRINGS

1. Helical coil springs

a) helical compression spring ;b)helical extension spring ;c)helical torsion spring.

- 1. Torsion bar springs
- 2. Leaf spring (beam spring)
- 3. Volute springs
- 4. Pneumatic spring
- 5. Belleville spring(coned disk spring)





- (a) Standard compression; fixed pitch; linear; constant rate; pushes.
- (b) Variable pitch; nonlinear; pushes; resists resonance.



 (c) Conical; linear or hardening; pushes; minimum solid height.



(d) Hourglass; nonlinear; pushes; resists resonance.





(b) Helical extension spring



(f) Standard closed-coil extension; linear after coils open; pulls.



(d) Helical torsion spring



(h) Helical torsion; linear; constant rate; twists.





(f) Bellville washer; high loads; nonlinear; pushes.

STRESS, DEFLECTION AND SPRING RATE OF AXIALLY LOADED HELICAL SPRINGS



$$T = F \times \frac{D}{2}$$
$$J = \frac{\pi d^4}{32}$$

shear stress in the spring wire due to torsion is

$$\tau_{\rm T} = \frac{\mathrm{Tr}}{\mathrm{J}} = \frac{\mathrm{F} \times \frac{\mathrm{D}}{2} \times \frac{\mathrm{d}}{2}}{\frac{\pi \mathrm{d}^4}{32}} = \frac{8\mathrm{FD}}{\pi \mathrm{d}^3}$$

Average shear stress in the spring wire due to force F is



Therefore, maximum shear stress the spring wire is

$$\tau_{\rm T} + \tau_{\rm F} = \frac{8{\rm FD}}{\pi {\rm d}^3} + \frac{4{\rm F}}{\pi {\rm d}^2}$$
or
$$\tau_{\rm max} = \frac{8{\rm FD}}{\pi {\rm d}^3} \left(1 + \frac{1}{\frac{2{\rm D}}{{\rm d}}}\right)$$
or
$$\tau_{\rm max} = \frac{8{\rm FD}}{\pi {\rm d}^3} \left(1 + \frac{1}{2{\rm C}}\right)$$
where, ${\rm C} = \frac{{\rm D}}{{\rm d}}$, is called the spring index.
finally,
$$\tau_{\rm max} = \left({\rm K}_s\right) \frac{8{\rm FD}}{\pi {\rm d}^3} \quad \text{where, } {\rm K}_s = 1 + \frac{1}{2{\rm C}}$$

The above equation gives maximum shear stress occurring in a spring. ${\bf K}_{\rm s}$ is the shear stress correction factor.

CURVATURE EFFECT

For springs where the wire diameter is comparable with the coil diameter, in a given segment of the spring, the inside length of the spring segment is relatively shorter than the outside length.

Hence, for a given magnitude of torsion, shearing strain is more in the inner segment than the outer segment

This unequal shearing strain is called the curvature effect.



Curvature effect decreases with the increase in spring index.

So more is the spring index (C = D/d) the lesser will be the curvature effect.

For example, the suspensions in the railway carriages use helical springs.

These springs have large wire diameter compared to the diameter of the spring itself.

In this case curvature effect will be predominantly high.

In the design of helical springs, the designer should use good judgement in assuming the value of the spring index C. The spring index indicates the relative sharpness of the curvature of the coil. A low spring index means high sharpness of curvature. When the spring index is low (C < 3), the actual stresses in the wire are excessive due to curvature effect. Such a spring is difficult to manufacture and special care in coiling is required to avoid cracking in some wires. When the spring index is high (C > 15), it results in large variation in the coil diameter. Such a spring is prone to buckling and also tangles easily during handling. A spring index from 4 to 12 is considered best from manufacturing considerations. Therefore, in practical applications, the spring index usually varies from 4 to 12. However, a spring index in the range of 6 to 9 is still preferred particularly for close tolerance springs and those subjected to cyclic loading.

To take care of the curvature effect, the earlier equation for maximum shear stress in the spring wire is modified as,

$$\tau_{\rm max} = (K_{\rm w}) \frac{8 {\rm FD}}{\pi {\rm d}^3}$$

Where, K_W is Wahl correction factor, which takes care of both curvature effect and shear stress correction factor and is expressed as,

$$K_{w} = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

Derivation of deflection in helical compression spring[®]

The equation of torsion is
$$\frac{M_l}{J} = \frac{G\theta}{l}$$
.

 θ = angle of twist in radians. For small angle, tan $\theta = \theta = \frac{y}{R}$ and l = length of wire = $(\pi D).i$



Knowing the deflection for a given load, one can easily estimate the no. of active turns requires using

$$i = \frac{yGd^4}{8FD^3} = \frac{yGd}{8FC^3}$$

Deflection in helical spring

Deflection in a helical spring, δ

 $y = \frac{8FD^{3}i}{Gd^{4}}$

Design parameter (spring rate or stiffness)

$$F_0 = \frac{F}{y} = \frac{Gd^4}{8D^3i}$$

- i : number of active coils
- F : Axial force
- D: Mean diameter of coil
- G: Modulus of rigidity
- D: diameter of spring wire
- F₀: spring stiffness ,spring rate

TERMINOLOGY OF HELICAL SPRINGS

The design of helical-coil springs involves selection of a material, and determination of the wire diameter, d, mean coil radius, R, number of active coils, N, and other spring parameters so that the desired force-deflection response is obtained, without exceeding the design stress under the most severe operating conditions.





Minimum free length of the spring: (Table 11.4)

If the ends are bent before grinding If the ends are either ground or bent If the ends are neither ground nor bent

$$l_{o} \ge (i+2)d + y + a$$

$$l_{o} \ge (i+1)d + y + a$$

$$l_{o} \ge (i)d + y + a$$

$$11.18(a)$$

$$11.18(b)$$

$$11.18(c)$$

where a = xdi, total clearance between working coils, mm

i is the number of active coils (Table 11.4)



Fig. 11.6: Value of x as a function of coil ratio (C)

DESIGN OF SPRINGS

- It should possess sufficient strength to withstand the external load.
- It should have the required load deflection characteristics
- It should not buckle under the external load.

DESIGN OF HELICAL SPRINGS

DESIGN OF HELICAL SPRINGS

- (i) For the given application, estimate the maximum spring force (F) and the corresponding required deflection (y) of the spring. In some cases, maximum spring force (F) and stiffness F0 which is (F/y) are specified.
- (ii) Select a suitable spring material and find out ultimate tensile strength (S_{ut}) from the data Calculate the permissible shear stress for the spring wire by following relationship: $\tau = 0.30 S_{ut}$ or $0.50 S_{ut}$
- (iii) Assume a suitable value for the spring index (C). For industrial applications, the spring index varies from 8 to 10. A spring index of 8 is considered as a good value. The spring index for springs in valves and clutches is 5. The spring index should never be less than 3.
- (iv) Calculate the Wahl factor by the following equation:

(Y)

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

Determine wire diameter (d) by Eq. $\tau = K \left(\frac{8FC}{\pi d^2}\right)$

DESIGN OF HELICAL SPRINGS

- (vi) Determine mean coil diameter (D) by the following relationship: D = Cd
- (vii) Determine the number of active coils (i) by Eq. y = $\frac{8FD^3i}{Gd^4}$ The modulus of rigidity (G) for steel wires is 81 370 N/mm².
- (viii) Decide the style of ends for the spring depending upon the configuration of the application. Determine the number of inactive coils. Adding active and inactive coils, find out the total number of coils it
- (ix) Determine the solid length of the spring by the following relationship: Solid length = $i_t d$
- (x) Determine the actual deflection of the spring by Eq. $y = \frac{8FD^3N}{CT^4}$

DESIGN PROCEDURE OF HELICAL SPRINGS

Given load F, spring index 'C', deflection 'y', allowable shear stress ' τ ' and the rigidity modulus 'G'.

Step 1. Design of wire diameter.

Shear stress

$$\tau = -\frac{1}{\pi d^2}$$

where
$$K = \text{curvature factor} = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

8FCK

Find wire diameter 'd'. select standard wire diameter. (Nearest higher value is to be adopted if calculated value is not standard one.)

Step 2. Mean coil diameterD = CdInside diameter of coil $D_i = D - d$ Outside diameter of coil $D_o = D + d$

MODULE-V

DESIGN OF BEARING

1. Terms used in Hydrodynamic Journal Bearing

A hydrodynamic journal bearing is shown in Fig. 1, in which O is the centre of the journal and O' is the centre of the bearing.

Let, D = Diameter of the bearing

d = diameter of journal

l= length of bearing



Fig. 1: Hydrodynamic Journal Bearing

The following terms used in hydrodynamic journal bearing are important from the subject point of view.

- 1. **Diametral clearance**. It the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance, c = D - d.
- 2. Radial clearance. It is the difference between the radii of the bearing and the journal.

Mathematically, radial clearance, $c_1 = R - r = \frac{D - d}{2} = \frac{c}{2}$.

- 3. **Diametral clearance ratio**. It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio, $\left|\frac{c}{d} = \frac{D-d}{d}\right|$.
- 4. **Eccentricity**. It is the radial distance between the centre (O) of the bearing and the displaced centre (O') of the bearing under load. It is denoted by e.
- 5. **Minimum oil film thickness**. It is the minimum distance between the bearing and the journal, under complete lubrication condition. It is denoted by h0 and occurs at the line of centres as shown in Fig. 1. Its value may be assumed as c / 4.

6. Attitude or eccentricity ratio. It is the ratio of the eccentricity to the radial clearance.

Mathematically, attitude or eccentricity ratio,
$$\varepsilon = \frac{e}{c_1} = \frac{c_1 - h_o}{c_1} = 1 - \frac{h_o}{c_1} = 1 - \frac{2h_o}{c}$$

7. Short and long bearing. If the ratio of the length to the diameter of the journal (i.e. I / d) is less than 1, then the bearing is said to be short bearing. On the other hand, if I / d is greater than 1, then the bearing is known as long bearing.

Notes : 1. When the length of the journal (I) is equal to the diameter of the journal (d), then the bearing is called square bearing.

2. Bearing Characteristic Number and Bearing Modulus for Journal Bearings

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of

three variables, i.e. $\frac{Zn}{p}, \frac{d}{c}, \frac{l}{d}$.

Therefore the coefficient of friction may be expressed as $\mu = \phi \left(\frac{Zn}{p}, \frac{d}{c}, \frac{l}{d} \right)$

Where,

 μ = Coefficient of friction,

 ϕ = A functional relationship,

Z = Absolute viscosity of the lubricant, in kg / m-s,

n = Speed of the journal in r.p.m.,

 $p = Bearing pressure on the projected bearing area in N/mm^2$, = Load on the journal $\div l \times d$

d = Diameter of the journal,

l = Length of the bearing, and

c = Diametral clearance.

The factor ZN / p is termed as bearing characteristic number and is a dimensionless number.

The variation of coefficient of friction with the operating values of bearing characteristic number (ZN / p) as obtained by McKee brothers (S.A. McKee and T.R. McKee) in an actual test of friction is shown in Fig. 2. The factor ZN/p helps to predict the performance of a bearing.



Fig. 2 Variation of coefficient of friction with ZN/p.

The part of the curve PQ represents the region of thick film lubrication. Between Q and R, the viscosity (Z) or the speed (N) are so low, or the pressure (p) is so great that their combination ZN / p will reduce the film thickness so that partial metal to metal contact will result. The thin film or boundary lubrication or imperfect lubrication exists between R and S on the curve. This is the region where the viscosity of the lubricant ceases to be a measure of friction characteristics but the oiliness of the lubricant is effective in preventing complete metal to metal contact and seizure of the parts. It may be noted that the part PQ of the curve represents stable operating conditions, since from any point of stability, a decrease in viscosity (Z) will reduce Zn / p. This will result in a decrease in coefficient of friction (μ) followed by a lowering of bearing temperature that will raise the viscosity (Z). From Fig. 2, we see that the minimum amount of friction occurs at A and at this point the value of Zn / p is known as bearing modulus which is denoted by K. The bearing should not be operated at this value of bearing modulus, because a slight decrease in speed or slight increase in pressure will break the oil film and make the journal to operate with metal to metal contact. This will result in high friction, wear and heating. In order to prevent such conditions, the bearing should be designed for a value of Zn / p at least three times the minimum value of bearing modulus (K). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of Zn / p = 15 K may be used. From above, it is concluded that when the value of ZN / p = 15 K may be used. p is greater than K, then the bearing will operate with thick film lubrication or under hydrodynamic conditions. On the other hand, when the value of ZN / p is less than K, then the oil film will rupture and there is a metal to metal contact.

3. Coefficient of Friction for Journal Bearings

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used.

Coefficient of friction
$$\mu = \left[\frac{33.25}{10^8} \times \frac{Zn}{p} \times \frac{d}{c}\right] + k$$
 (Eq. 19.5, pp. 19.3, Jalaludeen)

(When Z in N-s/m², or kg/m-s and p in N/mm²)

$$\mu = \left[\frac{33.25}{10^{10}} \times \frac{Zn}{p} \times \frac{d}{c}\right] + k \quad \text{(Eq. 19.6, pp. 19.3, Jalaludeen)}$$

(When Z in centipoise, or kg/m-s and p in kgf/cm²)

k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (i.e. 1 / d). (Refer Fig. 19.2, pp.19.24, Jalaludeen), and The design values can be taken from Table 19.5, pp. 19.13, Jalaludeen.

4. Critical Pressure of the Journal Bearing

The pressure at which the oil film breaks down so that metal to metal contact begins, is known as critical pressure or the minimum operating pressure of the bearing. It may be obtained by the following empirical relation, i.e. Critical pressure or minimum operating

pressure,
$$p_c = \frac{Zn}{4.75 \times 10^6} \left(\frac{d}{c}\right)^2 \left(\frac{l}{l+d}\right)$$
N/mm² (Eq. 19.15, pp. 19.6, Jalaludeen)

When, Z in N-s/ m^2

And
$$p_c = \frac{Zn}{475 \times 10^6} \left(\frac{d}{c}\right)^2 \left(\frac{l}{l+d}\right)$$
 kgf.cm² (Eq. 19.16, pp. 19.6, Jalaludeen), When, Z

centipoise

5. Sommerfeld Number

The Sommerfeld number is also a dimensionless parameter used extensively in the design of

journal bearings. Mathematically, $S = \frac{Zn}{60 \times 10^6 p} \left(\frac{d}{c}\right)^2$ (Eq. 19.8, pp. 19.4, Jalaludeen),

Table 19.7 to Table 19.10.

6. Heat Generated in a Journal Bearing

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing, $H_g = \mu WV$ watts.

(when the load on bearing W in Newtons and V in m/s)

And
$$H_g = \mu WV$$
 kgf-m/min or $\frac{\mu WV}{J}$ kcal/min (Eq. 19.10, pp. 19.4, Jalaludeen)

(when W in Newtons and the rubbing velocity V in m/s)

And
$$V = \frac{\pi dn}{60}$$
 m/s

After the thermal equilibrium has been reached, heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temperature difference, size and mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing $H_d = CA(t_b - t_a)$ (Eq. 19.12, pp. 19.5, Jalaludeen)

Where C= Heat dissipation coefficient (values can be obtained from pp. 19.5, Jalaludeen)

A = projected area = $l \times d$ in m²

 t_b = temp. of bearing in ^oC

 t_a = temp. of surroundings in °C

7. <u>Design Procedure for Journal Bearing</u>

The following procedure may be adopted in designing journal bearings, when the bearing load, the diameter and the speed of the shaft are known.

- 1 Determine the bearing length by choosing a ratio of I / d from Table 19.5, pp. 19.13, Jalaludeen.
- Check the bearing pressure, p = W / I.d (Eq. 19.9, pp. 19.4, Jalaludeen), from Table 19.5, pp. pp. 19.13, Jalaludeen, for probable satisfactory value.
- 3 Assume a lubricant from Table 19.11, pp. 19.26, Jalaludeen, and its operating temperature (t0). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high temperature installations such as steam turbines.
- 4 Determine the operating value of Zn / p for the assumed bearing temperature and check this value with corresponding values in Table 19.5, pp. 19.13, Jalaludeen to determine the possibility of maintaining fluid film operation.
- 5 Assume a clearance ratio c / d from Table 19.5, pp. 19.13, Jalaludeen.
- 6 Determine the coefficient of friction (μ) by using the relation as discussed in Art. 3.
- 7 Determine the heat generated by using the relation as discussed in Art. 6.
- 8 Determine the heat dissipated by using the relation as discussed in Art. 6.

9 Determine the thermal equilibrium to see that the heat dissipated becomes at least equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.

BALL AND ROLLER BEARING

Advantages and disadvantages of Roller bearing over sliding bearing

<u>Advantages</u>

- 1. Low starting and running friction except at very high speeds.
- 2. Ability to withstand momentary shock loads.
- 3. Accuracy of shaft alignment.
- 4. Low cost of maintenance, as no lubrication is required while in service.
- 5. Small overall dimensions.
- 6. Reliability of service.
- 7. Easy to mount and erect.
- 8. Cleanliness.

Disadvantages

- 1. More noisy at very high speeds.
- 2. Low resistance to shock loading.
- 3. More initial cost.
- 4. Design of bearing housing complicated.

Types of Rolling Contact Bearings

Following are the two types of rolling contact bearings:

- 1. Ball bearings; and
- 2. Roller bearings.

The ball and roller bearings consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Fig. 3. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and is usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads. The rolling contact bearings, depending upon the load to be carried, are classified as : (a) Radial bearings, and (b) Thrust bearings.

The radial and thrust ball bearings are shown in Fig. 4 (a) and (b) respectively. When a ball bearing supports only a radial load (WR), the plane of rotation of the ball is normal to the centre line of the bearing, as shown in Fig. 4 (a). The action of thrust load (WA) is to shift the plane of rotation of the balls, as shown in Fig. 4 (b). The radial and thrust loads both may be carried simultaneously.



Types of Radial Ball Bearings

Following are the various types of radial ball bearings:

1. <u>Single row deep groove bearing. A single row deep groove bearing as shown in Fig. 5 (a).</u>

During assembly of this bearing, the races are offset and the maximum numbers of balls are placed between the races. The races are then centered and the balls are symmetrically located by the use of a retainer or cage. The deep groove ball bearings are used due to their high load carrying capacity and suitability for high running speeds. The load carrying capacity of a ball bearing is related to the size and number of the balls.



Fig. 5 Types of Radial Ball Bearing

2. Filling notch bearing.

A filling notch bearing is shown in Fig. 5 (b). These bearings have notches in the inner and outer races which permit more balls to be inserted than in a deep groove ball bearing. The notches do not extend to the bottom of the race way and therefore the balls inserted through

the notches must be forced in position. Since this type of bearing contains larger number of balls than a corresponding un-notched one, therefore it has a larger bearing load capacity.

3. Angular contact bearing.

An angular contact bearing is shown in Fig. 5 (c). These bearings have one side of the outer race cut away to permit the insertion of more balls than in a deep groove bearing but without having a notch cut into both races. This permits the bearing to carry a relatively large axial load in one direction while also carrying a relatively large radial load. The angular contact bearings are usually used in pairs so that thrust loads may be carried in either direction.

4. Double row bearing.

A double row bearing is shown in Fig. 5 (d). These bearings may be made with radial or angular contact between the balls and races. The double row bearing is appreciably narrower than two single row bearings. The load capacity of such bearings is slightly less than twice that of a single row bearing.

5. <u>Self-aligning bearing.</u>

A self-aligning bearing is shown in Fig. 5 (e). These bearings permit shaft deflections within 2-3 degrees. It may be noted that normal clearance in a ball bearing are too small to accommodate any appreciable misalignment of the shaft relative to the housing. If the unit is assembled with shaft misalignment present, then the bearing will be subjected to a load that may be in excess of the design value and premature failure may occur. Following are the two types of self-aligning bearings:

(a) Externally self-aligning bearing, and (b) Internally self-aligning bearing.

In an externally self-aligning bearing, the outside diameter of the outer race is ground to a spherical surface which fits in a mating spherical surface in a housing, as shown in Fig. 5 (e). In case of internally self-aligning bearing, the inner surface of the outer race is ground to a spherical surface. Consequently, the outer race may be displaced through a small angle without interfering with the normal operation of the bearing. The internally self-aligning ball bearing is interchangeable with other ball bearings.

Types of Roller Bearings

Following are the principal types of roller bearings :

1. <u>Cylindrical roller bearings</u>. A cylindrical roller bearing is shown in Fig. 6(a). These bearings have short rollers guided in a cage. These bearings are relatively rigid against radial motion and have the lowest coefficient of friction of any form of heavy duty rolling-contact bearings. Such types of bearings are used in high speed service.





<u>2. Spherical roller bearings</u>. A spherical roller bearing is shown in Fig. 6 (b). These bearings are self-aligning bearings. The self-aligning feature is achieved by grinding one of the races in the form of sphere. These bearings can normally tolerate angular misalignment in the order

of $\pm 1\frac{1}{2}^{\circ}$ and when used with a double row of rollers, these can carry thrust loads in either

direction.

<u>3. Needle roller bearings</u>. A needle roller bearing is shown in Fig. 6 (c). These bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed. These bearings are used when heavy loads are to be carried with an oscillatory motion, e.g. piston pin bearings in heavy duty diesel engines, where the reversal of motion tends to keep the rollers in correct alignment.

<u>4. Tapered roller bearings</u>. A tapered roller bearing is shown in Fig. 6 (d). The rollers and race ways of these bearings are truncated cones whose elements intersect at a common point. Such type of bearings can carry both radial and thrust loads. These bearings are available in various combinations as double row bearings and with different cone angles for use with different relative magnitudes of radial and thrust loads.

THRUST BEARINGS

A thrust bearing is used to guide or support the shaft which is subjected to a load along the axis of the shaft. Such type of bearings are mainly used in turbines and propeller shafts. The thrust bearings are of the following two types :

1. Foot step or pivot bearings, and

2. Collar bearings.

In a foot step or pivot bearing, the loaded shaft is vertical and the end of the shaft rests within the bearing. In case of collar bearing, the shaft continues through the bearing. The shaft may be vertical or horizontal with single collar or many collars.

Footstep or Pivot Bearings

A simple type of footstep bearing, suitable for a slow running and lightly loaded shaft, is shown in Fig. If the shaft is not of steel, its end must be fitted with a steel face. The shaft is guided in a gunmetal bush, pressed into the pedestal and prevented from turning by means of a pin. Since the wear is proportional to the velocity of the rubbing surface, which increases with the distance from the axis (i.e. radius) of the bearing, therefore the wear will be different at different radii. It may be noted that the wear is maximum at the outer radius and zero at the centre. In order to compensate for end wear, the following two methods are employed.

The shaft is supported on a pile of discs. It is usual practice to provide alternate discs of different materials such as steel and bronze, as shown in Fig. (b), so that the next disc comes into play, if one disc seizes due to improper lubrication. It may be noted that a footstep bearing is difficult to lubricate as the oil is being thrown outwards from the centre by centrifugal force. In designing,

it is assumed that the pressure is uniformly distributed throughout the bearing surface.

Let W = Load transmitted over the bearing surface,

R = Radius of the bearing surface (or shaft),

A = Cross-sectional area of the bearing surface,

p = Bearing pressure per unit area of the bearing surface between rubbing surfaces.

 μ = Coefficient of friction, and

N = Speed of the shaft in r.p.m.

When the pressure in uniformly distributed over the bearing area, then $p = W/A = W/\pi R$

and the total frictional torque,

 $T = (2/3)\mu W R$

 \therefore Power lost in friction, P =(2 π N T)/60 ... (T being in N-m) Type your text